Substitution between Public and Private Consumption in Australian States

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Abstract

This paper employs a cointegrating panel approach based on Australian states to estimate the intratemporal elasticity of substitution between state government consumption expenditure and nondurable private consumption expenditure. Taken together with plausible values for the intertemporal elasticity of substitution, our estimates imply that public and private consumption are Edgeworth-Pareto complements.


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Views expressed in this paper are those of the authors and not necessarily those of the Reserve Bank. We acknowledge helpful comments from Paul Blacklow and seminar participants at the University of Tasmania. JEL classification: E21, E62, H31, H72. Correspondence: Graeme Wells, School of Economics and Finance, University of Tasmania, Private Bag 85, Hobart, Australia 7005. email: graeme.wells@utas.edu.au
1. **Introduction**

Central to the study of macroeconomics is an understanding of how government purchases of goods and services impact on aggregate economic activity. Though many data-based macroeconomic models predict that expansionary fiscal policy increases output in the short run, there is no clear empirical or theoretical consensus as to how changes in fiscal policy affect private consumption.

This paper undertakes an empirical analysis of the relationship between government and private consumption expenditure in Australia, utilising a framework in which both categories of consumption potentially yield utility for households. Our primary focus is on estimating the degree of substitution between government and private consumption. At the macro level this issue is important because the degree of substitutability (or complementarity) mutes (or amplifies) the income effects of fiscal policy. Substitutability is also relevant to micro-based policy analyses such as cost-benefit studies where private consumption is usually taken to be the utility metric. If government and private consumption were found to be perfect substitutes, for example, the appropriate metric would be the sum of the two.

Studies testing the relationship between private and public consumption have generally focused on time-series data for a single country, although there have recently been a number of studies based on panels of separate countries. The empirical analysis of this paper adds a new dimension to this line of research because, unlike the previous panel studies based on a cross-section of countries, the cross-sectional dimension in this paper consists of regions within a country, namely the six states of Australia. As far as we are aware, this paper is the first to use a panel approach to test the substitutability between private and public consumption for a single country.

A state-based study provides a number of advantages over the time-series analyses used in the past, on both theoretical and econometric levels. On the theoretical side, the way in which government spending is divided between the federal and state governments is of importance. Australian federal government spending is largely in areas such as federal administration, defence, customs, and telecommunications, most of which have a public good character. State governments have primary responsibility for expenditure in service-delivery areas such as health, education and public transport – services which are subject rival and potentially more directly related to private-sector spending. Hence the relationship between spending by state governments and the private sector may be a better indication of whether or not the private sector perceives government and private-sector spending to be substitutes.

Recently, much research has been conducted into panel models and the numerous benefits that panel analysis has in comparison to either a pure time-series analysis or cross-section analysis. These benefits have been discussed extensively by Baltagi and Kao (2000).
and Hsiao (1996) among others. One benefit is that a panel approach allows additional information from the cross-sectional dimension to help in estimating the model’s parameters. Further, panel data does not require a lengthy time series, so panel models can avoid problems such as structural breaks which often occur in long time series. More specifically for this paper, panel data analysis offers a significant advantage when testing for unit roots and cointegration. The power deficiencies of the conventional unit root and cointegration tests, in comparison to their panel counterparts, have been well documented. Adding the cross-sectional dimension to the time-series dimension increases the number of observations and, providing some cross-sectional homogeneity requirements are met, offers a significant advantage in the testing for nonstationarity and cointegration. This makes a panel approach an attractive and more powerful alternative to the numerous univariate studies conducted in the past.

Although testing the degree of substitutability between private and public consumption is not new, the approach taken in this paper is novel in that the relationship between private and public spending for a single country been not previously been analysed in a panel framework using regions within the country as the cross-section components. The way in which fiscal power in Australia is divided between the federal and state governments means that the present state-based panel approach may provide a sharper focus to estimation of the private-sector reaction to changes in public sector spending.

The remainder of the paper is structured as follows: Section 2 provides an overview of the relevant literature, locating our model within it. Section 3 implements a number of pre-tests preparatory to section 4 where a panel model, appropriate for the specific properties of the panel data, is estimated. Section 5 summarises the results, notes limitations and possible extensions.

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1 Fiorito and Kollintzas (2004) undertake a similar analysis to ours using annual data from 1970 to 1996 for 12 countries that subsequently joined the European Union. A distinction is made between ‘public goods’ (defence, public order and justice), ‘merit goods’ (health, education and other services), and private consumption expenditures. The state-based approach adopted in this paper has the advantage that state consumption expenditures correspond to Fiorito and Kollintzas’ ‘merit goods’. In the Australian federation there is no basis (other than by equal per-capita amounts) to allocate federal final expenditures (which correspond to ‘public goods’) to residents of individual states.
2. Literature Review

Bailey (1971) was the first to explore the macroeconomic implications of the degree of substitutability between government and private consumption expenditure. Prior to Bailey, most authors cast the government in a separate role, implying that private households ignored the goods and services supplied by the government and did not include them as an addition to their welfare. Later, Barro (1981) incorporated government spending into a general model of consumption allowing for the consumer’s utility to be directly affected by government purchases. Spurred by work of Bailey and Barro, a literature has developed to empirically test the relationship between government and private consumption.

The literature on this issue can be divided into several strands. In the first strand, estimation of the relationship between private and public consumption is achieved through exploiting intertemporal first-order conditions, or Euler-equations. A representative consumer maximises expected lifetime utility

\[ U_0 = E_t \sum_{t=0}^{\infty} \beta^t u(c_t^*) \]  

where \( u(.) \) is a concave utility function; \( c^* \) is effective consumption defined as an aggregate of real private consumption \( c \), and real government consumption \( g \); and \( \beta \) is the discount factor. Labour supply is inelastic.

For a given consumption aggregator the Edgeworth-Pareto criterion can be used to determine whether private and public consumption are complements or substitutes. This criterion states that private and public consumption are considered ‘net rivals’ if the marginal utility of one decreases as the quantity of the other increases, and ‘net complements’ if the reverse is true. Karras (1994), for example, assumes \( u(c^*) = u(c + \theta g) \) with \( u' > 0 \) and \( u'' < 0 \), so that the sign of \( \partial (\partial u / \partial c) \partial g = \theta u'' \) depends on the sign of \( \theta \). If \( \theta \) is positive (negative) then \( \partial (\partial u / \partial c) \partial g \) is negative (positive) implying that government and private consumption are substitutes (complements).

The representative consumer maximises utility subject to the economy-wide budget constraint given below in terms of effective consumption:

\[ a_{t+1} - b_{t+1} = \left[ (a_t - b_t) + w_t - c_t^* - (1-\theta) g_t \right] (1 + r) \]  

where it is assumed that lump-sum taxes have cancelled out, \( a_t \) is private financial wealth at the beginning of period \( t \), \( b_t \) is government debt, \( w_t \) is labour income, and \( r \) is the real interest rate which is assumed constant.
Choosing \( \{c^*_t\} \) to maximise (1) subject to (2) yields the Euler equation

\[
\frac{u'(c^*_t)}{\beta E_t u'(c^*_{t+1})} = 1 + r
\]

(3)

which can be linearised to give

\[
c_t = \alpha + \rho c_{t-1} - \theta g_t + \rho \theta g_{t-1} + \nu_t
\]

(4)

which, allowing for additive preference shocks, can be used to recover an estimate of \( \theta \). Kormendi (1983), Aschauer (1985), Ahmed (1986), Katsaitis (1987) and Karras (1994) are representative of this early approach. Analysing data for the United States, Kormendi and Aschauer found that government and private consumption expenditure are best described as substitutes. Ahmed found a similar relationship between private and public consumption for the United Kingdom; however Katsaitis found that for Canada, private and public spending were best described as complements. Karras (1994) is among the first to test the substitutability between private and government consumption for a number of countries, including Australia. The Australian results using annual data for 1950-1985 give an estimate of \( \hat{\theta} = -1.08 \), implying that public and private consumption are complements.

The second strand of literature incorporates both the intertemporal elasticity of substitution and the intratemporal elasticity of substitution. Studies in this strand – Amano and Wirjanto (1998) is an early example – attempt to determine whether private and public consumption are substitutes, complements or unrelated in an Edgeworth-Pareto sense based on the values of the intertemporal and intratemporal elasticities. To estimate the degree of substitutability, the method commonly proceeds in two steps. To begin, a representative consumer is assumed to maximise their expected lifetime utility function (1) where instantaneous utility is now defined as

\[
u(c^*_t) = \frac{v(c^*_t)^{1-\gamma}}{1-\gamma}, \frac{1}{\gamma} \neq 1, u(c^*_t) = \ln(v(c^*_t)) \frac{1}{\gamma} = 1
\]

(5)

Following Amano and Wirjanto (1998), a constant-elasticity-of-substitution utility function is considered as the aggregator for \( c^*_t \):

\[
v(c^*_t) = \left[ \phi c_i^{1-1/\alpha} + (1-\phi) g_t^{1-1/\alpha} \right]^{\sigma} \neq 1, v(c^*_t) = c_i^\sigma g_t^\sigma \text{ if } \sigma = 1
\]

(6)

From the above, \( 1/\gamma \) and \( \sigma \) represent the intertemporal and intratemporal elasticities of substitution respectively between private and public consumption. Amano and Wirjanto show

\footnote{See also Ogaki (1992) and Ogaki and Park (1997).}
that the sign of the partial cross-derivative, \( \partial (\partial u / \partial c) \partial g \), depends on both the intertemporal and intraperiod elasticities of substitution, and \( \text{sign}[u_{cg}(c,g)] = \text{sign}\left[ \frac{1}{\gamma} - \sigma \right] \).

If \( P_g / P_c \) is the purchase price of government consumption relative to the price of private consumption, then a static, or intraperiod, first-order condition of the model is found by equating the purchase price, with the marginal rate of substitution between government and private consumption. Amano and Wirjanto show that if the price and expenditure ratios are I(1), an estimate of \( \sigma \) can be obtained from the cointegrating regression

\[
\ln \left( \frac{c}{g} \right) = \alpha + \sigma \ln \left( \frac{P_g}{P_c} \right) + \varepsilon_t \tag{7}
\]

where \( \varepsilon_t \) is a stationary error process. In the second stage, estimation of the intertemporal elasticity of substitution, \( 1/\gamma \), and the discount factor, \( \beta \), is achieved by imposing the estimate of \( \sigma \) in the Euler equation

\[
E_t[\beta (\partial U / \partial c_{t+1}) / (\partial U / \partial c_t)] = 1 \tag{8}
\]

which is estimated using the Generalised Methods of Moments (GMM) procedure.

Using the above approach, Amano and Wirjanto (1998) find that the intratemporal and intertemporal elasticity of substitution between private and public consumption in the United States are similar in value implying the two goods are unrelated in an Edgeworth-Pareto sense. Esteve and Sanchis-Llopis (2005) employ a similar method to analyse the substitutability between private and public consumption in Spain. They find that the two variables are best described as Edgeworth-Pareto substitutes.

Both the first and second strands of the literature have been extended to a panel framework. For example, Ho and Nieh (2006) examine 23 OECD countries to determine whether private and public consumption are considered to be Edgeworth-Pareto complements, substitutes or unrelated goods, finding that private and public consumption are complementary goods in an Edgeworth-Pareto sense. Using panel cointegration methods, Kwan (2006) estimates the intratemporal elasticity of substitution for the nine East Asian countries. The results show a significant positive elasticity of intratemporal substitution between government and private consumption ranging from 0.57 and 1.05, depending on the estimator and the sample period.

There are only a small number of Australian studies on this topic. Henry and Olekalns (2001) include durables as well as nondurables in private consumption and, perhaps most significantly, take into account the possibility of structural breaks in the data. The inclusion of durables means that Henry and Olekalns’ setup does not allow for direct estimation of the substitution parameter; instead they estimate a VEC model and use the impulse response
functions for an increase in \( g \) to infer the degree of substitution\(^3\). They show that a long-run equilibrium relationship does exist between private and public consumption; however there is evidence of a significant structural shift during the period of financial deregulation. Consistent with Karras, Henry and Olekalns find that prior to 1983 private and public consumption for Australia were complementary however after financial deregulation they found that the relationship switched from one of complementarity to substitutability, although the latter estimate is most likely not statistically significant.

Evidence of structural breaks in Australian macroeconomic relations around the time of financial and exchange rate deregulation is not uncommon. It is plausible that private sector consumption decisions are affected by the lifting of liquidity constraints. Henry and Olekalns (2001) suggest that easing of liquidity restrictions may make it easier for consumers to borrow and thus substitute their own consumption for public consumption. Prior to deregulation, these opportunities were limited. They argue that it is not surprising that the relationship between private and public consumption in Australia altered during the early 1980s. Another possibility is that due to financial deregulation, the behaviour of the real interest rate changed. Henry and Olekalns however assume a constant rate of interest which, if this assumption is incorrect, may have led them to the conclusion that there was a change in consumption behaviour in the early 1980s.

A second recent Australian study is that of Harding (2007) who estimates the degree of substitution using the consumption aggregator \( c^* = c + \theta g \) in an Australian variant of the Christiano and Eichenbaum (1992) DSGE model. His estimate of \( \hat{\theta} = 0.2869 \), obtained using aggregate quarterly consumption and government expenditure data from 1978(1) to 2006(1), is not significantly different from zero.

From the discussion above, it is obvious that not only does the relationship between private and public consumption differ between countries but also the relationship between these two variables within a country can shift over time. Given the constantly changing economic environment and the importance of this issue for effective macroeconomic policy, continued study into this area is warranted.

The model in this paper is based on equation (7) where the intratemporal elasticity of substitution between public and private consumption is estimated in a panel framework. The present study breaks from the literature however as this is the first panel study where the cross-sectional dimension consists of regions within a country – as discussed previously, state

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\( ^3 \) Aristei and Pieroni (2008) also employ a VEC approach, with AIDS restrictions imposed on a 3 commodity demand system. Using quarterly 1964-2002 UK data they find that government consumption of ‘private’ goods has significantly differential effects on their three categories of private nondurable consumption.
government expenditures are concentrated on ‘private’ expenditures and so provide a natural partition for government activity in Australia.

3. Data and pretesting

Quarterly data for private and government consumption were obtained from the Australian Bureau of Statistics\(^4\). Government consumption, \(g\), consists of state and local government spending on goods and services, and private consumption consists of state spending on nondurable goods and services. All variables are seasonally adjusted, in constant prices and are taken on a state-by-state basis with the sample period extending from 1985:03 to 2007:01.

Included in the definition of ‘nondurables’ are: food, cigarettes and tobacco, alcoholic beverages, rent and other dwelling services, electricity, gas and other fuel, health, operation of vehicles, transport services, communications, recreation and culture, education services, hotels, cafes and restaurants, insurance and other financial services, and other goods. Included in the definition of ‘durable’ goods are: furnishings and household equipment, and purchase of vehicles. Net expenditure interstate is excluded. Expenditure on rent and dwelling services is the largest single component of total expenditure, averaging approximately 16-17 per cent of overall expenditure.

The proportion of private expenditure spent on nondurables and durables is similar across all six states. Taking as an example private expenditure in Tasmania, approximately 94 per cent of total expenditure is on what has been defined as nondurable goods and services. The two price series for each state, \(P_c\) and \(P_g\), are the implicit price deflators of government and nondurable private consumption respectively.

Given the relatively short time span of the data and the benefits associated with panel data analysis, the data are pooled into a panel framework. Before the panel model can be estimated, the properties of each component series need to be tested to ensure correct panel estimation methods are used.

Unit Root Tests

It has been well documented that most of the unit root tests for time series have low power—that is, they tend to accept the null of a unit root too often. The extension of unit root tests to the panel framework has allowed the additional information contained in the cross-sectional dimension to improve the power of unit root testing. Significant contributions include Levin, Lin and Chu (2002), Breitung (2000), Im, Pesaran and Shin (2003), Maddala and Wu (Fisher-Chi squared test) (1999), and Hadri (2000).

The tests developed by the above mentioned authors can be classified into two groups depending on whether the autoregressive coefficient is assumed to be homogeneous across cross-sections or not.

Consider the following AR(1) process for panel data:

\[ y_{it} = \alpha_i + \beta_i t + \rho_i y_{i,t-1} + \epsilon_{it} \]  

where \( i = 1, 2, \ldots, N \) cross-section units observed over \( t = 1, 2, \ldots, T \) periods; \( \alpha_i \) and \( \beta_i \) represent the cross-section specific intercept and trend terms, and \( \rho_i \) represents the autoregressive coefficients. If the autoregressive coefficients, \( \rho_i \), are assumed to be common across all cross-sections, then the Levin, Lin and Chu, Breitung and Hadri panel unit root tests are appropriate. The tests of Im, Pesaran and Shin, and Maddala and Wu relax this assumption, allowing the autoregressive coefficient to vary across cross-sections. The latter tests allow more flexibility when testing for a unit root and as such, the results of these tests will bear more weight than the results of the Levin, Lin and Chu, Breitung and Hadri tests.

### Table 1 Panel unit root tests

<table>
<thead>
<tr>
<th></th>
<th>( \ln(c/g) )</th>
<th>( \Delta \ln(c/g) )</th>
<th>( \ln \left( \frac{P_g}{P_c} \right) )</th>
<th>( \Delta \ln \left( \frac{P_g}{P_c} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPS</td>
<td>-1.09 (0.136)</td>
<td>-14.77 (0.00)</td>
<td>2.36 (0.99)</td>
<td>-17.24 (0.00)</td>
</tr>
<tr>
<td>Maddala and Wu</td>
<td>14.52 (0.27)</td>
<td>177.42 (0.00)</td>
<td>2.23 (0.99)</td>
<td>211.42 (0.00)</td>
</tr>
<tr>
<td>(ADF-Fisher)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LLC</td>
<td>0.45 (0.67)</td>
<td>2.45 (0.99)</td>
<td>1.20 (0.88)</td>
<td>-6.35 (0.00)</td>
</tr>
<tr>
<td>Breitung</td>
<td>-1.53 (0.06)</td>
<td>-8.05 (0.00)</td>
<td>3.86 (0.99)</td>
<td>-6.33 (0.00)</td>
</tr>
<tr>
<td><strong>Null: unit root</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hadri</td>
<td>8.31 (0.00)</td>
<td>-0.75 (0.77)</td>
<td>13.91 (0.00)</td>
<td>-1.42 (0.92)</td>
</tr>
<tr>
<td><strong>Null: does not have unit root</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Linear trend term and intercept term included in all tests</strong></td>
<td></td>
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<td></td>
<td></td>
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</tbody>
</table>

With the exception of the Levin, Lin and Chu test, all the panel unit root tests indicate that the two series are non-stationary I(1) processes.

The panel unit root tests used above are described in the literature as ‘first generation’ panel unit root tests. They are so called because each state within the component series is assumed to be independent of the other states. In the majority of empirical research conducted with nonstationary panel data in this field, the panel unit root tests used have assumed cross-
sectional independence. The assumption of cross-section independence in this paper, where the panel consists of regions within a single country, is perhaps more restrictive than in the studies that have preceded it. Not only do common international factors impact upon the states of Australia but the states are subject to common laws, have a common currency and a high degree of labour mobility. As such, the assumption of cross-sectional independence in this study is likely to be rejected. 

Overcoming this problem requires panel unit root tests that account for cross-sectional dependence. ‘Second generation’ panel unit root tests attempt to overcome this issue, however what has been accomplished has so far only been applied to large panels (see Breitung and Pesaran (2005) for an overview of these tests). Pesaran (2007) develops the most applicable panel unit root test in the presence of cross-sectional dependence. He proposes a cross-sectionally augmented Dickey-Fuller (CADF) regression which augments the standard Dickey Fuller regression with terms involving the level and first difference of the cross-sectional mean of the variables – in this way the test takes account of the common factor in the data. Unfortunately Pesaran provides critical values for panels with ten cross-sectional dimensions or more. As the critical values for a panel with ten cross-section dimensions vary widely depending on the time dimension, it is difficult to infer the necessary critical values for this paper and for that reason results are not presented. 

However, O’Connell (1998) notes that if cross-sectional correlation is present, panel unit root tests that assume cross-section independence are more likely to reject the null of a unit root. Given that the panel unit root tests used in this paper (which assume independence across cross-sections) almost always accept the null of a unit root, it is assumed that unit root tests that take into account cross-section dependence would also accept the unit-root null.

**Cointegration**

There is now a small literature developing on testing for cointegration in panel models. The key contributions to this field come from Pedroni (1999, 2001, 2004), and McCoskey and Kao (1998). Pedroni’s tests are based on Engle and Granger’s time series test for cointegration, where test statistics appropriate for panel residuals are simulated. More specifically, the following regression is run:

\[
y_{it} = \alpha_i + \beta x_{it} + \epsilon_{it}; \quad \left\{ \begin{array}{l} i = 1,2,...,N \\ t = 1,2,...,T \end{array} \right. \]

Both \( y_{it} \) and \( x_{it} \) are assumed to be I(1). Under the null hypothesis, it is assumed that the variables are not cointegrated and thus the residual series, \( \epsilon_{it} \), is nonstationary. Pedroni’s
approach is to obtain the residuals from (10) and then test whether the residuals are nonstationary by running the regression below for each component series:

$$\varepsilon_i = \rho \varepsilon_{i, t-1} + u_t$$  \hspace{1cm} (11)

or, in the augmented case,

$$\varepsilon_i = \rho \varepsilon_{i, t-1} + \sum_{j=1}^{p} \psi_j \Delta \varepsilon_{i, t-j} + u_t$$  \hspace{1cm} (12)

The null hypothesis of Pedroni’s test is no cointegration, $\rho_i = 1$. In the panel cointegration case as in panel unit root testing, there are two alternative hypotheses: the homogeneous alternative where $(\rho_i = \rho) < 1$ for all cross-sections, and the heterogeneous alternative $\rho_i < 1$ for all cross-sections.

Pedroni constructs appropriate critical values for several residual-based tests of the null of no cointegration, four based on pooling along the within-dimension and three based on pooling along the between-dimension. The within-dimension tests assume homogeneity of the autoregressive coefficient in the alternate hypothesis while the between-dimension tests assume heterogeneity of the autoregressive coefficient in the alternate hypothesis. If we denote $\rho_i$ the autoregressive coefficient of the residuals in the $i^{th}$ cross-section, then the homogenous and heterogeneous test are specified as follows:

**Within-dimension:**

$H_0 : \rho_i = 1 \text{ for all } i, \ H_0 : \rho_i = \rho < 1 \text{ for all } i.$

**Between-dimension:**

$H_0 : \rho_i = 1 \text{ for all } i, \ H_0 : \rho_i < 1 \text{ for all } i.$

Table 2 shows Pedroni’s seven panel test statistics, which evaluate the null hypothesis of no cointegration against both the homogeneous and the heterogeneous alternatives. On examination of the p-values, six out of the seven tests strongly reject the null hypothesis of no cointegration. It can be concluded that the consumption and price series used in this paper form a cointegrating relationship.
### Table 2 Pedroni residual-based Cointegration Tests

<table>
<thead>
<tr>
<th>Null Hypothesis: No cointegration</th>
<th>First $H_A$ - common AR coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistic</td>
</tr>
<tr>
<td>Panel $\psi$</td>
<td>1.75</td>
</tr>
<tr>
<td>Panel $\rho$</td>
<td>-17.91</td>
</tr>
<tr>
<td>Panel PP</td>
<td>-11.77</td>
</tr>
<tr>
<td>Panel ADF</td>
<td>-4.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Second $H_A$ - individual AR coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group $\rho$</td>
<td>-14.68</td>
</tr>
<tr>
<td>Group PP</td>
<td>-10.93</td>
</tr>
<tr>
<td>Group ADF</td>
<td>-4.36</td>
</tr>
</tbody>
</table>

### 4. Results

In this section, the intratemporal cointegrating model outlined in section 2 is estimated by employing panel estimation techniques. Given that panel models deal with time and cross-section dimensions, estimation is more complicated than estimating a pure time-series or cross-section regression. The panel techniques used in this paper need to account for a number of panel-specific problems including different residual variances for each of the component series as well as contemporaneous correlation between the residuals. Panel estimation is further complicated because the estimation methodology needs to allow for the variables being cointegrated.

Two estimation methods are used. The first is a generalised least squares (GLS) method that accounts for both cross-section heterogeneity as well as conditional correlation between the contemporaneous residuals for each cross-section. The second estimation method is Dynamic Ordinary Least Squares (DOLS). First introduced in a paper by Saikkonen (1991), the DOLS estimation method has shown to be an efficient estimation technique for cointegrated models.
Generalised Least Squares (GLS)

The model to be estimated is

$$\ln \left( \frac{c_i}{g} \right)_{it} = \alpha_i + \sigma \ln \left( \frac{P_g}{P_c} \right)_{it} + \epsilon_{it}, \quad i = 1, 2, \ldots, 6$$

As indicated we choose a fixed effects model in which the intercept term varies over the individual cross-sections but the coefficient of concern, $\sigma$, is constant. Hence $\alpha_i$ captures the effects of those variables that are unique to each cross-section yet constant over time and $\sigma$ gives the intratemporal elasticity of substitution that is constant across states and time. When $T$ (the number of time series data) is large and $N$ (the number of cross-sectional units) is small, there is likely to be little difference in the value of the parameters estimated by a fixed-effects model and a random-effects model; also, if the error term and the regressors are correlated, the estimators in the fixed-effects model are unbiased.

The estimation of (13) needs to take two additional factors into account. First, given the differences between each state, allowance must be made for differences in the residual variances – that is, the model needs to correct for heteroskedasticity in the residuals. Secondly, given the close connection between the states in terms of inflation and interest rates, contemporaneous correlation between the residuals needs to be considered also.

We assume a cross section seemingly unrelated regression with

$$E \left( \epsilon \ln \left( \frac{P_g}{P_c} \right) \right) = \Omega_N$$

where elements on the diagonal of $\Omega$ are not necessarily equal. Serial correlation is assumed to be zero. This estimation method is appropriate in this case where, as previously mentioned, the unique qualities of each state would lead to different-sized variances, and regional spillovers and the interactions between the states would lead to a correlation between the states. As such, a fixed effects model that accounts for both cross-section heteroskedasticity and contemporaneous correlation in the residuals is estimated and the results are presented below:

$$\ln \left( \frac{c_i}{g} \right)_{it} = \hat{\alpha} + \hat{\sigma} \ln \left( \frac{P_g}{P_c} \right)_{it}$$

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Estimate</td>
<td>Std. Err.</td>
<td></td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>1.49</td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td></td>
<td></td>
<td>(603.67)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.69)</td>
</tr>
</tbody>
</table>
Using this GLS estimation method, the intratemporal elasticity of substitution between private and public consumption is significant with a value of 0.09\(^5\).

**Dynamic Ordinary Least Squares (DOLS)**

Although the above estimation method may be appropriate for a number of reasons, the estimation of this particular panel equation is further complicated by the fact that the two variables used in the model are cointegrated. Saikkonen (1991) discusses asymptotic efficiency of the estimated parameters in this context. His model, later augmented by Kao and Chiang (2000) to the panel framework, attempts to correct for asymptotic inefficiency when the variables in a regression are cointegrated. In Kao and Chiang’s influential paper, the asymptotic distributions of three panel estimation methods are analysed; ordinary least squares OLS, fully modified OLS (FMOLS) and dynamic OLS (DOLS). Kao and Chiang find that the DOLS method outperforms both the FMOLS and OLS with respect to asymptotic efficiency when estimating panels with cointegrated variables.

As in the GLS method, the DOLS model is a fixed-effects model that takes into account both heteroskedasticity and contemporaneous correlation in the residuals but the equation is augmented to include leads and/or lags of the change in the independent variable:

\[
\ln \left( \frac{c}{g} \right)_{i,t} = \alpha + \sigma \ln \left( \frac{P_g}{P_c} \right)_{i,t} + \sum_{j=-q}^{q} \phi_{i,j} \Delta \ln \left( \frac{P_c}{P_g} \right)_{i,t+j} + \epsilon_{it}, \quad i = 1, 2, \ldots, 6, \quad t = 1, 2, \ldots, 87 \quad (16)
\]

The reason behind the augmentation of the estimated equation is summarised by Saikkonen (1991, p. 14)

The idea is essentially to remove the asymptotic inefficiency of the least-squares estimator by using all the stationary information of the system to explain the short-run dynamics of the cointegration regression. Increasing the amount of such stationary information may reduce the relevant error covariance matrix of the cointegration regression and thereby improve the asymptotic efficiency.

The results of the model estimated using DOLS are shown below, with the \( \phi \) -coefficients retained being determined on the basis of significance as indicated by \( t \)-ratios. It can be seen that the use of DOLS has no effect on the estimated value of intercept term or the fixed

\[^5\text{We have not calculated standard errors for the cross-section fixed effects. However the estimates are: NSW (0.12), Vic (0.06), QLD (0.05), SA (-0.02), WA(-0.07), TAS (-0.015).}\]
effects\textsuperscript{6}, but the estimate of the intratemporal elasticity of substitution increases from 0.09 to 0.17.

\[
\ln \left( \frac{c_i}{g_i} \right)_{t,t} = \alpha + \sigma \ln \left( \frac{P_{c_i}}{P_{g_i}} \right)_{t,t} + \delta \Delta \ln \left( \frac{P_{c_{i+1}}}{P_{g_{i+1}}} \right)_{t,t+1} \\
1.49 \quad 0.17 \quad 0.22 \\
(618.5) \quad (4.72) \quad (5.04)
\]

\( \bar{R}^2 = 0.92, DW = 0.92 \)

Diagnostic testing of an estimated model is essential to ensure accuracy in the model’s interpretation. Unfortunately, correcting for problems such as autocorrelation and heteroskedasticity in panel data has not been the subject of extensive research. Following from the univariate tests, the low value of the Durbin-Watson statistic, 0.92, implies that some form of positive autocorrelation is present, but as far as we are aware Newey-West, or HAC (heteroskedasticity and autocorrelation consistent) standard errors, which correct for autocorrelation, have not yet been extended to the panel framework.

5. Conclusions

The purpose of this paper is to investigate the substitutability between private and public consumption expenditure in Australia. In order to achieve this, private and public consumption expenditure for each of the six Australian states were analysed in a panel framework in order to determine the intratemporal (or static) elasticity of substitution between them.

Using the available panel unit root and cointegration tests and an efficient estimation method for cointegrated panel models, an intratemporal elasticity of substitution of 0.17 was found. Bearing in mind that in the Amano and Wirjanto (1998) framework the sign of the partial cross-derivative, \( \partial (\partial u / \partial c) / \partial g \) depends on both the intertemporal and intraperiod elasticities of substitution, and \( sign[u_{s,c} (c, g)] = sign \left[ \frac{1}{\gamma} - \sigma \right] \), further information is required to assess the substitution relationship between public and private consumption in Australia.

The intertemporal elasticity of substitution \( \frac{1}{\gamma} \) has not been estimated in the paper. As far as we are aware the only recent Australian study to estimate an intertemporal elasticity of substitution is Cashin and McDermott (2003). Their aggregate model has three goods –

\textsuperscript{6} The estimated values of the cross-section fixed effects are the same as in fn.54.
exportables, importables and non-tradables. Utility for the representative ‘consumer’ (including both the private-sector and the government) is defined over imports and non-tradables in the same way as in Amano and Wirjanto (1998), and the same two-step estimation procedure is applied. Using annual Australian data from 1970 to 1995, they estimate $\frac{1}{\gamma} = 0.88$ which if taken at face value, and together with our estimate of the intratemporal elasticity of substitution of 0.17, would imply that government and private consumption are complements. Alternatively, one might draw on the DSGE literature where estimates of the coefficient of relative risk aversion (i.e. $\gamma$) range between 0.5 and 2.5. Given our estimate of $\sigma$, values of $\gamma$ in this range would still yield the result a complementarity result.

This result may not be robust, however, for a number of reasons. There is some evidence that our DOLS estimate does not fully account for serial correlation, and we have only allowed for contemporaneous cross-correlation between the states. Most likely a richer correlation structure is warranted. Second, our model may be overly simplistic in that the price ratio used in this study is not, for government consumption expenditure, reflective of the price the representative consumer actually faces. The Amano and Wirjanto (1998) setup is solving a ‘social planning’ problem with a unitary government, no distorting taxes, and where, presumably, the representative consumer is the median voter. Australian reality differs from this setup in potentially important ways. Perhaps the most fruitful extension to the work presented in this paper, however, would be to use the same state-based consumption data to undertake a VEC analysis of the interaction between government and private consumption. Such an approach would avoid reliance on the ‘social planning’ assumptions that underpin our present estimates.
6. References


