A Multi-Regional Computable General Equilibrium Model for New Zealand

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Abstract

Computable General Equilibrium (CGE) models are tools used routinely for policy analysis. New Zealand has so far lacked a comprehensive multi-regional CGE model that would facilitate examination of economic issues at the regional level. Assessing the regional impact of external shocks and understanding the interdependence of our regions, and the implications of that interdependence for the national economy, are some of the applications such a model would be useful for. A multi-regional CGE model for New Zealand is being developed at Victoria University. This project is taking advantage of recently collated regional data and accelerating computational power to produce an innovative model that takes a fresh look at the issues involved in building and using a CGE model.

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Contents

List of Figures

List of Tables

1 Introduction

Computable General Equilibrium (CGE) models are tools used routinely for policy analysis[2]. They are macroeconomic models which use general equilibrium theory from microeconomics to model the behaviour of the components that make up the macroeconomy. They can then be used to analyse how a shock to the aggregate economy or to some component of it affects the other components (and via feedbacks the aggregate economy).

General-purpose multi-sectoral CGE models for New Zealand were developed at Victoria University in the 1980's and 1990's — see [10, 11, 12, 16]. Since that time the field of CGE modelling has developed in response to new data becoming available, new issues gaining prominence, and the continual rise in computing power. In light of this, we should ask: are we able to take advantage of these developments to build an up-to-date CGE model for New Zealand that will complement those already in use? This paper asserts an affirmative answer to this question.

Recently there has been a strong focus on quantifying the similarities, differences, and linkages between our regions. A wealth of regional data is now available from various sources. Our assertion is that we now have sufficient information to build a multi-regional, multi-sectoral CGE model for New Zealand. We also believe that there are key advantages to be gained in building a new model with a 'bottom-up' theoretical structure rather than simply adding a 'top-down' regional disaggregation module to an existing CGE model. This paper presents the basic multi-regional CGE model developed so far, that forms the foundation of a PhD thesis to be submitted to Victoria University for examination.

2 Methodological Approach

There are three aspects that characterize the approach taken in developing a multi-regional CGE model:

1. The method of regional disaggregation

2. The theoretical structure

3. The method of implementation

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[2]see for example [13]

For each aspect a choice has to be made between two options, and each choice cannot necessarily be made independently of the other choices.

The first choice, over how to handle regions, is a choice between a top-down disaggregation of economy-wide simulation results, and a bottom-up method that treats each region as though it is an economy in its own right, trading with other regions in much the same way that countries trade.

The choice of theoretical structure reflects the choice over the first aspect. Simply stated, if bottom-up regional modelling is chosen, many variables will carry a region subscript along with the numerous other superscripts and subscripts common to CGE models. If top-down modelling is chosen, no region subscripts are needed on the main model variables but a new set of regional variables and equations are added that allocate the aggregate impact of shocks to regions according to pre-determined regional shares.

The theoretical structure also reflects the method of implementation: if the model is to be implemented as a linearized system, then the equations of the model will usually be presented in their linearized, or 'percentage change' form. If the model will be implemented as a system of non-linear equations, the structural, or 'levels' form of the equations will be listed.

The method of implementation therefore involves choosing between a linearized model or a levels model. The two major competing software packages for implementing CGE models are GEMPACK and GAMS. GEMPACK is much more suited to implementing linearized models while GAMS is ideal for implementing levels models. Therefore the implementation method depends on the choice of programming environment. GEMPACK could be described as more 'user focussed' as it is more user friendly and more intuitive to use. GAMS on the other hand is more 'development focussed' as it is more flexible and has capabilities far beyond 'just' solving CGE models.

A common assumption when one is considering how to handle the regional modelling is that bottom-up modelling is only feasible when separate input-output data tables are available for each region to be modelled. New Zealand does not currently have such regional input-output data available. We therefore cannot build a bottom-up model based on such data. There is however a method that allows us to have a bottom-up regional structure while only possessing enough data for a top-down disaggregation. It is this method that will be described in this paper.

The methodological approach taken is essentially this:

1. Construct the model with a theoretical structure common to bottom-up multi-regional models. A key source of insight for this aspect is the FEDERAL multi-regional model for Australia.[9]

2. Use regional data to disaggregate a national input-output table into regions. This is akin to making similar assumptions that are made when one is doing a top-down regional disaggregation. The regional module for the generic multi-sectoral ORANI-G model [6] gives guidance on this.

3. Implement the model in levels form in GAMS for reasons alluded to above. The GAMS model library contains many models implemented in this way.[3]

The remainder of this paper gives details on how the above approach is used to construct the multi-regional CGE model for New Zealand.

3 Theoretical Structure

The model has four sectors — the household sector, the production sector, the public sector, and the foreign sector. The behaviour of the agents within each of these sectors will be briefly described, followed by a discussion of the constraints put on their behaviour by the imposition of the competitive general equilibrium framework.

3.1 Households

The household sector primarily provides labour services to producers and consumes goods and services made by them. There is one representative household agent per region. The labour supply choice is endogenous, arising from a decision of how to split the household's time endowment between labour and leisure. Households also own the stock of currently installed capital in their own region and so receive its rental income. Any part of household income that is not spent becomes private savings. The share of income that is not spent is usually exogenously imposed. The household agent in each region then has a set of inter-related decisions to make:

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[3]Also, see [7]

1. a decision over how many commodities and how much leisure to consume

2. given the previous decision, what the types of commodities will constitute its commodity consumption demand

3. and given that decision, how much of each type of commodity to demand from each source (i.e. each domestic region plus importers)

The decisions are modelled as a utility-maximization problem for the first decision, followed by expenditure-minimization problems for the latter decisions. These nested household decisions are summarized in figure 1.

At the top level of the decision nest, the household agent in region R chooses the level of total composite commodity Q and the level of leisure N to consume to maximize utility subject to their expenditure constraint. The utility function is restricted to be of the Constant Elasticity of Substitution (CES) class of functions. The functional form of the demands that solve the optimization problem depend on the assumed value of the elasticity of substitution σ.[4] We therefore express the demand functions using a functional notation:

$$Q^d = Q^d(P_Q, P_L, NomWel, NomSav) \tag{1}$$

$$N^d = N^d(P_Q, P_L, NomWel, NomSav) \tag{2}$$

Note that each variable in the equations above are for a given region R. However, the region subscript has been suppressed for simplicity. The interpretation of the first equation for example is that the demand for total composite commodity Q by the household agent in region R is a function of the commodity price and wage rate *faced by that agent*. I.e. there are region-specific wage rates and commodity prices. $NomWel$ and $NomSav$ are nominal wealth (value of regional endowments) and savings, and appear in the demand functions because each household is limited in its expenditure (out of wealth) by the amount $NomWel - NomSav$.

Although the model is implemented in levels form, it is easier to interpret demand equations by looking at their linearized form, which can be written algebraically for a given σ. The linearized form of the above demand

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[4]If $\sigma = 0$, utility is of the Leontief form; if $\sigma = 1$, utility is Cobb-Douglas; we exclude the possibility of perfect substitutes by assuming σ is finite.

functions are:

$$q^d = ws - \sigma \cdot p_Q - (1 - \sigma)\left(X_Q \cdot p_Q + X_N \cdot p_L\right)$$

$$n^d = ws - \sigma \cdot p_L - (1 - \sigma)\left(X_Q \cdot p_Q + X_N \cdot p_L\right)$$

The lowercase variables are the percentage changes of the levels variables. For example q is $100 \cdot dQ/Q$ and ws is the percentage change in $NomWel - NomSav$. The X_Q and X_N are weights that are approximately expenditure shares.

From the linearized equations we can easily see the effect on demands of a change in expenditure out of wealth, and how the effects of price changes depend on σ. In particular we can see that:

$$\varepsilon_Q = -\sigma - (1 - \sigma) \cdot X_Q$$

$$\eta_Q = -(1 - \sigma) \cdot X_N$$

where ε_Q is the own-price elasticity of total composite commodity demand
η_Q is the cross-price elasticity of total composite commodity demand (with respect to the wage rate)

Below the top level, the decision nest consists of a commodity composition choice and a source choice. The source choice is split into two parts — the choice between domestic and imported sources, and then the choice over domestic regions. The multi-level decision structure is essential to enable different elasticities of substitution to apply at each decision point[5]. E.g. we might wish to assume that the elasticity of substitution between commodities from the domestic regions is higher than the elasticity between domestic and imported sources. The optimizing choice at each point below the top-level decision is an expenditure minimization problem where the household agent in each region R seeks to minimize the cost of forming the composite required at the decision point directly above. For example,

$$Q^{Gd} = Q^{Gd}(Q^d, P_{Q^H} \mid_{H \in COM}) \qquad\qquad \forall G \in COM \qquad (3)$$

are the demand functions that minimize the cost of forming the total composite commodity Q where Q is a CES combination of each type of commodity Q^G.

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[5]that is, if we want to avoid complicating matters by using CRESH functions

The notation $P_{Q^H} |_{H \in COM}$ is a shorthand for P_{Q^1}, P_{Q^2}, ... where $COM = \{1, 2, ...\}$.

The linearized form of the above equation is:

$$q^{Gd} = q^d - \sigma \left(p_{Q^G} - \sum_{H \in COM} X_{Q^H} \cdot p_{Q^H} \right) \qquad \forall G \in COM$$

Again, the lower case letters are percentage change variables and the X_{Q^G}'s are weights approximately equal to the share of composite commodity G in expenditure on total composite commodities.

The implications of price changes on the composition of consumption demand can be seen from the linearized demand functions: if the price of composite commodity G rises relative to an index of all the composite commodity prices[6], then the demand for composite commodity G will fall by an amount determined by the value of σ. Assuming for example $\sigma < 1$ means that the demand for each composite commodity will be inelastic to changes in its price relative to its substitutes' prices.[7]

The lower level demand functions for source choice, which are analogous to (3), are:

$$
\begin{aligned}
Q^{Gdom\,d} &= Q^{Gdom\,d}(Q^{Gd}, P_{Q^{Gdom}}, P_{Q^{Gimp}}) & \forall G \in COM \quad (4) \\
Q^{Gimp\,d} &= Q^{Gimp\,d}(Q^{Gd}, P_{Q^{Gdom}}, P_{Q^{Gimp}}) & \forall G \in COM \quad (5) \\
Q^{GXd} &= Q^{GXd}(Q^{Gdom\,d}, P_{Q^{GZ}} |_{Z \in REG}) & \forall G \in COM, \forall X \in REG \quad (6)
\end{aligned}
$$

where 'dom' signifies 'domestic sources' and 'imp' signifies 'imported'.

3.2 Production

The production sector produces goods and services for consumption by households, the government, and foreigners (i.e. for exporting), and for use within the sector as an input into production. It relies on currently installed capital as well as labour and intermediate inputs (domestically produced or imported) for production. As capital by definition takes time to build (that is, more time than consumption goods and intermediate inputs), capital formation takes place within the sector as a concurrent activity to production for

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[6]I.e. if $p_{Q^G} > \sum_{H \in COM} X_{Q^H} p_{Q^H}$, and $q^d = 0$

[7]This interpretation is from [1] and [2]

current usage. Investment is financed by private and public savings and allocated to industries and regions according to a user-selected criterion. Other activities that also occur within the sector are the transformation of domestically produced commodities into exports, and the transformation of imports into imported commodities, ready for domestic use.

3.2.1 Current Production

Producers of goods and services are classified by industry to reflect differences in production technique across industries. Each industry produces only one, unique type of commodity for current use. These are also differentiated by region. I.e. there is one agent for each industry J in each region R that engages in current production of commodity G, where $G = J$. Thus there are as many varieties of commodity G as there are regions, and we refer to a particular variety as commodity G from region R. The full decision nest of the agent for industry J in region R is detailed in figure 2.

At the top level of the decision nest, the industry agent chooses the cost-minimizing combination of total composite commodity Q and composite factor F to produce a given level of output of its commodity. which is commodity G where $G = J$. The cost-minimization problem is similar to the expenditure minimization problems of the household agents — costs are minimized subject to a CES function, which is the production function in this case. The demand functions that solve the firm's optimization problem are:

$$
\begin{align}
Q^d &= Q^d(P_Q, P_F, Q^{GRs}) \tag{7} \\
F^d &= F^d(P_Q, P_F, Q^{GRs}) \tag{8}
\end{align}
$$

This time there is one set of these equations for each industry in each region, but again the subscripts on all the variables have been suppressed for simplicity. Equation (8) for example relates the demand for composite factor by industry J in region R to the prices *they* face for composite intermediate inputs Q and composite factor F, along with the desired level of output of their commodity Q^{GRs} (where $G = J$). The commodity composition and source composition choices follow the pattern described for households. That is, there are equations identical to (3) - (6) for each industry in each region for current production.

Composite factor F is formed in a similar manner to composite Q. The important feature of this choice is that the firm can only demand labour and capital from within its own region, and indeed this is what gives the firm its

regional characteristic. Additionally, the firm can only use capital specific to its industry.[8]

3.2.2   Capital Formation

At the same time as producing commodities for current usage, firms construct capital for future use. The common convention that no labour or capital are directly employed in the construction of new capital is followed; the only costs are purchases of goods and services (which would have used labour and capital in their production). The process of capital formation is modelled to be identical to the way firms form their composite intermediate input for current production — see figure 3. For a given level of new capital K^{new} constructed by, and specific to, industry J in region R, the agent chooses a cost-minimizing combination of commodities Q^G to form K^{new} from a CES production function. There are therefore a set of equations analogous to equations (3) - (6) for each industry in each region for capital formation.

3.2.3   Allocation of Investment

The amount of new capital created by each industry in each region depends on an allocation of investment decision that is made independently of the technology of capital formation discussed above. A given level of aggregate investment is allocated to industries and regions in one of two ways, chosen by the user:

1. the investment demand of each industry in each region is fixed at exogenously determined levels.

2. the investment demand of each industry in each region is fixed at exogenously determined proportions of real aggregate investment.

The ORANI [2, 6] and FEDERAL [9] approach of endogenous allocation of investment is yet to be fully implemented in this model. Once it has been, choosing this approach will cause investment demands to adjust to keep expected rates of return on investment equalized in the face of shocks.

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[8]Labour is not specific to industries which is why L only has a region R subscript in the diagram in figure 2 but K has subscripts J and R since capital is industry and region specific.

3.2.4 Exporters and Importers

Exporter and importer agents act as simple conduits between the domestic and foreign sectors. By including them in the model we can clearly see the transformation process involved in converting domestic commodities into exports and foreign commodities into imported commodities. At this stage these transformation processes are costless. It also enables us to establish both the prices relevant to the domestic economy (domestic currency prices received for exports and paid for imported commodities) and the prices relevant for the foreign sector (foreign currency prices paid for exports and received for imports).

There is one exporter agent for each commodity G, and the costless transformation assumption means that they simply demand the same volume of the relevant domestic commodity (Q^G) as the plan to export (EXP^{Gs}). I.e. for each exporter agent G:

$$Q^{Gd} = EXP^{Gs} \tag{9}$$

Each exporter then has a source choice decision to make. Since we assume that imported commodities are not immediately exported again without any value-added from the domestic economy, the only relevant sources are the domestic regions. Each exporter then has a set of demand functions analogous to (6) for households.

Similar to exporters, there is one importer agent of each commodity G who demands imports (IMP^G) to convert into imported commodities (Q^{Gimp}) for selling onwards to domestic users. The assumption of costless transformation implies that for each importer agent G:

$$IMP^{Gd} = Q^{Gimp\ s} \tag{10}$$

Note that the importer and exporter agents lack a regional dimension. This captures the idea that, as far as the foreign sector is concerned, exported commodities sourced from different domestic regions are identical, and have identical purchase prices. Similarly, as far as a domestic user of an imported commodity is concerned, commodities arriving at different ports are identical, and also have the same purchase prices.

3.3 Government

The public sector purchases goods and services and derives revenue from taxation. Any excess of government revenue over spending becomes pub-

lic savings. Currently the model only provides for government consumption expenditure, commodity taxes, duties and export subsidies. The sector's description will be enhanced later by modelling of government investment expenditure, income taxes, and GST (separately to commodity tax rather than included as at present). The sector is also currently described by the behaviour of a single agent that lacks a regional dimension. There is therefore potential for future developments involving specification of local governments' revenue and expenditure.

3.3.1 The Composition of Government Consumption

For a given level of real government consumption expenditure, the government agent has a commodity composition choice and a source choice analogous to that of households and firms — see figure 4. The set of demand functions resulting from the decision at the top of the decision nest are:

$$Q^{Gd} = Q^{Gd}(Real\ G, P_{Q^H}\mid_{H \in COM}) \qquad\qquad \forall G \in COM \quad (11)$$

Remember again that P_{Q^G} is the purchase price of composite commodity G faced by the government. There is also a set of equations analogous to (4) - (6) that describes the government's source choice of commodities.

Making the level of *Real G* exogenous is the standard way of dealing with government expenditure in CGE models. It is usual to not allow any substitution at least at the top level of the decision nest to reflect the idea that the government is very unresponsive to price changes when considering its composition of expenditure. It also seems reasonable to assume that the government will be less willing than firms to substitute between sources of commodities.

3.3.2 Commodity Taxes, Import Duties and Export Subsidies

Exogenously fixed *ad valorem* commodity tax rates are imposed on each domestic user. For simplicity the rates faced by the government and exporters are assumed to equal zero. Commodity tax rates can vary by industry and use (current production or capital formation) but not region where the industry agent is located. For simulation purposes, the commodity tax rates faced by households have a regional dimension so that households in different regions may face different tax rates for the same commodity from the same source.

While the majority of commodity tax paid by households is GST, this allows for regional differences in other components, such as petrol tax rates.

Ad valorem import duty rates and export subsidy rates are exogenously specified for each commodity G. Duty is paid by importer agents and is passed on to users through the price of imported commodities. I.e. the basic price of imported commodities includes duty while the c.i.f. import price does not. Similarly, export subsidies are received by exporter agents and place a wedge between their basic price (the price they receive, including the subsidy) and the f.o.b. export price (paid by the foreign sector).

3.4 Foreign Sector

The foreign sector agent demands domestic goods and services (exports) and supplies the domestic economy with its own goods and services (imports). A trade surplus is equivalent to borrowing by foreigners and is financed by (domestic) private and public savings.

Imports of each commodity are supplied perfectly elastically at the foreign currency (c.i.f.) import price for that commodity, so each import supply is determined by the demand of the relevant importer in equilibrium.

The agent demands exports of each commodity EXP^G given foreign currency (f.o.b.) export prices $P_{EXP^G}^{for\$}$ according to the (inverse) export demand curve for each commodity:

$$P_{EXP^G}^{for\$} = \left(EXP^{Gd}\right)^{-1/\varepsilon_{EXP^G}} \cdot \psi_{EXP^G} \tag{12}$$

where ε_{EXP^G} is the elasticity of export demand for commodity G
         ψ_{EXP^G} is a co-efficient reflecting the height of the export
                demand curve for commodity G

3.5 Conditions for Competitive General Equilibrium

Having set out the key relationships that govern the behaviour of the agents, we now turn to the constraints imposed on them by the assumption of competitive equilibrium. The constraints fall into one of two groups of conditions:

1. Market clearing conditions — there is no excess demand (or supply) in any market.

2. Zero pure profit conditions — there is no excess of revenue over costs in any productive activity.

Each of these will be discussed in turn.

3.5.1   Market Clearing

All interaction between the agents occur through the operation of the following markets:

Commodity markets one market for each commodity coming from each source[9]

Export markets one market for each commodity exported

Import markets one market for each commodity imported

Investment markets one market for each type of new capital (i.e. for each industry in each region)

Capital markets one market for each type of currently installed capital (i.e. for each industry in each region)

Labour markets one market for each type of labour (i.e. for each region)

Savings market one market where finance for investment and trade surpluses (lending to foreigners) is available.

Market clearing in each market implies the quantity demanded equals the quantity supplied in each one.

Since there is one market for each commodity from each source (domestic regions and importers), market clearing means:

$$Q^{GSs} = \sum_{all\ users} Q^{GSd} \qquad\qquad \forall G \in COM, \forall S \in SRC \qquad (13)$$

Market clearing for factors similarly implies that the demand for currently installed capital specific to industry J in region R is equal to the supply — i.e. the household's endowment of that industry-specific, region-specific capital. Since labour is region-specific but not industry-specific, there are only as many labour market clearing conditions as there are regions. Each condition requires the labour supply of the household agent in region R to equal the total labour demand by all industries in that region.

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[9]includes imported goods markets

The export and import markets are the channels through which exporters and importers trade with the foreign sector, and their market clearing conditions simply state that the demand for export G equals its supply, and the demand for import G equals its supply.

The investment market is similarly the channel through which investment demand drives capital formation. Market clearing implies that the allocation of investment determines the industry-by-region distribution of capital formation.

Finally, the savings market is where finance is traded between lenders and borrowers. Only one equilibrium condition arises from the activity in this market: that total savings equals total borrowing. This condition is not explicitly included in the model due to Walras' Law — clearing of this market is implied by the satisfaction of all the other market clearing conditions.

3.5.2   Zero Pure Profits

Along with market clearing, competitive general equilibrium requires zero pure profits in all production activities. These include the production of commodities for current usage, the production of new capital, and the transformation processes that convert domestic commodities into exports and imports into imported goods. Zero pure profits in each of these activities implies that revenue equals costs for each one.

For example for each industry J in region R producing commodity G of variety R (where $G = J$), zero pure profits in equilibrium means:

$$P_{Q^{GR}} \cdot Q^{GRs} = P_Q \cdot Q^d + P_F \cdot F^d \tag{14}$$

3.6   Purchase Price Definitions

There are a large number of purchase price variables included in the model to facilitate the nested demand functions of agents as described above. Each purchase price has an equation that defines it as a function of either other purchase prices or of basic prices and tax, duty, or export subsidy rates.

For example, the equation that defines the purchase price of total composite commodity Q faced by the household agent in region R is:

$$P_Q \cdot Q^d = \sum_{G \in COM} P_{Q^G} \cdot Q^{Gd} \tag{15}$$

I.e. P_Q is a weighted sum of the purchase prices of each commodity G, P_{Q^G}, where the sum weights are the proportions of household expenditure that goes to each commodity.

Moving down the decision nest for the household agent in region R, there are equations defining the purchase prices at each level:

$$P_{Q^G} \cdot Q^{Gd} = P_{Q^{Gdom}} \cdot Q^{Gdom\,d} + P_{Q^{Gimp}} \cdot Q^{Gimp\,d}$$
$$\forall G \in COM \quad (16)$$

$$P_{Q^{Gdom}} \cdot Q^{Gdom\,d} = \sum_{X \in REG} P_{Q^{GX}} \cdot Q^{GXd} \qquad \forall G \in COM \quad (17)$$

$$P_{Q^{GS}} = P_{Q^{GS}} \cdot CommTaxPower_{Q^G}$$
$$\forall G \in COM, \forall S \in SRC \quad (18)$$

The variable $CommTaxPower_{Q^G}$ in equation (18) is one plus the rate of commodity tax on purchases of commodity G (from any source) imposed on households in region R.

The foreign currency purchase price of import G faced by the relevant importing agent is defined as a function of the basic price of import G (the foreign currency c.i.f. price received by the foreign sector) by the equation:

$$P_{IMP^G}^{for\$} = DutyPower_{IMP^G} \cdot P_{IMP^G} \qquad (19)$$

The nominal exchange rate ϕ can be used to convert this into a domestic currency purchase price:

$$P_{IMP^G}^{dom\$} = \phi \cdot P_{IMP^G}^{for\$} \qquad (20)$$

Similarly the domestic currency f.o.b. purchase price of export G faced by the foreign sector is (implicitly) defined as a function of the basic price of export G (the domestic currency price received by the relevant exporting agent) by the equation:

$$P_{EXP^G} = ExportSubsidyPower_{EXP^G} \cdot P_{EXP^G}^{dom\$} \qquad (21)$$

and the nominal exchange rate ϕ can be used to convert this into a foreign currency purchase price:

$$P_{EXP^G}^{for\$} = \frac{1}{\phi} \cdot P_{EXP^G}^{dom\$} \qquad (22)$$

3.7 Defining Equations for Macro Aggregates, Indices, etc.

There are also a large set of equations that define various aggregate measures and indices that facilitate different closure assumptions and provide useful summary measures. Most of the equations are very straight-forward and so are omitted here. The variables they define include:

- nominal and real aggregate household consumption, investment, government consumption

- nominal and real regional consumption expenditure, income, and wealth

- various valuations of exports, imports, and the trade balance — in domestic or foreign currency terms, in basic prices or in c.i.f. and f.o.b. prices, and in nominal or real terms.

- measures of nominal and real GDP using expenditures, incomes, or value-added.

- price indices for deriving the real measures above such as aggregate and regional consumer price indices, investment price indices, trade price indices, the GDP deflator, etc.

- labour market measures including definitions of regional and economy-wide labour demand and supply, and price indices for regional and economy-wide real wages.

- government fiscal measures including definitions of aggregate commodity tax and duty revenue, and the government fiscal balance (= − borrowing requirement).

- measures of aggregate private, public, and total savings, and measures of the demand for funds (aggregate investment + trade balance)

3.8 Numeraire

The model focuses on the real economy and as such contains no monetary theory of price determination. The level of each price variable in the model must then be interpreted as that price *relative to* some undetermined price level — the numeraire. The usual approach is to exogenously specify one of

the price variables, so that all other prices are relative to it. The nominal exchange rate ϕ is commonly used for this purpose, although other useful candidates include the economy-wide average wage, the consumer price index, or the GDP deflator. For the standard implementation of the model, the nominal exchange rate is made the numeraire by the equation:

$$\phi = 1 \tag{23}$$

4   Model Implementation

4.1   Matrix Form Representation of the Whole Model

The above sections 3.1 - 3.8 have presented and discussed the key equations of the model. If j is the number of industries and r is the number of regions specified, then the total number of core model equations (i.e. excluding those discussed in section 3.7) is equal to $1 + 9r + 18j + 28jr + 2jr^2 + 12j^2r + 4j^2r^2$. A three-industry three-region version of the model would therefore consist of 1036 key equations. Doubling the number of regions and industries increases the number of key equations 9-fold.

The number of variables v always exceeds the number of equations e, so some of the variables need to be exogenously determined. If $\mathbf{V}$ is the vector of all model variables, we can think of $\mathbf{V}$ as partitioned into e endogenous variables $\mathbf{Y}$ and $v - e$ exogenous variables $\mathbf{X}$. The entire system of equations that constitute the model as implemented in GAMS can then be represented in matrix form as:

$$\mathbf{D}(\mathbf{V}) = \begin{pmatrix} \mathbf{F}(\mathbf{V}) \\ \mathbf{E}(\mathbf{V}) \end{pmatrix} = \begin{pmatrix} \mathbf{F}(\mathbf{Y}, \mathbf{X}) \\ \mathbf{E}(\mathbf{0}, \mathbf{X}) \end{pmatrix} = 0 \tag{24}$$

where the vector function $\mathbf{F}(\mathbf{V})$ embodies all the equations of the theoretical structure (sections 3.1 - 3.8) and $\mathbf{E}(\mathbf{V})$ consists of the equations that exogenously fix the subset $\mathbf{X}$ of $\mathbf{V}$. For example if the capital endowment specific to industry J in region R, K_{indJR}, is to be fixed exogenously at the level determined in the base data, $[K_{indJR}]_i$, for all industries and regions then $j \times r$ rows of $\mathbf{E}(\mathbf{V})$ will imply:

$$K_{indJR} - [K_{indJR}]_i = 0 \qquad \forall J \in IND, R \in REG$$

4.2 Implementing The Model In GAMS

The system $\mathbf{D}(\mathbf{V}) = 0$ is a square system of v variables in v non-linear equations which can be solved using the Newton Method. Provided with an initial solution $\mathbf{V}_0$, the Newton algorithm involves iterations according to:

$$\mathbf{V}_{n+1} = \mathbf{V}_n - \mathbf{J}_n^{-1}\mathbf{D}(\mathbf{V}_n) \tag{25}$$

where $\mathbf{J}$ is the Jacobian matrix of first partial derivatives. I.e.,

$$\mathbf{J}_n = \frac{\partial}{\partial \mathbf{V}'_n}\mathbf{D}(\mathbf{V})$$

GAMS can be used to solve systems of equations such as $\mathbf{D}(\mathbf{V}) = 0$ by inputting bounds on $\mathbf{V}$, specifying an initial solution $\mathbf{V}_0$, and using a solver such as PATH. This solver uses pivot methods guided by an advanced merit function to converge on the solution.[3]

The process of implementation is summarized in the following steps:

1. Import national input-output data and use it to construct a model-consistent economy-wide input-output data matrix

2. Import regional GDP figures and use these to construct a regional shares data matrix

3. Use the input-output data matrix and the regional share data to establish the initial solution to the model — that is, the initial equilibrium

4. Use the initial equilibrium and assumed elasticity parameters to calibrate the demand parameters

5. Specify a model closure and a "base run" simulation setting (the meaning of this will be discussed shortly)

6. Use the calibration, the closure, and the simulation setting to forward-solve the model to obtain the base case solution (which should be identical to the initial equilibrium)

7. Change the model closure if desired and specify a simulation setting (i.e. a shock)

8. Use the calibration, the closure, and the simulation setting to forward-solve the model to obtain the post-shock solution

9. Use the post-shock solution and the base case solution to derive the changes and percentage changes in the variables

The methodology involved in each step will be outlined below. Much of the software-specific detail is omitted to help the reader focus on the overall process. These details are available on request from the author.

4.2.1 Construction Of The Economy-wide Input-output Data Matrix

Official national input-output data is published by Statistics New Zealand. The latest official data is for 1995-6 — see [14]. These tables are used to generate an economy-wide data matrix for the model. As an example, table 1 shows the data matrix for a three-industry version of the model. Where necessary, cells in this table will be referred to as IO(*row,column*).

4.2.2 Construction Of The Regional Shares Data Matrix

Statistics New Zealand have released measures of regional gross domestic product for 15 regions, with the latest figures available being for 2003 — see [15]. These measures are based on the summation of gross value added (GVA) measured in current prices across 17 industries in each region. This data can be used to derive regional shares of GVA in each industry. Aggregation of the table can be done prior to calculating these shares if a model with a smaller number of regions and/or industries is being implemented. For some industries, these shares could be determined by assumption rather than by the official data. For example, if is known that production of industry J only occurs in region R then the regional share of industry J's GVA for region R can be set at one and the other regions' shares at zero.

Along these lines, a regional shares data matrix for a three-industry, three-region version of the model is shown in table 2. This is the primary data set used to disaggregate economic activity across regions. Where necessary, cells in this table will be referred to as RegShare(*Region, Industry*) to remind the reader that this is *Region*'s share of (the GVA of) *Industry*.

21

4.2.3 Establishing the Initial Equilibrium Solution

This section describes the general procedure used to obtain initial values for all model variables (including exogenous variables) that is consistent with competitive general equilibrium. A single GAMS program, "data.gms", performs the data aggregation operations discussed above along with these procedures described below.

Prices Because the model is in theory homogenous of degree zero in prices, we are free to arbitrarily choose the level of prices. The flows in the IO data matrix are value flows — they have price and quantity components. If we assume all *domestic* prices are equal to one then the flows can be interpreted as quantity flows instead. The prices relevant for *foreigners*, that is the c.i.f. import prices and f.o.b. export prices, may not necessarily be equal to one, but these do not affect the usage of the IO data matrix.

Flows to Current Production Each commodity row and industry column of the IO data matrix is effectively split using the regional shares matrix. The flow of commodity G (from all domestic sources) to industry J in region R is assumed to be IO(G, J) × RegShare(R, J). Similarly the flow of commodity G from region R to industry J (in all regions) is IO(G, J) × RegShare(R, J). The initial equilibrium value ($[\,\cdot\,]_i$) of the demand for commodity G from region X by industry J in region R for current production (1) is assumed to be:

$$[Q_{indJR}^{GX(1)d}]_i = IO(G, J) \times RegShare(X, G) \times RegShare(R, J)$$

The other cells in the industry J column of the IO matrix are used in a similar way to derive the initial values:

$$[Q_{indJR}^{Gimp(1)d}]_i = IO(IMP\ G, J) \times RegShare(R, J)$$

$$[L_{indJR}^{d}]_i = IO(LAB, J) \times RegShare(R, J)$$

$$[K_{indJR}^{d}]_i = IO(CAP, J) \times RegShare(R, J)$$

Flows to Capital Formation The investment column IO($\cdot$, INV) needs to be split into industries and then into regional industries. The first task is achieved by splitting the column according to each industry's share of GVA

as implied in the aggregate input-output data. The regional shares matrix is then used analogously to the above to split each industry J column and derive initial equilibrium quantities from the flows data. For example, the initial equilibrium value ($[\cdot]_i$) of the demand for commodity G from region X by industry J in region R for capital formation (2) is assumed to be:

$$[Q_{indJR}^{GX(2)d}]_i = IO(G, INV) \times RegShare(X, G) \times [ind_{JR}\text{'s share of } Nominal\ I]_i$$

where

$$[ind_{JR}\text{'s share of } Nominal\ I]_i = [ind_J\text{'s share of } GVA]_i \times RegShare(R, J)$$

Flows to Households   The consumption column IO($\cdot$, CON) is split according to the regional distribution of labour demand (and supply), which depends on the distribution of labour demand across industries and the distribution of each industry across the regions. Then following a similar pattern to that of industries discussed above, we derive:

$$[Q_{hshR}^{GXd}]_i = IO(G, CON) \times RegShare(X, G) \times [\text{Region } R\text{'s share of } Labour\ force]_i$$

where

$$[\text{Region } R\text{'s share of } Labour\ force]_i = \sum_{J \in IND} [ind_J\text{'s share of } L^d]_i \times RegShare(R, J)$$

and

$$[ind_J\text{'s share of } L^d]_i = \frac{IO(LAB, J)}{\sum_{L \in IND} IO(LAB, L)}$$

Flows to Government   Since the government is assumed to have no regional characteristics, the IO($\cdot$, GOV) column is not split. The regional distribution of government demand for each commodity is consistent with the splitting of the commodity (industry) rows according to the regional distribution of each industry. I.e.

$$[Q_{gov}^{GRd}]_i = IO(G, GOV) \times RegShare(R, G)$$

Flows to Exporters   Similarly, the IO($\cdot$, EXP) column is not split and the regional distribution of commodity demand for export is:

$$[Q_{expG}^{GRd}]_i = IO(G, EXP) \times RegShare(R, G)$$

Time Endowment, Leisure, and Private Savings The endowment of *TIME* that each household agent has available to divide between work and leisure is estimated using the simple assumption that each household puts 20% of its time endowment $TIME_{hshR}$ towards employment. This corresponds to an average of approximately 34 hours work per week, 52 weeks per year. In the initial equilibrium then we have:

$$[TIME_{hshR}]_i = 5 \times [L_{hshR}^s]_i$$

The consumption of leisure can then be calculated as:

$$[N_{hshR}^d]_i = [TIME_{hshR}]_i - [L_{hshR}^s]_i$$

The raw input-output data gives no guidance as to average propensities to consume or save, so initial values for APC_{hshR} and APS_{hshR} have to come from an external source.

Commodity Taxes These tax flows are pro-rated over the flows of commodities (both from domestic industries and imports) to form the TAX matrix. For example, the tax flow in IO(TAX, J) is split so that the tax paid by industry J on flows of commodity G is:

$$TAX(G, J) = IO(TAX, J) \times \frac{IO(G, J) + IO(IMP\,G, J)}{\sum_{H \in COM} IO(H, J) + IO(IMP\,H, J)}$$

The TAX matrix is then used to calculate the commodity tax rates. for example, the commodity tax rate on commodity G faced by the household in region R is:

$$[CommTaxRate_{Q_{hshR}^G}]_i = \frac{TAX(G, CON)}{IO(G, CON) + IO(IMP\,G, COM)}$$

Import Duties and Export Subsidies Initial values for these cannot be derived from the raw input-output data. Some external source of information needs to be used to set $[DutyRate_{IMP^G}]_i$ and $[ExportSubsidyRate_{EXP^G}]_i$.

Other Initial Values All other initial values of the variables can be derived from the initial values discussed above using the model equations. For example, $[Q_{hshR}^{Gdom(1)d}]_i$ can be calculated using the purchase price definition associated with this quantity — equation (17). Likewise $L_{hshR}^s]_i$ can be determined by the labour market clearing condition.

4.2.4 Model Calibration

Having established an initial solution to the model, the parameters of the various demand functions can be derived. Calibration essentially means finding the parameter values that are consistent with the initial equilibrium solution. The GAMS program "calibration.gms" takes all the demand functions as equations and all the parameters as variables and solves the system given the initial equilibrium solution and substitution elasticity values where CES functions are used.

For example if the demand functions for $Q_{indJR}^{G(1)d}$ and $Q_{indJR}^{GX(1)d}$ were derived from Leontief and CES functions respectively, GAMS would solve

$$[Q_{indJR}^{G(1)d}]_i = a_{Q_{indJR}^{G(1)d}} \cdot [Q_{indJR}^d]_i$$

$$[Q_{indJR}^{GX(1)d}]_i = \frac{[Q_{indJR}^{Gdom(1)d}]_i}{v_{Q_{indJR}^{Gdom(1)}}} \cdot \left(\sum_{Z \in REG} a_{Q_{indJR}^{GZ(1)d}} \cdot \left(\frac{a_{Q_{indJR}^{GX(1)d}} \cdot [P_{Q_{indJR}^{GZ(1)}}]_i}{a_{Q_{indJR}^{GZ(1)d}} \cdot [P_{Q_{indJR}^{GX(1)}}]_i} \right)^{\frac{\rho_{Q_{indJR}^{Gdom(1)}}}{\rho_{Q_{indJR}^{Gdom(1)}} - 1}} \right)^{\frac{-1}{\rho_{Q_{indJR}^{Gdom(1)}}}}$$

for $a_{Q_{indJR}^{G(1)d}}$, $a_{Q_{indJR}^{GX(1)d}}$, and $v_{Q_{indJR}^{Gdom(1)}}$ given the initial equilibrium values and the ρ parameters[10], which are related to the elasticity of substitution parameters σ by $\rho = \frac{\sigma-1}{\sigma}$.

4.2.5 Model Closure

The user can specify which variables are exogenous and which are endogenous provided the resulting equation system is square and consistent[11]. A suggested short-run closure for the model is shown in table 3.

4.2.6 Walras' Law and Macro Balance

When specifying the closure, or making changes to it, careful thought has to be given to the implications of Walras' Law. To prevent the model from being over-identified, the market clearing equation for the savings market is excluded. On one side of this market stand the households and government

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[10]and the constraint $\sum_{X \in REG} a_{Q_{indJR}^{GX(1)d}} = 1$, which "determines" the $v_{Q_{indJR}^{Gdom(1)}}$ parameters

[11]care needs to be taken for example that a quantity and its relevant price are not both set exogenously as this may result in the system becoming unsolvable

with funds[12] available for lending. On the other side stand the investor agent demanding finance for capital formation and the foreign agent demanding finance for their trade deficit (a domestic trade surplus). If all the other markets are in equilibrium, then the equation system automatically implies that the savings market is in equilibrium too, *for any vector of equilibrium prices.*

The operation of Walras' Law can then be seen as the source of the macroeconomic balancing identity $S \equiv I + Trade\ Balance$. Once one side of the identity is determined, the other side is determined as well. When we make a closure assumption, we have to decide which component will adjust to the others. The standard short run closure mentioned above, for example, implicitly assumes that nominal investment will adjust to savings. If all the APC's are fixed, private savings is also fixed. Government behaviour is essentially exogenous with fixed real expenditure levels and tax rates, so the government balance is inflexible. The savings side of the identity therefore has no flexibility, so the other side must be flexible to adjust to economic shocks. With exogenously fixed import prices and export demand curves, the trade balance is not flexible so investment needs to absorb any shocks. If I^d_{indJR} is fixed exogenous for each industry J and region R then the capital construction costs are what adjust. I.e. $Nominal\ I$ will adjust. If instead we set ind_{JR}'s share of $Real\ I$ exogenously, we need to ensure that $Real\ I$ is endogenous so that it can adjust.

If we wanted to set $Real\ I$ exogenously, then some component of savings (or the trade balance) needs to be endogenized. The only way to make savings flexible at this stage is to make one of the APC's endogenous. This has the perverse effect that a shock will only affect the savings of one region. A possible remedy would be to fix each region's share of aggregate savings (and make all the APC's endogenous) so that the shock's effect will be spread over all the regions.

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[12]wealth not consumed by households and revenue not spent by government. Obviously it is possible that the government runs a budget deficit in which case they are on the other side of the market. Similarly the foreign sector may run a trade surplus in which case they also change sides. Households changing sides of the market is also possible in theory but highly undesirable in reality.

4.2.7 Base Run and Model Checking

Having calibrated the model and set the closure assumption, the model is ready to be forward solved. In GAMS, it is a single program "simulation.gms" that runs all simulations. Prior to running the program, a run-type and a shock have to be chosen. There are four run-types:

1. base run

2. homogeneity test

3. convergence test

4. simulation

During a base run the program uses the initial equilibrium as the starting point to solve the model, and begins to look for a solution. It should be expected that the program will terminate without any iterations because it is starting at the solution.

The homogeneity test is a test of the model structure to ensure it is homogenous of degree zero in prices. If all domestic prices are doubled, all nominal variables should double and all real variables and foreign prices should not change. The homogeneity test can easily be conducted by changing the value of the numeraire.

The convergence test involves perturbing the starting point for the algorithm to see if it still converges, and if so whether it still converges to the same solution.[13]

4.2.8 Running Simulations

To run a simulation, the "simulation.gms" program takes a specified shock and applies it to the exogenous variables. It then uses the initial equilibrium as the starting point to solve the model. Note that the base run and the homogeneity and convergence tests are just special cases of simulations.

A shock is specified as percentage changes in some exogenous variables. GAMS uses the specified shock to calculate the post-shock value of the exogenous variables and proceeds to find a model solution. Having obtained this, the program calculates the change and percentage change in all the

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[13]One has to be careful not to perturb the initial guess too much otherwise the convergence test may fail — there is no guarantee that the system is globally convergent.

model variables from the unperturbed initial equilibrium, and reports these along with the post-shock solution.

5 Discussion and Conclusion

We have seen how a model can be constructed with an apparently bottom-up theoretical structure yet be provided with data only usually sufficient for a top-down disaggregation. The assumptions made here are therefore no better than those of a top-down model. However, the structure allows for types of simulations that are usually only possible with a bottom-up model — in particular, supply-side shocks at the regional level. We are also able to see the implications of regional immobility of factors.

As part of the solution, GAMS calculates an array of total flows of each commodity from each source (including importers) to each destination (including exporters). The availability of this array from data sources (i.e. regional input-out tables) would make building a purely bottom-up model possible. Without that data, the best we can do, and have done, is derive it using aggregate input-output data, regional GDP figures, and some assumptions. The inclusion of this array in the model makes it possible in theory to specify some regional commodity flows exogenously and allow the model to calculate the relevant regional shares. This is an example of how additional information can be used to extend the model. So far only regional GDP figures have been used. There is plenty of other regional data that could be used to improve the model's representation of the NZ economy.

The model that has been presented in this paper is a work-in-progress towards a PhD thesis at the School of Economics and Finance, Victoria University of Wellington. Components yet to be introduced into the model include ORANI-style investment allocations, GST and income taxes. Extensions of the model to be developed will address the issues of factor immobility and bottlenecks arising from the physical networks that connect our regions. Example simulation results for the three-industry, three-region implemented model discussed above are available from the author on request.

6 Appendix

This appendix lists the important variables of the model where the meaning is not obvious from the variable name, and some associated notation for reference.

hsh_R	representative agent for households in region R
ind_{JR}	representative agent for industry J in region R
inv	investment agent
exp_G	exporter agent for commodity G
imp_G	importer agent for commodity G
gov	government agent
row	foreign sector agent (rest of the world)
COM	set of commodities
IND	set of industries
REG	set of domestic regions
SRC	set of sources ($= REG \cap \,'imp'$)
$\varepsilon_{Q^{GEXP}}$	elasticity of export demand for commodity G
$\psi_{Q^{GEXPd}_{row}}$	height co-efficient of export demand curve for commodity G
ϕ	nominal exchange rate (domestic \$ per foreign \$)
APC_{hshR}	average propensity to consume of hsh_R
APS_{hshR}	average propensity to save of hsh_R
EXP^{Gd}_{row}	exports of commodity G demanded by row
EXP^{Gs}	supply of exports of commodity G by exp_G
F^d_{indJR}	composite factor demanded by ind_{JR}
I^d_{indJR}	investment demand for ind_{JR}
IMP^{Gd}_{impG}	imports of commodity G demanded by imp_G
IMP^{Gs}	supply of imports of commodity G by row
IPI	investment price index
K^d_{indJR}	demand for capital by ind_{JR}
K_{indJR}	capital endowment specific to production by ind_{JR}
K^{new}_{indJR}	new capital constructed by ind_{JR}
K^{t+1}_{indJR}	future capital endowment specific to production by ind_{JR}
L^d_{indJR}	demand for labour by ind_{JR}
L^s_{hshR}	supply of labour by hsh_R
N^d_{hshR}	leisure demanded by hsh_R

$NomExp_{hsh_R}$	nominal expenditure of hsh_R
$NomInc_{hsh_R}$	nominal income of hsh_R
$NomSav_{hsh_R}$	nominal savings of hsh_R
$NomWel_{hsh_R}$	nominal wealth of hsh_R
P_{EXP^G}	basic price of exports of commodity G (domestic \$)
$P^{dom\$}_{EXP^G_{row}}$	domestic currency price for exports of commodity G paid by row
$P^{for\$}_{EXP^G_{row}}$	foreign currency price for exports of commodity G paid by row
P_{IMP^G}	basic price of imports of commodity G (foreign \$)
$P^{dom\$}_{IMP^G_{imp_G}}$	domestic currency price for imports of commodity G paid by imp_G
$P^{for\$}_{IMP^G_{imp_G}}$	foreign currency price for imports of commodity G paid by imp_G
$P_{F_{indJR}}$	price of composite factor paid by ind_{JR}
$P_{K_{indJR}}$	rental rate on K_{indJR} earned by hsh_R
$P_{K^{new}_{indJR}}$	construction cost of new capital K^{new}_{indJR}
$P_{L_{hshR}}$	wage rate earned by hsh_R
$P_{Q_{hshR}}$	price of total composite commodity paid by hsh_R
$P_{Q_{indJR}}$	price of total composite commodity paid by ind_{JR}
$P_{Q^G_{exp_G}}$	price of composite commodity G paid by exp_G
$P_{Q^G_{gov}}$	price of composite commodity G paid by gov
$P_{Q^G_{hshR}}$	price of composite commodity G paid by hsh_R
$P_{Q^{G(1)}_{indJR}}$	price of composite commodity G paid by ind_{JR} for current production
$P_{Q^{G(2)}_{indJR}}$	price of composite commodity G paid by ind_{JR} for capital formation
$P_{Q^{Gdom}_{gov}}$	price of domestic commodity G paid by gov
$P_{Q^{Gdom}_{hshR}}$	price of domestic commodity G paid by hsh_R
$P_{Q^{Gdom(1)}_{indJR}}$	price of domestic commodity G paid by ind_{JR} for current production
$P_{Q^{Gdom(2)}_{indJR}}$	price of domestic commodity G paid by ind_{JR} for capital formation
$P_{Q^{Gimp}}$	basic price of imported commodity G
$P_{Q^{Gimp}_{gov}}$	price of imported commodity G paid by gov
$P_{Q^{Gimp}_{hshR}}$	price of imported commodity G paid by hsh_R
$P_{Q^{Gimp(1)}_{indJR}}$	price of imported commodity G paid by ind_{JR} for current production
$P_{Q^{Gimp(2)}_{indJR}}$	price of imported commodity G paid by ind_{JR} for capital formation
$P_{Q^{GR}_{exp_G}}$	price of commodity G from region R paid by exp_G
$P_{Q^{GR}_{gov}}$	price of commodity G from region R paid by gov
$P_{Q^{GX}_{hshR}}$	price of commodity G from region X paid by hsh_R
$P_{Q^{GX(1)}_{indJR}}$	price of commodity G from region X paid by ind_{JR} for current production
$P_{Q^{GX(2)}_{indJR}}$	price of commodity G from region X paid by ind_{JR} for capital formation

Q^d_{hshR}	total composite commodity demanded by hsh_R
Q^d_{indJR}	total composite commodity demanded by ind_{JR}
Q^{Gd}_{expG}	composite commodity G demanded by exp_G
Q^{Gd}_{gov}	composite commodity G demanded by gov
Q^{Gd}_{hshR}	composite commodity G demanded by hsh_R
$Q^{G(1)d}_{indJR}$	composite commodity G demanded by ind_{JR} for current production
$Q^{G(2)d}_{indJR}$	composite commodity G demanded by ind_{JR} for capital formation
$Q^{Gdom\,d}_{gov}$	domestic commodity G demanded by gov
$Q^{Gdom\,d}_{hshR}$	domestic commodity G demanded by hsh_R
$Q^{Gdom(1)d}_{indJR}$	domestic commodity G demanded by ind_{JR} for current production
$Q^{Gdom(2)d}_{indJR}$	domestic commodity G demanded by ind_{JR} for capital formation
$Q^{Gimp\,d}_{gov}$	imported commodity G demanded by gov
$Q^{Gimp\,d}_{hshR}$	imported commodity G demanded by hsh_R
$Q^{Gimp(1)d}_{indJR}$	imported commodity G demanded by ind_{JR} for current production
$Q^{Gimp(2)d}_{indJR}$	imported commodity G demanded by ind_{JR} for capital formation
$Q^{Gimp\,s}$	supply of imported commodity G by imp_G
Q^{GRd}_{expG}	commodity G from region R demanded by exp_G
Q^{GRd}_{gov}	commodity G from region R demanded by gov
Q^{GRs}	output (supply) of commodity G in region R (by ind_{JR} with $G = J$)
Q^{GXd}_{hshR}	commodity G from region X demanded by hsh_R
$Q^{GX(1)d}_{indJR}$	commodity G from region X demanded by ind_{JR} for current production
$Q^{GX(2)d}_{indJR}$	commodity G from region X demanded by ind_{JR} for capital formation
$Real\ G$	real aggregate government consumption expenditure
$Real\ I$	real aggregate investment
$TIME_{hshR}$	time endowment of hsh_R

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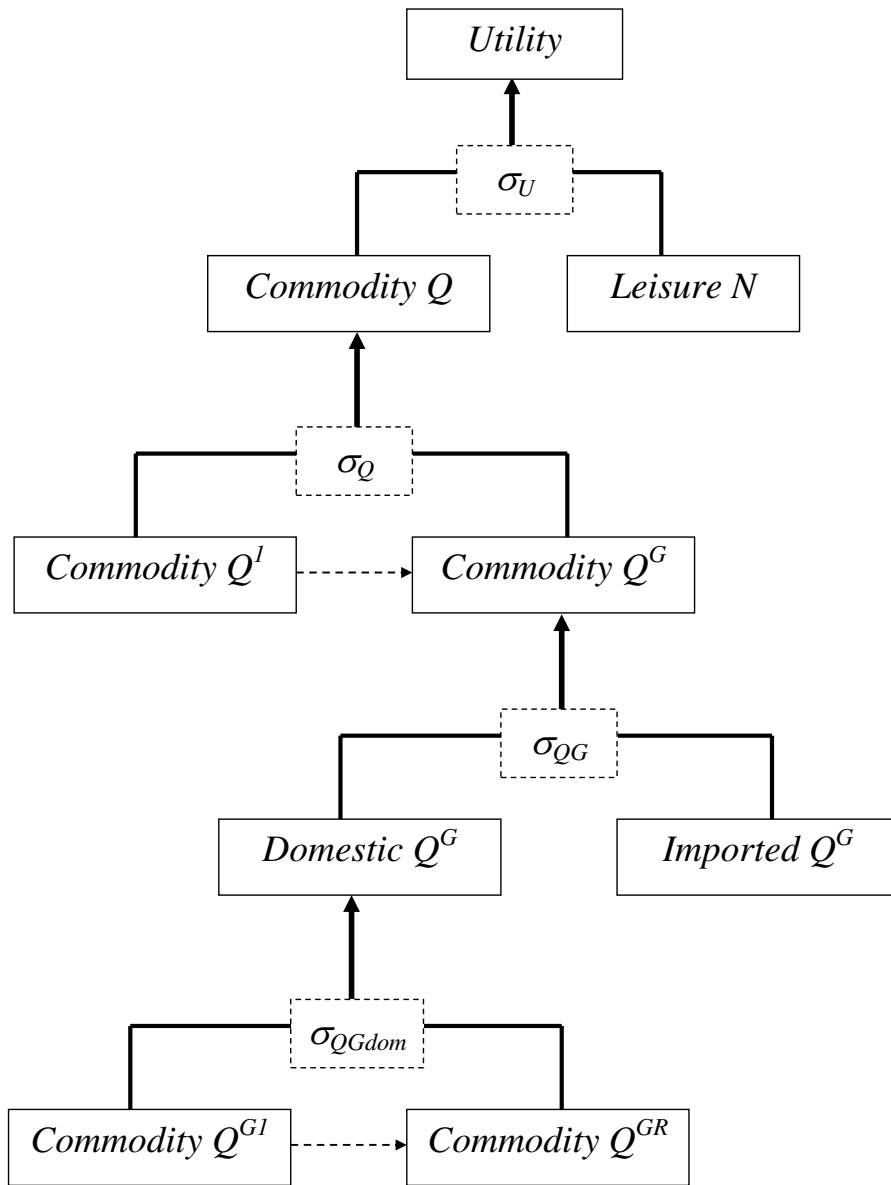
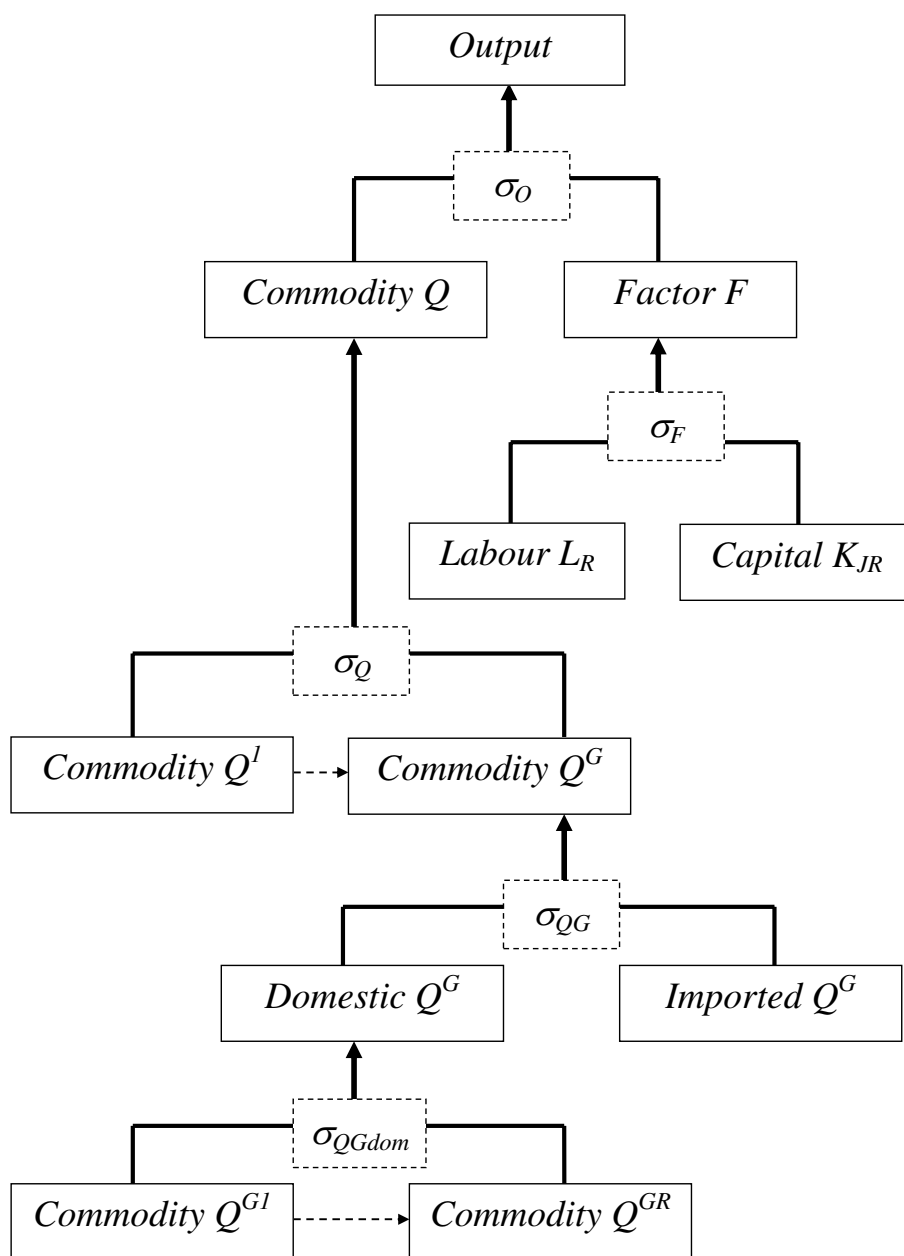Figure 1: decision nest of household agent in each region

Figure 2: decision nest of industry J agent in each region for current production

New Capital K^{new}

σ_K

Commodity Q^I - - - → Commodity Q^G

σ_{QG}

Domestic Q^G Imported Q^G

σ_{QGdom}

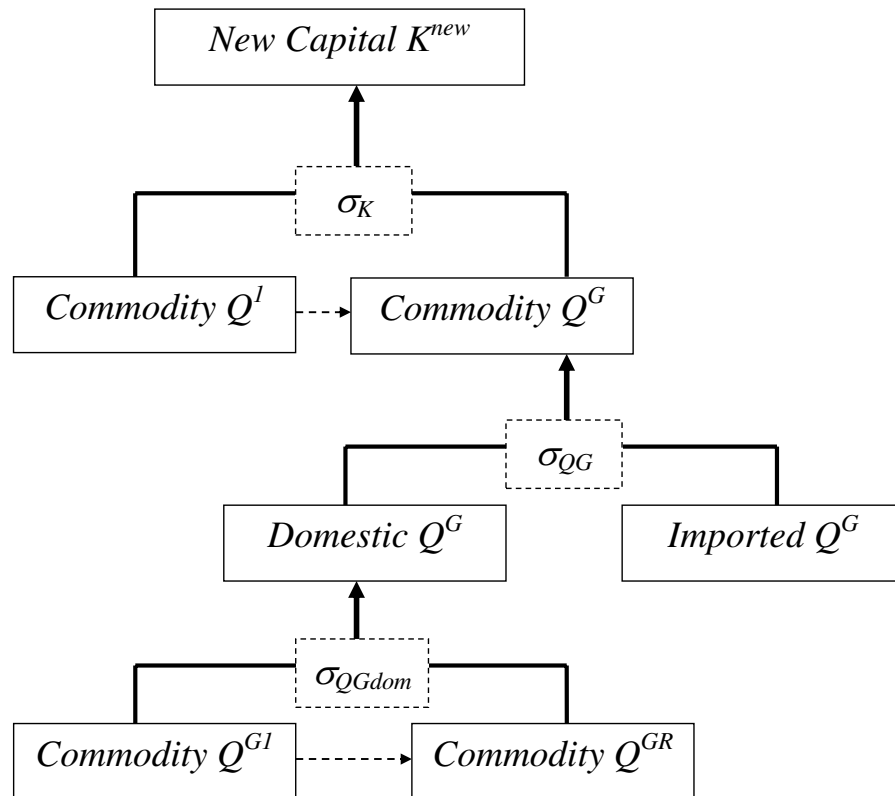Commodity Q^{GI} - - - - → Commodity Q^{GR}

Figure 3: decision nest of industry J agent in each region for capital formation
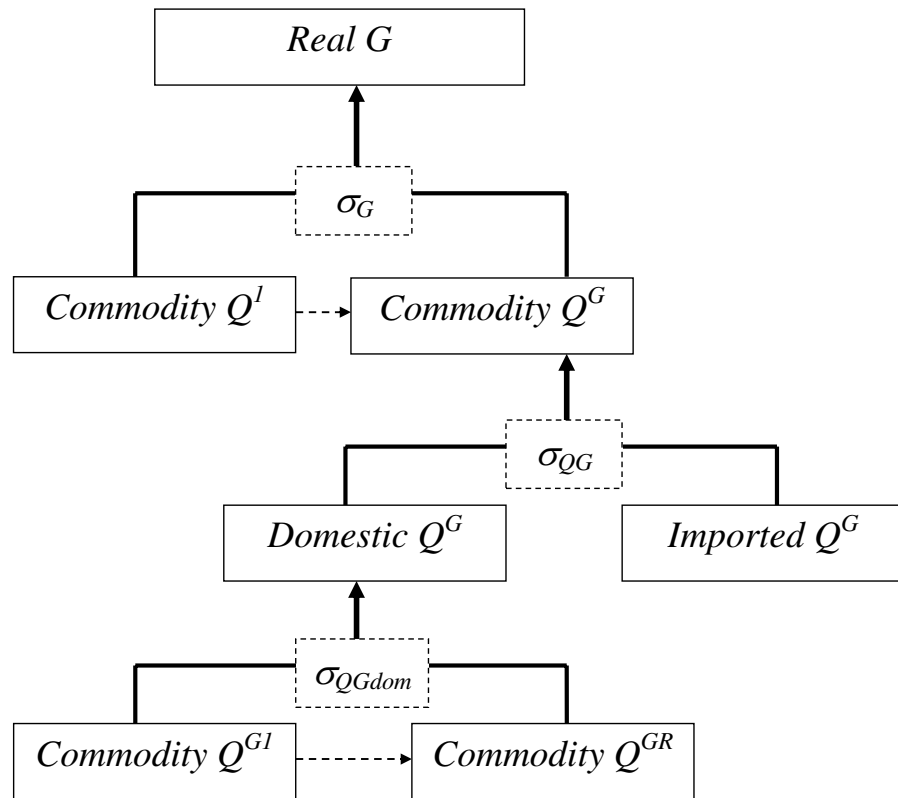
Figure 4: decision nest of government for consumption

raw data coilumn	table4 map	table4 map	table4 map	129	133	131+132	128
raw data row	1	2	3	CON	INV	GOV	EXP
table4 map	1						0
table4 map	2	DATA					0
table4 map	3						0
table3 map	IMP1						0
table3 map	IMP2						0
table3 map	IMP3						0
212	TAX			0	0	0	
215	LAB			0	0	0	
216+217	CAP						

Table 1: IO data matrix

	Auckland	Other North Island	South Island
Goods	0.279	0.463	0.258
Refined Petrol	0	1	0
Services	0.390	0.416	0.194

<div align="center">Table 2: RegShare data matrix</div>

Exogenous Variable	Dimension
P_{IMP^G}	COM
K_{indJR}	$IND \times REG$
$TIME_{hshR}$	REG
APC_{hshR}	REG
I^d_{indJR}	$IND \times REG$
$CommTaxPower_{Q^G_{hshR}}$	$COM \times REG$
$CommTaxPower_{Q^{G(1)}_{indJ}}$	$COM \times IND \times REG$
$CommTaxPower_{Q^{G(2)}_{indJ}}$	$COM \times IND \times REG$
$DutyPower_{IMP^G}$	COM
$ExportSubsidyPower_{EXP^G}$	COM
$\psi_{Q^{GEXPd}_{row}}$	COM
$Real\ G$	1

<div align="center">Table 3: A short-run closure for the model</div>