Simultaneous Auctions at Separate Markets

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Abstract

In this paper, we study an auction game in which simultaneous English auctions take place at several separate markets, and each market has multiple identical units of a good to sell. A bidder who knows her own private valuation of the good and the valuation probability distribution over other bidders has to select one market to participate and each bidder needs at most one unit of the good. We analyze the bidders' mixed-strategy Bayesian Nash equilibrium and develop an algorithm to solve the equilibria of these games with finite types and finite markets. Furthermore, we examine the expected prices and the average surplus gained by the agents with a successful bid among markets.

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1 Introduction

We study an auction game in which simultaneous English auctions take place at several separate markets with an exogenous fixed supply of homogenous goods available for sale in each market. Each individual demands at most one unit of good. We refer a bidder's valuation of the good as her type and bidders' type distribution is public information ex ante. After observing her own type, each bidder decides which market to participate. Once all bidders choose their intended markets, an English auction is conducted in each market. We develop an algorithm to solve the Bayesian Nash equilibrium for the above game with finite types and finite markets. We then show that for such a class of games, there exist no Bayesian Nash equilibria in pure strategies. More importantly, we prove the existence and uniqueness of the mixed Bayesian Nash equilibrium.

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Furthermore, we examine the expected trading prices and the average surplus gained in the auctions.

There are two important features in our model. Firstly, each bidder has to choose which market to enter before bidding. Secondly, once all potential bidders enter their target markets, an English auction starts in each market simultaneously and each bidder bids truthfully. This implies that each individual can only choose one market to bid for the goods. Such situations are common in cities or towns where two or more flower/fish markets coexist. Thus each merchant must decide to go to which flower/fish market to bid for their goods. Other potentially related examples include medical students deciding which clinical medicine to pursue, scholars deciding which journal to submit their paper, firms deciding to supply components/parts either for Apple or Dell, and so on. The above examples share a common property in that, the payoffs from one choice decreases with the number of agents who choose the same target. From this point of view, our paper is also closely related to the literature on congestion and crowding games.

Although there are some studies on multi-unit auctions, most of them restrict attention to the case of one market. For example, Wilson (1979), Klemperer and Meyer (1989). Back and Zender (1993) on simultaneous auctions; and Weber (1983), Maskin and Riley (1992) and Bulow and Klemperer (1996) on sequential auction, to name a few. In contrast to the above literature, our paper discusses simultaneous auctions in multiple markets. As we assume individuals bid truthfully in auctions, the most important strategy for each bidder is the choice among multiple markets. This deviation from standard auction literature makes our paper more close to Kranton and Minehart (2001). There is still one crucial distinction between Kranton and Minehart (2001) and our model. In contrast to the assumption that each buyer observes her type after deciding which market to participate made in Kranton and Minehart (2001), we suppose that a buyer learns her type before entering her intended market. In our model, each buyer needs at most one unit of the good and may only participate in one market. As the payoff a buyer receives for participating a particular market decreases with the total number of buyers choosing the same market, our model is closely related to those in congestion games (Rosenthal, 1973) or crowding games (Milchtaich, 1996). While both congestion and crowding games possess Nash equilibria in pure strategies, we show that in our model, there exists only mixed Nash equilibria. The nonexistence of pure Nash equilibrium comes from the incomplete information on buyers' valuations: in standard congestion or crowding models, agents' valuations are public information while in our model, buyers only observe the ex ante valuation distribution.

The arguments of our algorithm proceeds as follows. At the first stage, for a buyer with the lowest valuation to randomize among multiple markets, she must receive the same expected surplus from each market. Note that the only way that such a buyer obtain positive surplus is that the total number of other buyers entering the same market she participates is less than the supply of that market. This implies that the probability that excess supply occurs must be the same among all markets. From this observation we may obtain for each market the probability that a typical potential bidder will enter that market. For the second stage and we look at a buyer with valuation next to the lowest. For such a buyer to be indifferent among all market, we require that her expected payoffs are the same from each market. Note that her expected gains from bidding consist of two parts: the trade executed at price zero and the trade executed at the price equal to the lowest valuation. From the first stage we know the probability that the former trade occurs is the same across all markets, it is straightforward to see that the probability that the latter trade occurs must be the same across all markets as well. By this condition we may compute for each market the probability that a buyer with the lowest valuation will enter that market. By applying the above procedure from buyers with the lowest valuation to buyers with the highest valuation, we may compute the mixed strategies for buyers of each type.

The remaining part of this paper is organized as follow. We construct the model in section 2 by introducing an auction-market choice game with incomplete information. In the next section, we characterize the symmetric Bayesian equilibrium by a unique mixed-strategy equilibrium in which all markets are selected with a positive probability by all types of bidders. The last section concentrates on developing an algorithm to calculate the unique symmetric equilibrium and give an example where two types of bidders are playing a market choice game between two markets.

2 The Model

Now we consider a market-choice Bayesian game. Suppose there are n bidders and the set of bidders is denoted by $N = \{1, \ldots, n\}$. There is a single product available to bidders and each bidder wishes to purchase one unit of the goods. Each bidder has a private valuation of the goods and this valuation is not known by the others. Let bidder i's private information be t_i, $t_i \in T_i$. Without loss of generality, we assume $T_i = T$ for all $i \in N$ and T is a finite set. Let $T = \{v_1, v_2, \ldots, v_h\}$ where $v_1 < v_2 < \ldots < v_h$. So a bidder's lowest (highest) type of possible valuations for a unit of the goods is v_1 (v_h). In this paper, a bidder with private type t_i is also addressed by a t_i-type bidder. Suppose each bidder has a belief on other bidders' types and we model those beliefs with a common prior and ex-ante symmetry. The information bidder i has regarding the types of other bidders is captured by a distribution $F_i(\mathbf{t}_{-i} \mid t_i)$ where $\mathbf{t}_{-i}$ denotes $(t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)$. Let F_i be the set of all possible f_i and $\mathbf{F} = \Pi_{i \in N} F_i$. Furthermore, we assume that the bidders' types are independent and nature draws each bidders' type according to a common prior probability distribution $f(t)$, $f : T \to [0, 1]$. In which case $F_i(\mathbf{t}_{-i} \mid t_i) = F_i(\mathbf{t}_{-i}) = f(t_1) \times \ldots \times f(t_{i-1}) \times f(t_{i+1}) \times \ldots \times f(t_n)$ for any $i \in N$. Suppose $f(v_k) = \rho_k$ for $k = 1, \ldots, h$ and $\sum_{k=1}^{h} \rho_k = 1$.

2.1 Markets and Bidders' Payoffs

There are m markets and the set of markets is denoted by M, $M = \{1, \ldots, m\}$. Markets may vary in the supply amount and we assume that market j has q_j units of the goods to sell. Each bidder is allowed to enter at most a market to bid. Let bidder i's decision on market choice be $a_i \in M$ and other bidders' decisions is denoted by $\mathbf{a}_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$. $N_j = \{i \mid i \in N$ and $a_i = j\}$ denotes the set of bidders who have chosen market j to bid. We assume that auctioneers in all markets use ascending price auction to sell the goods and all units of the goods have to be sold at the same price. The price is incremented until the market clears. Thus the equilibrium price of market j, p_j, is the $q_j + 1$ highest bidder's valuation if $|N_j| \geq q_j + 1$; or $p_j = 0$ when $|N_j| \leq q_j$.

For bidder i, her payoff u_i can be described as follows,

$$u_i(a_i, \mathbf{a}_{-i}, t_i) = \begin{cases} t_i - p_{a_i} & \text{if } t_i \geq p_{a_i}, \\ 0 & \text{otherwise.} \end{cases}$$

We can see that market j's equilibrium price p_j is determined by the types of bidders who choose to bid at market j and the quantity of the goods selling in market j. One may notice that the q_j and $q_j + 1$ highest bidder's valuations at market j may possibly be the same. In that case, we assume that bidder i always has a successful bid if $t_i > p_{a_i}$, and has a successful bid with probability $\frac{q_{a_i} - \left|\{i | i \in N,\ a_i = j \text{ and } t_i > p_{a_i}\}\right|}{\left|\{i | i \in N,\ a_i = j \text{ and } t_i = p_{a_i}\}\right|}$ (and an unsuccessful bid with probability $1 - \frac{q_{a_i} - \left|\{i | i \in N,\ a_i = j \text{ and } t_i > p_{a_i}\}\right|}{\left|\{i | i \in N,\ a_i = j \text{ and } t_i = p_{a_i}\}\right|}$) if $t_i = p_{a_i}$. By this assumption, a bidder whose valuation is equal to the market equilibrium price should have no surplus no matter whether she has acquired a unit of the goods or not.

Example 1 *Suppose there are five bidders 1, 2, 3, 4, 5 and two markets A and B. If bidder 1, 2 and 3 have decided to go to market A and bidder 4 and 5 go to market B. If bidders' valuations are 5, 4, 4, 6, 8 respectively and market A has two units to sell and market B has one to sell. The outcome in market A will be bidder 1 successfully purchases a unit of the goods, and bidder 2 and 3 each has a half chance to get a unit of the goods. The equilibrium price in market A is 4 so bidder 1 has a surplus of 1 and both bidder 2 and 3 has no surplus. Bidder 5 will has a successful bid with an equilibrium price 6 in market B.*

3 Symmetric Mixed-strategy Bayesian Nash Equilibrium

Each bidder's decision on market choice depends on her speculation about the chance of a successful bid, as well as the price to pay. A static market-choice Bayesian game can be denoted by a quadruple $(N, M, \langle u_i \rangle, \mathbf{F})$. A pure strategy for bidder i is a function $s_i(t_i) : T \to M$, where for each type $t_i \in T$, $s_i(t_i)$ specifies the action from the feasible market set M. A mixed strategy for bidder

i is a mapping $\sigma_i : T \rightarrow \Delta(M)$, where $\Delta(M)$ is the set of probability distributions over M. We study symmetric Bayesian equilibria (both pure-strategy and mixed-strategy) of this game and we represent those equilibria as a pure strategy $s(.)$, or a mixed strategy $\sigma(.)$.

Given a pure-strategy $s(.)$, the expected payoff to bidder i with type t_i when other bidders use pure strategy s and i chooses action a_i is

$$U_i(a_i, s, t_i) = \sum_{\mathbf{t}_{-i} \in T^{n-1}} u_i(a_i, \mathbf{a}_{-i}(s, \mathbf{t}_{-i}), t_i) F_i(\mathbf{t}_{-i}),$$

where $\mathbf{a}_{-i}(s, \mathbf{t}_{-i}) = (s(t_1), \ldots, s(t_{i-1}), s(t_{i+1}), \ldots, s(t_n))$ denotes the action profile chosen by other bidders when their valuation vector is $\mathbf{t}_{-i}$ and their actions are composed with the strategy s.

Similarly, the expected payoff to a bidder i with type t_i when other bidders use a given mixed strategy σ and i chooses action a_i is

$$U_i(a_i, \sigma, t_i) = \sum_{\mathbf{t}_{-i} \in T^{n-1}} \sum_{\mathbf{a}_{-i} \in M^{n-1}} u_i(a_i, \mathbf{a}_{-i}, t_i) \boldsymbol{\gamma}(\mathbf{a}_{-i}, \sigma, \mathbf{t}_{-i}) F_i(\mathbf{t}_{-i}),$$

where $\boldsymbol{\gamma}(\mathbf{a}_{-i}, \sigma, \mathbf{t}_{-i})$ is the probability distribution over $\mathbf{a}_{-i}$ induced by the types of the rest of bidders ($\mathbf{t}_{-i}$) and composed with the strategy σ. An action a_i is a v_k-type bidder's best response to the strategy σ being played by other bidders if $U_i(a_i, \sigma, v_k) \geq U_i(a'_i, \sigma^*, v_k)$, $\forall a'_i \in M$. A strategy σ^* comprises a symmetric Bayesian equilibrium if $\sigma^*(t_i)$ is a best response of bidder i, for each type $t_i \in T$ to the strategy σ^* being played by other bidders. Thus we know that the following condition holds:

$$U_i(a_i, \sigma^*, t_i) \geq U_i(a'_i, \sigma^*, t_i),\ \forall a'_i \in M,\ a_i \in \mathbf{supp}(\sigma^*(t_i)).$$

In addition, we also address a bidder's expected payoff in terms of the following form

$$U_i(a_i, \sigma, t_i) = \sum_{v \in T \cup \{0\}, v < t_i} prob(p_{a_i} = v \mid \sigma)(t_i - v).$$

3.1 Existence of Symmetric Equilibrium

In this section, we aim to prove that every market is included in every bidder's symmetric equilibrium strategy with a positive probability. First we give a lemma to show that if a certain market is excluded from a certain type bidder's equilibrium strategy, then it is also excluded from the equilibrium strategies of bidders with higher types.

Lemma 2 *For any symmetric equilibrium strategy $\sigma(\cdot)$, if there exists a type $v_{\widehat{k}}$ such that either $\widehat{k} = 1$ or $\widehat{k} > 1$ and $\mathbf{supp}(\sigma(v_k)) = M$ for any $k \in \{1, \ldots, \widehat{k}-1\}$, then $\mathbf{supp}(\sigma(v_k)) \subseteq \mathbf{supp}(\sigma(v_{\widehat{k}}))$ for any $k \in \{\widehat{k}+1, \ldots, h\}$.*

Proof. Consider the case that $\widehat{k} = 1$. Suppose that there exists a market j such that $j \in M$ and $j \notin \mathbf{supp}(\sigma(v_1))$. Since $j \notin \mathbf{supp}(\sigma(v_1))$, we know that

$$U_i(j, \sigma, v_1) \leq U_i(j', \sigma, v_1)$$
$$\Longrightarrow prob(p_j = 0 \mid \sigma) \leq prob(p_{j'} = 0 \mid \sigma)$$

for $\forall\ j' \in \mathbf{supp}(\sigma(v_1))$.

If a bidder i with private type v_2 takes an action $a_i = j$, her expected payoff is

$$\begin{aligned}
U_i(j, \sigma, v_2) &= prob(p_j = 0 \mid \sigma)v_2 + prob(p_j = v_1 \mid \sigma)(v_2 - v_1) \\
&= prob(p_j = 0 \mid \sigma)v_2 \\
&\leq prob(p_{j'} = 0 \mid \sigma)v_2 \\
&< prob(p_{j'} = 0 \mid \sigma)v_2 + prob(p_{j'} = v_1 \mid \sigma)(v_2 - v_1) \\
&= U_i(j', \sigma, v_2)
\end{aligned}$$

The probability of $p_j = v_1$ is zero because no v_1-type bidder appear at market j implies equilibrium price can not be v_1. Also we know $prob(p_{j'} = v_1 \mid \sigma) > 0$ because $j' \in \mathbf{supp}(\sigma(v_1))$. $U_i(j, \sigma, v_2) < U_i(j', \sigma, v_2)$ implies that j is strictly dominated by j' for any bidder with private type v_2. Thus $j \notin \mathbf{supp}(\sigma(v_2))$. Using this argument, we know that a v_k-type bidder i's expected payoff by taking action j can be described as follows:

$$\begin{aligned}
U_i(j, \sigma, v_k) &= prob(p_j = 0 \mid \sigma)v_k + \sum_{v \in T, v < v_k} prob(p_j = v \mid \sigma)(v_k - v_1) \\
&= prob(p_j = 0 \mid \sigma)v_k \\
&\leq prob(p_{j'} = 0 \mid \sigma)v_k \\
&< prob(p_{j'} = 0 \mid \sigma)v_k + \sum_{v \in T, v < v_k} prob(p_{j'} = v \mid \sigma)(v_k - v_1) \\
&= U_i(j', \sigma, v_k),
\end{aligned}$$

for $\forall\ j' \in \mathbf{supp}(\sigma(v_{k-1}))$. We have proven the case $\widehat{k} = 1$.

Now consider $\widehat{k} > 1$. Suppose that there exists a market j such that $j \in M$ and $j \notin \mathbf{supp}(\sigma(v_{\widehat{k}}))$. For a bidder i with private type $v_{\widehat{k}+1}$, we know

$$\begin{aligned}
U_i(j, \sigma, v_{\widehat{k}+1}) &= \sum_{v \in T \cup \{0\}, v < v_{\widehat{k}+1}} prob(p_j = v \mid \sigma)(v_{\widehat{k}+1} - v) \\
&= \sum_{v \in T \cup \{0\}, v < v_{\widehat{k}}} prob(p_j = v \mid \sigma)(v_{\widehat{k}+1} - v) + prob(p_j = v_{\widehat{k}} \mid \sigma)(v_{\widehat{k}+1} - v_{\widehat{k}}) \\
&= \sum_{v \in T \cup \{0\}, v < v_{\widehat{k}}} prob(p_j = v \mid \sigma)(v_{\widehat{k}+1} - v) \\
&\leq \sum_{v \in T \cup \{0\}, v < v_{\widehat{k}}} prob(p_{j'} = v \mid \sigma)(v_{\widehat{k}+1} - v) \\
&< U_i(j', \sigma, v_{\widehat{k}+1}),
\end{aligned}$$

for $\forall\ j' \in \mathbf{supp}(\sigma(v_{\widehat{k}}))$. Using the same argument reputedly, we can infer that $U_i(j,\sigma,v_k) < U_i(j',\sigma,v_k)$ for $\forall k \in \{\widehat{k}+1,\ldots,h\}$. Q.E.D. ■

If a market is excluded from some bidders' equilibrium strategies, the above lemma leads to a paradox: Since all higher type bidders will not bid at this market, it becomes more attractive to higher type bidders. We will prove that every market, under symmetric equilibrium strategy, must be given a positive possibility of participation.

Proposition 3 *For any symmetric Bayesian Nash equilibrium σ in a market-choice game,* $\mathbf{supp}(\sigma(t)) = M$ *for any $t \in T$.*

Proof. First, we consider the case where $t = v_1$. By Lemma 2 we know that $j \notin \mathbf{supp}(\sigma(v_k))$ for $k \in \{2,\ldots,h\}$ if $j \notin \mathbf{supp}(\sigma(v_1))$. It means that if a bidder with the lowest type does not bid at market j, then no bidder will bid at market j. However, for a bidder i who deviates the strategy σ to choose market j to bid will receive a full surplus of her valuation t_i, with probability one. That makes $a_i = j$ strictly dominate other strategy. Therefore we know that all markets should be included in a v_1-type bidder's equilibrium strategy, so $M \subseteq \mathbf{supp}(\sigma(v_1))$. Now we check the case of $t = v_2$. Using the same argument we know a market j has only v_1-type bidders if $j \notin \mathbf{supp}(\sigma(v_2))$. A v_k-type bidder i's expected payoff is

$$U_i(j',\sigma,v_k) = prob(p_{j'} = 0 \mid \sigma)v_k + \sum_{v\in T, v<v_k} prob(p_{j'} = v \mid \sigma)(v_k - v),$$

where $j' \in \mathbf{supp}(\sigma(v_k))$. If a v_k-type bidder i deviates and uses strategy $a_i = j$, her expected payoff will be

$$U_i(j,\sigma,v_k) = prob(p_j = 0 \mid \sigma)v_k + [1 - prob(p_j = 0 \mid \sigma)](v_k - v_1),$$

which is strictly larger than the expected payoff under equilibrium strategy. This contradiction leads up to that $M \subseteq \mathbf{supp}(\sigma(v_2))$. We can use the same argument to apply to the cases where $t_i = v_2, v_3, \ldots$. Q.E.D. ■

We have learnt that a symmetric Bayesian equilibrium must be a mixed-strategy equilibrium. In the next section we will show how to calculate a mixed-strategy equilibrium.

4 Algorithm of Solving Equilibrium

In this section we have introduced an algorithm to solve the symmetric equilibria in market-choice games. Consider an equilibrium σ^* and let $\sigma^*(v_k) = (\alpha_{k,1}, \alpha_{k,2}, \ldots, \alpha_{k,m})$, where $\alpha_{k,j}$ denotes the probability that a v_k-type bidder goes to market j to bid. For any type $v_k \in T$, we know $\sum_{j=1}^{m} \alpha_{k,j} = 1$. For a bidder i who knows she has the lowest type, $t_i = v_1$, i can only gain a positive surplus only when she can purchase the good at the price of 0. So if a v_1-type bidder i

goes to market j, her expected payoff will be $U_i(j, \sigma^*, v_1) = prob(p_j = 0) \times v_1$. We know the $p_j = 0$ only when $|N_j| \leq q_j$. Thus i's best response a_i^* satisfies:

$$prob(\left|N_{a_i^*}\right| \leq q_{a_i^*}) \geq prob(|N_j| \leq q_j),\ \forall j \in M. \tag{1}$$

Let $\alpha_j = \sum_{k=1}^{h} \alpha_{k,j} f(v_k)$ be the probability that a typical bidder who plays strategy σ^* bids at market j. If bidder i chooses to bid at market j, she knows $prob(|N_j| \leq q_j) = \sum_{x=0}^{q_j-1} \binom{n-1}{x}(1-\alpha_j)^{n-x-1}(\alpha_j)^x$. Therefore $prob(|N_j| \leq q_j) = prob(|N_{j'}| \leq q_{j'})$ for j, $j' \in \mathbf{supp}(\sigma^*(v_1))$. We know that $\mathbf{supp}(\sigma^*(v_1)) = M$. Thus we have $m-1$ equations with m unknowns $a_1, \ldots, a_m$. We also know $\sum_{j=1}^{m} \alpha_j = 1$. The solutions of these simultaneous $(n-1)th$-degree polynomial equations in m unknowns can be very complex. However, we can prove that there is a unique solution.

Once the average participation probability for market j has been solved, we can recursively solve $\sigma^*(v_1)$, $\sigma^*(v_2)$, $\ldots$ by comparing the expected payoff of type $v_2, v_3, \ldots$ bidders in different markets. Finally, $\sigma^*(v_h)$ can be solved by calculate the difference between $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ and $\sigma^*(v_1), \sigma^*(v_2), \ldots, \sigma^*(v_{h-1})$.

5 Conclusion

We study an auction game in which simultaneous English auctions take place at several separate markets with an exogenous fixed supply of homogenous goods available for sale in each market. We find all bidders use a mixed-strategy in which all markets are chosen by a positive probability under symmetric Bayesian equilibrium. We know the symmetric equilibrium is unique and can be solved by a recursively process in terms of a typical bidder with a possible private type. We believe that the implication of this research provides applied settings in a range of areas, such as career choice, constituency nomination strategy, adoption of innovation, and so on. We are also interested to bring them into this model in future work.

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