Does natural rate variation matter? Evidence from New Zealand

Michael Kirker

Economics Department, Reserve Bank of New Zealand,

2 The Terrace, Wellington, 6011, New Zealand.

Tel: +64 4 471 3792, Fax: +64 4 473 1209.

email address: michael.kirker@rbnz.govt.nz

Abstract

Most small new Keynesian models are estimated on demeaned data, implicitly assuming time-invariant natural rates. Using a small open-economy new Keynesian model of New Zealand, I examine the implications this assumption has on the model's properties. From the Bayesian estimation, I find noticeable variation in the natural rates: the natural real rate of interest, inflation target, potential output, and neutral real exchange rate. I compare a time-varying version of the model against a time-invariant version. My results show that ignoring time-varying natural rates can impact on the model's fit to the data, its structural persistence, and impulse responses.

Keywords: natural rates; inflation target; open economy

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1 Introduction

The natural rates (or equilibrium values) are an important structural feature in all small new Keynesian models. Typically in the literature, small new Keynesian policy models are estimated on demeaned data (see Cho and Moreno 2006, Liu 2006, and Buncic and Melecky 2008 as examples). Using demeaned data implicitly assumes that the natural rates are constant over the entire sample period. While this might be a reasonable assumption for very short samples, it has the potential to bias the dynamics encapsulated within the model.

Often, these natural rates are important concepts from the perspective of monetary policy. For example, the natural real rate of interest provides a benchmark level for the real interest rate. If monetary policy sets the real interest rate above the natural rate, contractionary pressure will be placed upon the economy. And the further away the real interest rate is from the natural real rate, the more pressure there is. [1]

In this paper, I examine the bias created by assuming time-invariant nautral rates within the framework of a small open-economy new Keynesian model. I do this by comparing a version of the model in which the natural rates are time varying, and another version in which the natural rates are time invariant. I use the term 'natural rates' to collectively refer to the natural real rate of interest, inflation target, potential output, and neutral real exchange rate.

[1] See Archibald and Hunter (2001) for a further discussion on the natural real rate from a policy perspective.

Recent work by Sbordone (2007) and Benati (2008) found that allowing for persistence in the inflation target reduced the degree of intrinsic persistence within a hybrid new Keynesian Phillips curve.[2] Which means that if we use demeaned inflation data, variation in the inflation target could be misconstrued as intrinsic persistence in the inflation dynamics, suggesting monetary policy must work harder to control inflation. If this situation holds for inflation, it could also hold for other natural rates.

Early research has attempted to estimate the natural real rate of interest for New Zealand using various techniques (see Plantier and Scrimgeour 2002, Basdevant et al 2004, and Schmidt-Hebbel and Walsh 2007 as examples). The results from this earlier research suggests there has been significant time variation in the natural real rate of interest.

And in 1999, the Policy Target Agreement (PTA) — that encapsulates the agreed objectives of monetary policy — was amended to include the objective of avoiding unnecessary instability in output, interest rates, and the exchange rate while pursuing of its price stability objective. At that time, Reserve Bank governor Dr Brash said the change did not alter the way the Reserve Bank implemented monetary policy, and it was in line with the way policy had been developing (Christchurch Press 1999). This suggests that historically, the implicit inflation target (that is consistent with the actions and behaviour of the Reserve Bank) may have differed from

2 Sbordone (2007) concludes that inflation deviations from trend show no intrinsic persistence once persistence in the trend is allowed for.

the (explicit) midpoint of the target band. Therefore, assuming a constant inflation target or simply taking the midpoint of the inflation target band (both common approaches) may not be an appropriate approach when modelling the New Zealand economy.

The closed economy literature has developed many techniques for estimating time-varying natural rates.[3] These range from simple filtering techniques, to semi-structural approaches (e.g. Laubach and Williams 2003), to fully structural DSGE estimates (e.g. Giammarioli and Valla 2004). Small open new Keynesian models that do not rely on demeaned data, typically employ a DSGE style approach and express the natural real rate of interest in structural terms (such as in Gali and Monacelli 2005, and Monacelli 2005).

However, as Giammarioli and Valla (2004) point out, structural estimates of the natural rates are often highly dependent upon the microfoundations of the model. Though lacking in economic intuition, semi-structural estimates that combine a small macro model with statistical filtering techniques, tend to produce more robust estimates of the natural rates.

I use a modified semi-structural version of the new Keynesian model by Berg et al (2006) that allows for the natural rates to be time-varying or time-invariant. In the time-varying version of the model the natural rates are modelled as random-walk processes, and can be estimated using the Kalman smoother. I examine the bias of

[3] Typically these approaches only focus on potential output and the natural real rate of interest.

working with time-invariant natural rates by comparing the dynamics and fit of the two versions of the model.

The estimation of the natural rates from the time-varying model shows noticeable trends over our sample period (1992Q1-2008Q1). Therefore, using a gaps model with demeaned data (where the natural rates are time-invariant) maybe a poor assumption for New Zealand. Comparing the time-varying and time-invariant models reveals that using demeaned data does result in noticeable changes to the model's fit and dynamics. According to the posterior odds ratio, the time-varying model is a significantly better fit. The distributions of the persistence parameters for the Phillips Curve and Monetary Policy rule are slightly lower in the time-varying model. Finally, the impulse responses of the time-varying model are generally smaller and less persistent than the impulse responses of the time-varying model.

The remainder of this paper is organised as follows. Section 2 outlines the model used in this analysis. Section 3 gives details on the data and estimation method. Section 4 discusses the results from the estimation of the model and the robustness tests. Section 5 provides a brief conclusion.

2 The model

The model used in this analysis is adapted from the small open-economy new Keynesian model developed by Berg et al (2006). I follow the structure of their model closely, with a few notable exceptions. I make adjustments to the Phillips curve so that it is expressed in terms of annualised inflation, rather than in a combination of annual and annualised inflation (this brings it into line with the majority of models in the literature). I also redefine the real exchange rate (and associated parameters) such that an appreciation of the New Zealand dollar increases the real exchange rate.

The one significant difference between my model and those in the literature is that it explicitly model time variation in the natural real rate of interest, inflation target, growth rate of potential output, and neutral real exchange rate. I refer to this model as the *time-varying model*. Using the Kalman smoother the historical paths of the unobservable natural rates in the model can be backed-out. In addition, I develop a restricted *time-invariant model*, in which the natural rates mentioned above are assumed to be constant over time. Using these two models, I am able to isolate the effects of allowing for time-varying natural rates within the model.

2.1 Domestic economy

I specify an IS relationship for the output gap in equation 1. Similar specifications of the IS relationship can be found in Svensson (2000), Leitemo and Söderström (2005), and Buncic and Melecky (2008).

$$x_t = (1-\beta_x)E_t x_{t+1} + \beta_x x_{t-1} - \beta_r \tilde{r}_{t-1} - \beta_z \tilde{z}_{t-1} + \beta_f x_t^f + \varepsilon_t^x \tag{1}$$

The IS relationship states that today's output gap (x_t) is dependent upon its expected value next period ($E_t x_{t+1}$) and its lagged value (x_{t-1}). The output gap is also affected by last period's real interest rate gap ($\tilde{r}_{t-1}$), last period's real exchange rate gap ($\tilde{z}_{t-1}$), and by the foreign demand conditions (the foreign output gap, x_t^f).

The real interest rate gap ($\tilde{r}_t$), real exchange rate gap ($\tilde{z}_t$), and output gap (x_t), are defined as the difference between the observed levels (r_t, z_t, and y_t) and their natural rates (the natural real rate of interest r_t^*, neutral real exchange rate z_t^*, and potential output y_t^*).[4]

$$\tilde{r}_t = r_t - r_t^* \tag{2}$$
$$\tilde{z}_t = z_t - z_t^* \tag{3}$$
$$x_t = y_t - y_t^* \tag{4}$$

For the time-varying model, the natural real rate of interest (r_t^*) and the neutral level of the real exchange rate (z_t^*), both follow a random walk process.

$$r_t^* = r_{t-1}^* + \varepsilon_t^{r*} \tag{5}$$
$$z_t^* = z_{t-1}^* + \varepsilon_t^{z*} \tag{6}$$

For the time-invariant model, r_t^* and z_t^* are constant at the rates $\bar{r}$ and $\bar{z}$ respectively (equations 5 and 6 are replaced with 5' and 6').

$$r_t^* = \bar{r} \tag{5'}$$

$$z_t^* = \bar{z} \tag{6'}$$

In the model, the level of potential output (y_t^*) grows at an annualised rate of g_t^*.

[4] The real interest rate is calculated using the Fisher equation: $r_t = i_t - E_t \pi_{t+1}$.

$$400(\Delta y_t^*) = g_t^* \tag{7}$$

The growth rate (g_t^*) above follows a random walk process in the time-varying model:

$$g_t^* = g_{t-1}^* + \varepsilon_t^{g*} \tag{8}$$

And is constant (at the rate $\bar{g}$) in the time-invariant model:

$$g_t^* = \bar{g} \tag{8'}$$

Unlike the specification of potential output commonly used in semi-structural models (see Berg et al 2006, Laubach and Williams 2003, and Giammarioli and Valla 2004 as examples), the specification used here does not distinguish between permanent and temporary shocks to the change in potential output. Fuentes and Gredig (2007) found that in the case of Chile, this simpler specification cannot be statistically rejected in favour of a more general specification that distinguishes between permanent and temporary shocks (and nests the specification used by Berg et al 2006). [5]

Inflation within the domestic economy is modelled using a hybrid new Keynesian Phillips curve (9) in a similar fashion to Svensson (2000) and Giordani (2004). The current level of annualised inflation (π_t) depends on expected future inflation

[5] A version of the model that did explicitly model both permanent and temporary shocks was estimated. However, the model had very weak identification of which shocks were permanent, and which were temporary. As such, the results were very similar.

$(E_t \pi_{t+1})$, the previous period's inflation rate (π_{t-1}), last period's output gap (x_{t-1}), and the change in import prices (captured by the change in the real exchange rate, Δz_t).

$$\pi_t = (1-\alpha_\pi)E_t\pi_{t+1} + \alpha_\pi \pi_{t-1} + \alpha_x x_{t-1} - \alpha_z \Delta z_t + \varepsilon_t^\pi \tag{9}$$

To complete the core structure of the domestic economy and anchor inflation to a stable level, a monetary policy reaction function is defined using the following forward-looking Taylor-type rule that features interest rate smoothing (10):

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i)\left[r_t^* + E_t \pi_{t+1}^T + \gamma_\pi E_t \left(\pi_{t+4}^A - \pi_{t+4}^{A,T}\right) + \gamma_x x_t\right] + \varepsilon_t^i \tag{10}$$

The monetary authority moves the nominal interest rate (i_t) away from its natural rate — the natural rate of nominal interest ($r_t^* + E_t \pi_{t+1}^T$) — in response to deviations in expected annual inflation from its annual target $\left(E_t(\pi_{t+4}^A - \pi_{t+4}^{A,T})\right)$, and the contemporaneous output gap.

The (annualised) inflation target (π_t^T) represents the implicit inflation target of the monetary authority — the inflation target that is consistent with the actions and behaviour of the monetary authority. In the time-varying model, the implicit inflation target follows a random walk process. [6]

$$\pi_t^T = \pi_{t-1}^T + \varepsilon_t^{\pi T} \tag{11}$$

[6] Sbordone (2007) uses a random walk process for trend inflation, which is very similar to the inflation target in my model.

And in the time-invariant model, the inflation target is constant.

$$\pi_t^T = \bar{\pi} \tag{11'}$$

The annual inflation rate (π_t^A), and annual inflation target $\left(\pi_t^{A,T}\right)$ are given by the following identities:

$$\pi_t^A = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \tag{12}$$
$$\pi_t^{A,T} = (\pi_t^T + \pi_{t-1}^T + \pi_{t-2}^T + \pi_{t-3}^T)/4 \tag{13}$$

2.2 *Exchange rate relationship*

Typically in the literature, open-economy models rely on an uncovered interest rate parity (UIP) condition to model the exchange rate. However, most empirical studies find the UIP condition to be a poor fit to actual data (see Froot and Thaler 1990). Therefore, like Berg et al (2006), I use a modified UIP condition (given in equation 14).

$$z_t = z_{t+1}^e + (r_t - r_t^f + \rho_t^*)/4 + \varepsilon_t^z \tag{14}$$

Where z_t is the real exchange rate, z_{t+1}^e is the expected real exchange rate next period, ρ^* is the equilibrium risk premium, and ε_t^z is a shock to the risk premium.

Expectations (z_{t+1}^e) are a weighted average between rational expectations ($E_t z_{t+1}$) and adaptive expectations (z_{t-1}) as defined in equation 15. When $\delta_z = 1$, expectations are fully rational and we obtain the standard UIP condition.

$$z_{t+1}^e = \delta_z E_t z_{t+1} + (1 - \delta_z) z_{t-1} \tag{15}$$

The equilibrium risk premium (ρ_t^*) is defined as:

$$\rho_t^* = 4[z_t^* - \delta_z E_t z_{t+1}^* - (1-\delta_z) z_{t-1}^*] - r_t^* + r_t^{f*} \tag{16}$$

Where z_t^* is the neutral level of the real exchange rate, r_t^* is the neutral real interest rate of the domestic economy, and r_t^{f*} is the foreign neutral real interest rate.

2.3 Foreign economy

Following Berg et al (2006), the foreign economy is modelled as a closed-economy version of the domestic economy. I also detrend all the foreign observable series, therefore, all of the natural rates in the foreign economy equations are equal to zero.

$$x_t^f = (1-\beta_x^f) E_t x_{t+1}^f + \beta_x^f x_{t-1}^f - \beta_r^f r_{t-1}^f + \varepsilon_t^{x,f} \tag{17}$$

$$\pi_t^f = (1-\alpha_\pi^f) E_t \pi_{t+1}^f + \alpha_\pi^f \pi_{t-1}^f + \alpha_x^f x_{t-1}^f + \varepsilon_t^{\pi,f} \tag{18}$$

$$i_t^f = \gamma_i^f i_{t-1}^f + (1-\gamma_i^f)\left(\gamma_\pi^f E_t \pi_{t+4}^{A,f} + \gamma_x^f x_t^f\right) + \varepsilon_t^{i,f} \tag{19}$$

3 Estimation

For each model the parameters are estimated using Bayesian estimation. Bayesian estimation has a number of advantages including allowing us to compare the fit of models, and allowing us to use prior information we may have to help ‘pin down’ weakly identified parameters. I used the IRIS toolbox for MATLAB to compute the Bayesian estimation of the two models. [7]

[7] The IRIS toolbox was created by Jaromír Beneš, and is available from: `http://www.iris-toolbox.com/`

3.1 Data

I estimate the models using quarterly data for New Zealand (the domestic economy) and the United States (a proxy for the foreign economy) from 1992Q1 to 2008Q1. This sample range captures New Zealand's inflation targeting history after allowing for a short (but volatile) disinflationary period after the adoption of inflation targeting in late 1989.

For the domestic economy (New Zealand), output (y_t) is measured as the log of seasonally-adjusted real GDP. The nominal interest rate (i_t) is defined as the 90-day bank bill rate. Inflation (π_t) is an annualised measure derived from the consumer price index (CPI).

CPI data prior to 1999Q3 is adjusted to exclude components that relate to interest charges. This ensures a consistent definition of CPI over the sample. Also, I correct for an outlier in 2001Q1 when the government moved from charging market-rate to income-based rents on state housing (thereby producing a sharp one-off fall in the rent component of CPI for that quarter). [8]

I use the United States to proxy the foreign economy, and detrend all the observable series using an HP filter. [9] The foreign output gap (x_t^f) is calculated on the log of seasonally-adjusted real GDP. The foreign interest rate (i_t^f) is calculated using the

[8] The model was also estimated using the unadjusted CPI series. However, this had very little impact on the results.

[9] see Buncic and Melecky 2008 for a similar use of the United States as a proxy for the foreign economy.

90-day bank bill rate. And foreign annualised inflation (π_t^f) is calculated using core CPI (excluding food and energy). [10]

The real exchange rate series (z_t) is derived as:

$$z_t = 100 \times log\left(e_t \times \frac{CPI_t^{NZ}}{CPI_t^f}\right)$$

Where e_t is the nominal exchange rate (\$US/\$NZ), and the CPI measures for New Zealand (CPI_t^{NZ}) and the United States (CPI_t^f) are the measures used in the inflation rate calculations above.

3.2 Identification

Because I have extended the small open-economy model to explicitly model the natural rates, the model may not identify, or only weakly identify, the value of some parameters (the model has 11 shock terms but only seven observable series). In these cases, our priors become important in anchoring the parameter values. To test the identification of the parameters in the model, I use the Fisher information matrix.

The Fisher information matrix examines the full information likelihood function to test if it is weakly or not identified in each dimension. The analysis reveals that for the time-varying model, there are 11 (out of 29) dimensions in which the full information likelihood is weakly identified, and none that are unidentified. I look

[10] This core CPI measure is the inflation measure that the Federal Reserve focuses on.

at the parameters that have a particularly large weighting contributing to each of the weakly identified dimensions, and those that have relatively large weights in multiple dimensions to identify those parameters for which the model will struggle to identify. From this criteria, the following parameters can be considered to be weakly identified:

> The sensitivity of the output gap to the real exchange rate gap (β_z), the sensitivity of the output gap to the foreign demand conditions (β_f), monetary policy's responsiveness to deviations in annual inflation from its target (γ_π), the standard deviation of shocks to the growth rate of potential output (σ_{g*}), the standard deviation of shocks to the inflation target ($\sigma_{\pi T}$), and the standard deviation of shocks to the neutral real exchange rate (σ_{z*}).

It is therefore, particularly important that we understand the impact our choice of priors for these parameters have on the parameter estimates.

3.3 Priors

Table 1 provides a summary of the prior distributions that I specify for the Bayesian estimation of the time-varying model.[11] The choice of priors for the main economic equations were influenced by a range of previous models of the New Zealand economy and other small open economies. For the natural rate equations, priors

[11] A table of the priors and posteriors for the time-invariant model along with plots of the posterior distributions for both models is available in the working paper version. Where the two models share the same parameters, the same priors were used.

were formed based on analysis of the time series used in the Reserve Bank's Forecasting and Policy System (FPS) model.

For the domestic Phillips curve, I set the prior on α_x, the effect of the output gap on inflation, equal to 0.1. In other small open-economy literature with similar Phillips curve specifications, this parameter value ranges in size from 0.0011 (Buncic and Melecky 2008) to 0.22 (Harjes and Ricci 2008). It is therefore difficult to form a tight prior on what an appropriate value should be.

I set the prior on γ_π, monetary policy's responsiveness to expected inflation deviations from target, equal to 2. This value matches that used by Berg et al (2006) and Harjes and Ricci (2008). However, the prior on γ_π is more defuse than in Harjes and Ricci (2008). Also, I set the sensitivity to the output gap (γ_x) equal to 1 (higher than in Berg et al 2006, and Harjes and Ricci 2008), noting that the Reserve Bank is required to give consideration to the output gap under clause 4b of its Policy Targets Agreement.

The priors chosen for the domestic parameters that relate to the foreign economy (e.g. sensitivity of the output gap to foreign demand conditions, β_f) and real exchange rate (e.g. sensitivity of domestic inflation to an increase in the real exchange rate, α_z) are set relatively low compared to other literature. The motivation for this is based on the fact that although the United States is a relatively important trading partner for New Zealand, its share of exports and imports is not large enough to provide a significantly good proxy for all of New Zealand's trade. Therefore, it is expected that New Zealand would be fairly insensitive to the United States in our small and simple model. If we were to replace the United States data in the model with composite data of all of New Zealand's trading partners, it is likely that larger (more sensitive) priors would be more appropriate.

To find suitable means for the priors on the standard deviation of shocks to the annualised growth rate of potential output (σ_{g*}), and the standard deviation of shocks to the inflation target ($\sigma_{\pi T}$), I fit the random walk equations from the model to the potential output series used in FPS and the midpoint of the inflation target series. The choice of means for the priors ($\sigma_{g*} = 0.1$ and $\sigma_{\pi T} = 0.15$) are close to the values obtained from the estimations.

Finally, I set the mean of the prior on σ_{z*}, the standard deviation of shocks to the neutral real exchange rate, equal to 1. It is difficult to find other estimates to inform the prior, but I expect that the shocks to the real exchange rate would be significantly larger than the shocks to to neutral real exchange rate. Choosing $\sigma_{z*} = 1$ sets the ratio of standard deviations between neutral real exchange rate shocks and real exchange rate shocks (σ_{z*}/σ_z) to 0.5.

4 Results

4.1 Posteriors

The means and 90 percent confidence intervals from the posterior distributions of the time-varying model parameters are also presented in table 1. Overall, the means of the posteriors seem reasonable for the parameters in the model. Below, I draw

Table 1
Priors and posteriors of the time-varying model

Parameter		Prior Mean	Prior S. D.	Prior Dist.	Prior Range	Posterior Mean	Posterior 90% CI
				Domestic economy			
β_x	IS: weight on lag	0.4	0.15	Beta	[0,1]	0.726	[0.618, 0.834]
β_r	IS: effect of real int rate gap	0.1	0.05	Gamma	[0,∞)	0.065	[0.018, 0.108]
β_z	IS: effect of real exch rate gap	0.01	0.005	Gamma	[0,∞)	0.006	[0.002, 0.011]
β_f	IS: effect of foreign output gap	0.05	0.015	Gamma	[0,∞)	0.042	[0.022, 0.061]
α_π	PC: weight on lag	0.5	0.15	Beta	[0,1]	0.194	[0.099, 0.284]
α_x	PC: effect of the output gap	0.1	0.035	Gamma	[0,∞)	0.056	[0.028, 0.083]
α_z	PC: effect of change in real exch rate	0.075	0.05	Gamma	[0,∞)	0.031	[0.003, 0.061]
γ_i	MP: smoothing parameter	0.7	0.2	Gamma	[0,∞)	0.778	[0.688, 0.874]
γ_π	MP: responsiveness to inflation	2	0.5	Gamma	[0,∞)	2.148	[1.263, 2.983]
γ_x	MP: responsiveness to output gap	1	0.3	Gamma	[0,∞)	0.808	[0.439, 1.154]
				Exchange rate			
δ_z	UIP: forward looking weight	0.75	0.15	Gamma	[0,∞)	0.475	[0.403, 0.542]
				Foreign economy			
β_x^f	IS: weighting on lag	0.4	0.15	Beta	[0,1]	0.54	[0.436, 0.655]
β_r^f	IS: effect of real int. rate gap	0.1	0.05	Gamma	[0,∞)	0.033	[0.006, 0.060]
α_π^f	PC: weight on lag	0.5	0.15	Beta	[0,1]	0.172	[0.086, 0.252]
α_x^f	PC: effect of the output gap	0.1	0.035	Gamma	[0,∞)	0.047	[0.024, 0.073]
γ_i^f	MP: smoothing parameter	0.7	0.2	Beta	[0,1]	0.775	[0.704, 0.850]
γ_π^f	MP: responsiveness to inflation	1.75	0.5	Gamma	[0,∞)	1.844	[1.004, 2.517]
γ_x^f	MP: responsiveness to output gap	1	0.3	Gamma	[0,∞)	1.295	[0.853, 1.738]
				Standard deviations of shocks			
σ_x	Std dev: output shock	0.5	Inf	Inv. G.	[0,∞)	0.363	[0.235, 0.487]
σ_π	Std dev: inflation shock	0.5	Inf	Inv. G.	[0,∞)	0.616	[0.393, 0.826]
σ_i	Std dev: interest shock	0.5	Inf	Inv. G.	[0,∞)	0.415	[0.260, 0.577]
σ_{g*}	Std dev: growth rate shock	0.1	Inf	Inv. G.	[0,∞)	0.097	[0.028, 0.172]
$\sigma_{\pi T}$	Std dev: inflation target shock	0.15	Inf	Inv. G.	[0,∞)	0.098	[0.035, 0.164]
σ_{r*}	Std dev: neutral real rate shock	0.2	Inf	Inv. G.	[0,∞)	0.241	[0.048, 0.525]
σ_z	Std dev: real exch. rate shock	2	Inf	Inv. G.	[0,∞)	1.066	[0.535, 1.551]
σ_{z*}	Std dev: neutral real exch. Rate shock	1	Inf	Inv. G.	[0,∞)	1.319	[0.266, 2.426]
σ_x^f	Std dev: foreign output shock	0.5	Inf	Inv. G.	[0,∞)	0.18	[0.124, 0.238]
σ_π^f	Std dev: foreign inflation shock	0.5	Inf	Inv. G.	[0,∞)	0.379	[0.248, 0.508]
σ_i^f	Std dev: foreign interest shock	0.5	Inf	Inv. G.	[0,∞)	0.274	[0.185, 0.362]

attention to a few of the interesting results.

The IS relationship of the domestic economy shows a relatively high degree of persistence ($\beta_x = 0.726$), despite the prior suggesting the IS relationship was slightly more forward looking. On the other hand, the domestic Phillips curve shows very little persistence ($\alpha_\pi = 0.194$) in annualised inflation.

The domestic economy is relatively insensitivity towards to the foreign economy and the real exchange rate. The mean of the posterior distributions for these parameters in both the IS relationship and Phillips curve are lower than the mean of the prior distributions.

The standard deviation of shocks to the inflation target ($\sigma_{\pi T} = 0.098$) is lower than my prior of 0.15. This suggests the inflation target has been relatively more stable than the midpoint of the inflation target band.

Unexpectedly, the posteriors mean of the standard deviation of shocks to the neutral real exchange rate ($\sigma_{z*} = 1.319$) is larger than the standard deviation of shocks to the UIP condition ($\sigma_z = 1.066$).

4.2 Fit

Table 2 reports the log marginal likelihoods from the two models. From these values, we can computer the Bayes factor of 107098 ($= exp(11.58)$). [12]

The Bayes factor reveals that the fit of the time-varying model to the data is pre-

[12] The Bayes factor comparing model M_i to M_j is defined as: $BF_{ij} = p(y|M_i)/p(y|M_j)$.

Table 2
Model fit

Model	Log marginal likelihoods $log(p(y\|M_i))$
Time varying	-303.94
Time invariant	-315.52

ferred over the fit of the time-invariant model (the Bayes factor is greater than one). The value of the Bayes factor also gives us the strength of the factor's preference for the time-varying model. The Bayes factor suggests that in order for us to choose the time-invariant model over the time-varying model, our prior probability for the time-invariant model would need to be 107098 times greater than our prior probability for the time-varying model.

Such a large Bayes factor seems almost implausible given the similarities between the two models. However, according to Sims (2003), when the set of models being compared is too sparse, the results from the Bayesian model comparison will tend to be implausibly sharp. This 'misbehaviour' is a result of the discrete collection of models serving as a poor proxy for a continuous parameter space. If we were to introduce more models that spanned the region between the time-varying and time-invariant models, it is unlikely the Bayes factor would favour a particular model so strongly above all the rest.

4.3 Model estimated natural rates

Using the Kalman smoother, it is possible to extract the paths of the unobservable variables within a model. I use this approach to find the time-varying estimates of the natural real rate of interest, inflation target, output gap (driven by the models estimate of potential output), and neutral real exchange rate. The results are plotted in figure 1.

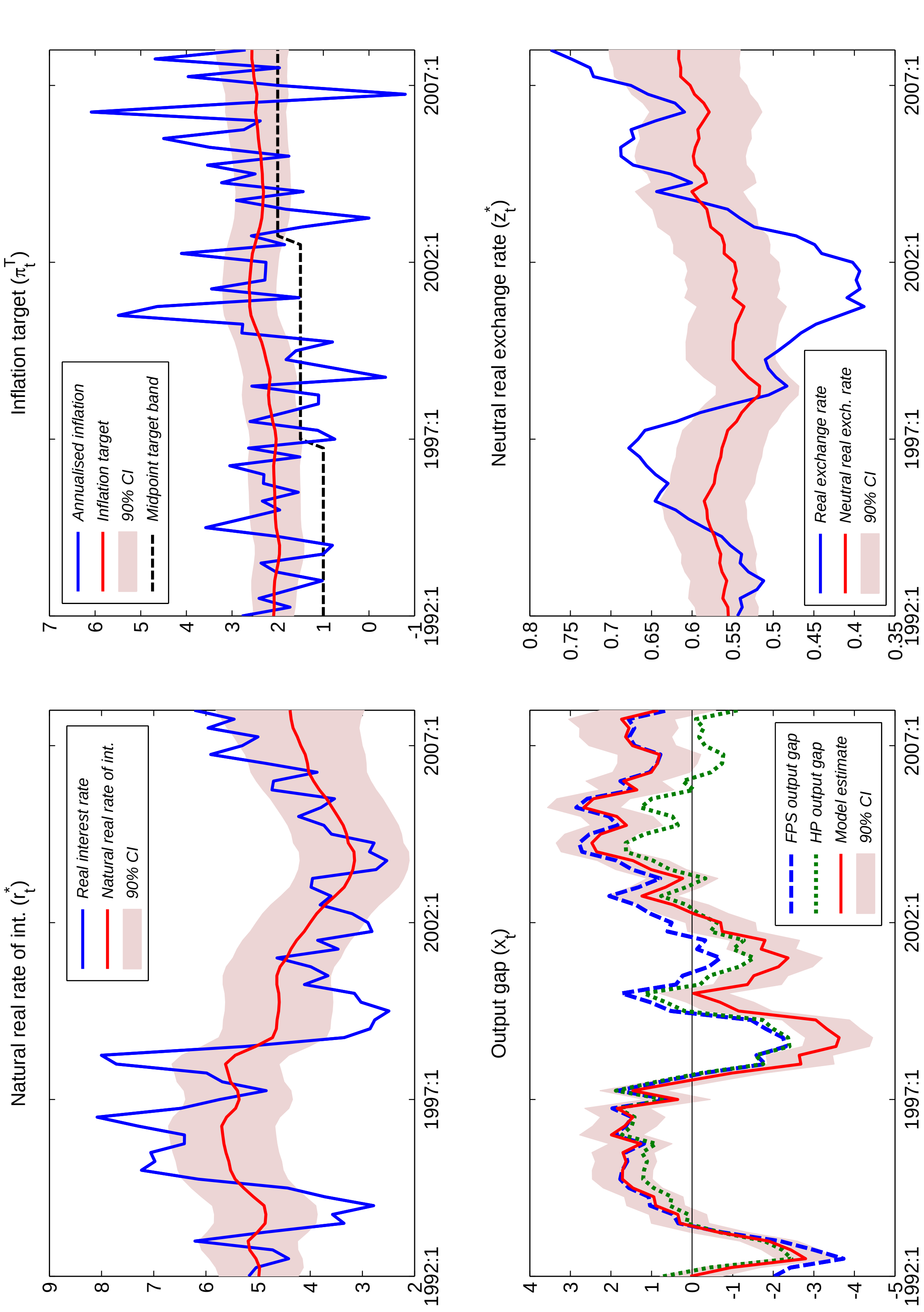
The estimated natural real rate of interest in figure 1 shows that between 1992 and 1998 the natural real rate of interest was relatively stable around 5.25 percent. However, after 1998 the natural real rate began trending downwards, reaching almost 3 percent in 2004, before rising to its current level of around 4.3 percent.

Between 1992 and 2004, the trend and magnitude of the natural real rate estimate broadly match those found by Basdevant et al (2004) who used several statistical and semi-structural approaches to estimate the natural real rate over this time period. However, my results contrast with Schmidt-Hebbel and Walsh (2007) who, using a semi-structural approach with a backwards-looking close-economy new Keynesian model, estimate that the natural real rate of interest has been relatively stable around 5 percent (with only small, but persistent deviations) between 1986 and 2006.

The rising natural real rate since 2004 suggests that if policymakers had assumed a time-invariant natural real rate, they would have over predicted the contractionary strength of monetary policy.

According to the time-varying model, the (annualised) inflation target of Reserve Bank has been relatively stable over the whole sample period (figure 1). Prior to 2000, the Reserve Bank was targeting annualised inflation at a rate slightly above 2 percent. Around 2000, the inflation target increased to a new rate of 2.5 percent,

Fig. 1. Model estimates of the natural rates



where it has stayed close to for the remainder of the sample period.

Figure 1 also shows the output gap estimate from the time-varying model alongside the output gap used in FPS and the output gap estimated by an HP filter. Between 1998 and 2003, the model's estimates of the output gap is significantly lower than the output gap used in FPS. However, across the rest of the sample, it suggests an output gap very similar to that used in FPS. The calculation of the output gap used in FPS is done over a longer data set than used in this paper. This explains the discrepancy between the estimate of the output gap at the start of our sample from both the model and HP filter (close to zero percent), and the output gap according to the measure used for FPS (-2 percent).

The neutral real exchange rate is the exchange rate at which no pressure is put on the domestic output gap or inflation rate (see figure 1). The neutral rate is estimated to have been steadily increasing since 2002. At the beginning of 2008, the real exchange rate between New Zealand and the United States was slightly over 20 percent above its neutral level.

4.4 Dynamics

I investigate the implications that time-varying natural rates have on a model's dynamics using two main measures. First, I examine how the structural persistence parameters differ between the time-invariant and time-varying models. This follows similar analysis from previous literature which investigates inflation persistence and its implication for monetary policy. And second, I compare the impulse responses of the time-varying and time-invariant models to a variety of shocks.

4.4.1 Persistence Parameters

Recent international literature (such as Sbordone 2007 and Benati 2008) has suggested that the inflation persistence we usually observe in a hybrid new Keynesian Phillips curve may overstate the true level of structural inflation persistence. Sbordone (2007) defines structural inflation persistence as the persistence that is a structural feature of the economy, and not a consequence of the way monetary policy has been conducted. This distinction is important for policymakers as trend inflation (or the inflation target) is ultimately determined by policymakers's actions and therefore, is not taken as given when setting monetary policy. For the model, I extend the analysis to compare the posterior distribution of the three parameters that control persistence in the domestic economy's inflation rate (α_π), output gap (β_x), and interest rate (γ_i), to assess if allowing for time-variation in the natural rates has any significant impact on the persistence within the model.

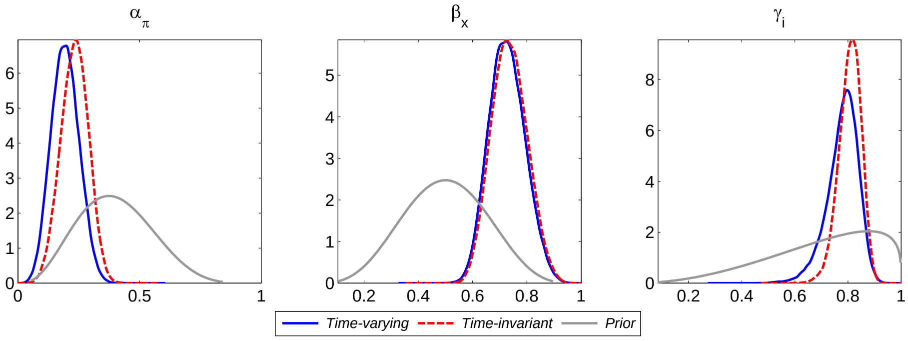
The first panel in figure 2 shows the posterior distributions of the persistence parameter α_π, the weighting on the lagged inflation term in the Phillips curve. We can see that the posterior distribution for the time-varying model is slightly lower than the posterior distribution under the time-invariant model. This means that allowing for time-variation in the natural rates, decreases the persistence of inflation, but only slightly. While this result are similar to Sbordone (2007), the time-varying model still shows some low levels of persistence.

The difference between my results and those in Sbordone (2007) comes from the fact that trend inflation (the inflation target) in Sbordone (2007) tracks actual inflation quite closely, much more so than the case for New Zealand (see figure 1). Therefore, the inflation target is my model has significantly less persistence that that in Sbordone (2007). By not having as much persistence in the inflation target, deviations in inflation from its target will naturally be more persistent.

From the plot in the second panel of figure 2 we can see that the posterior distribution of β_x (the persistence parameter in the IS relationship) is virtually identical under the time-varying and time-invariant models. This means the persistence of the output gap in the model is not significantly affected by the introduction of time-varying natural rates.

Finally, in the third panel of figure 2, I examine the posterior distribution of the interest rate smoothing parameter (γ_i). The results show that the posterior distribution of γ_i under the time-varying model is centered slightly lower than the distribution under the time-invariant model. Therefore, allowing for time-variation in the natural rates reduces the persistence of interest rates. In terms of the Taylor-type rule, the monetary authority giving slightly less emphasis to interest rate smoothing and is slightly more aggressive to deviations in inflation from its target and the output gap, when the natural rates are allowed to vary with time.

Fig. 2. Posterior distributions of persistence parameters



4.4.2 Impulse Responses

To computer the impulse responses, I subject the models to a one unit shock to the major domestic and foreign shock terms. Figure 3 show the impulse responses of both models to domestic shocks (domestic output, inflation, interest rate, and real exchange rate shocks), while figure 4 shows the impulse responses to foreign shocks (foreign output, inflation, and interest rate shocks).

The results show that for both domestic and foreign shocks, the impulse responses of the time-varying model generally have less persistence and lower magnitudes compared to the time-invariant model. However, the individual results can vary widely depending upon which shock and variable is examined.

In response to a domestic output shock (figure 3), the output gap, interest and exchange rate impulses of the time-varying model show a more cyclical response. The domestic inflation impulse of the time-invariant model show a rather unusual response, with inflation initially driven lower. This appears to be a result of the time-invariant model's Phillips curve being more forward-looking (and thus more sensitive to monetary policy's response), less sensitive to the domestic output gap, and more sensitive to the real exchange rate. For all four variables, the impulse responses for the time-varying model displays less persistence to the shocks by converging to zero faster.

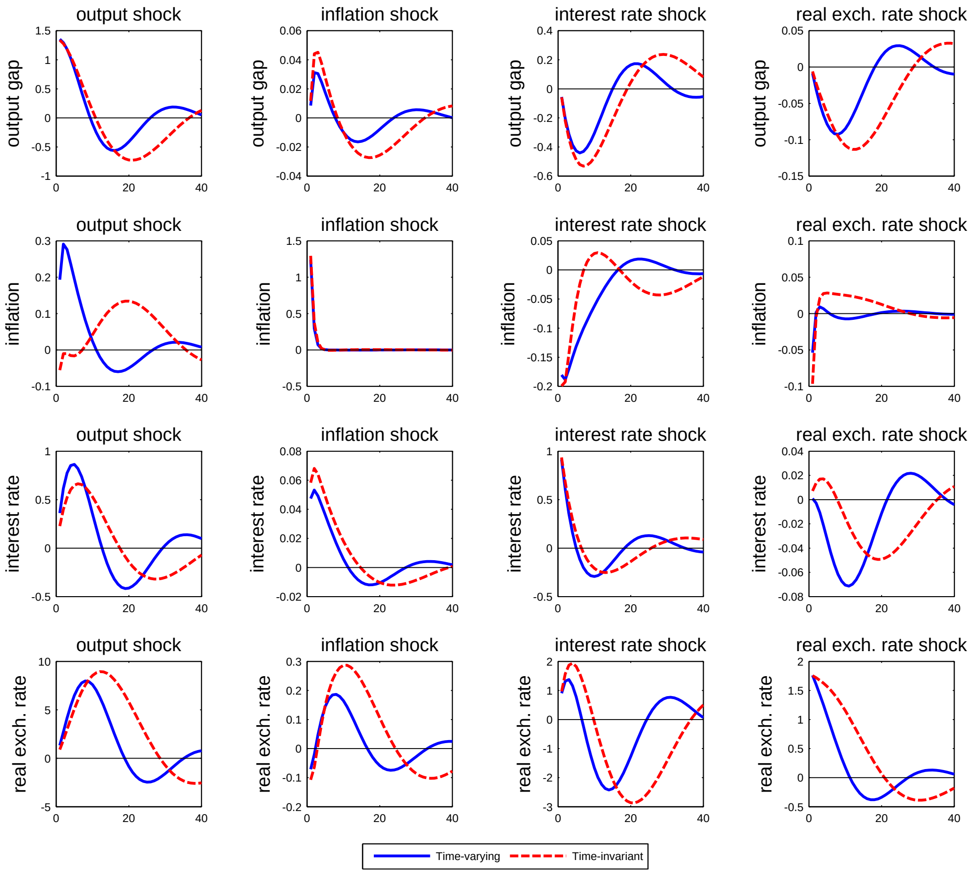
For both domestic inflation and interest rate shocks, the time invariant model shows more persistence in the domestic output gap and real exchange rate. The time-varying model is back close to the natural rate after 40 quarters, while the time-invariant model takes longer to converge. The magnitudes of the cyclical peaks in the domestic output gap and real exchange rate responses to these shocks are also noticeably larger in the time-invariant model.

For a one unit shock to the real exchange rate, the domestic economy has very different impulse responses under the two models. The time-varying model has noticeable less persistence, and is close to converging after 40 periods. The actual paths of the impulses also show a more cyclical profile under the time-varying model.

In response to foreign shocks, the impulse responses of the domestic economy are mixed (see figure 4). For a foreign output shock, the time-varying model shows a more pronounced cyclical pattern in domestic output, interest rates and the real exchange rate. Despite this, both the time-varying and time-invariant models show similar persistence to the shock, with both models back close to their natural rates after 40 periods.

In response to a foreign inflation shock, both models show very different impulses. However, the magnitudes of the impulses reveal that the domestic economy under

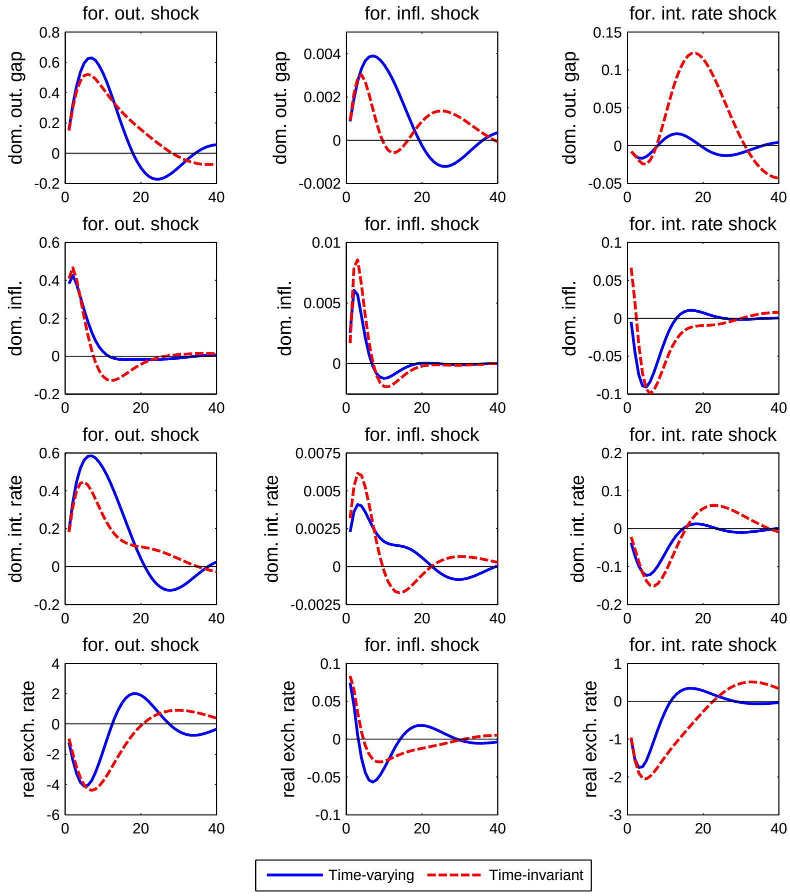
Fig. 3. Impulse responses to domestic shocks



both models is fairly insensitive to foreign inflation shocks. This is mainly a result of the limited effect the foreign inflation shock has on the real exchange rate (one of the two channels the foreign economy affects the domestic economy). Foreign interest rates move to offset the inflation, resulting in very little chance in the foreign real interest rate (and hence, a limited impact on the real exchange rate).

The impulse responses of the two models show a significant difference for the domestic output gap's response to a foreign interest rate shock. However, the time-invariant model shows more persistence, taking over 40 periods to converge for all the domestic variables. For all of three foreign shocks, the impulses of the foreign

Fig. 4. Impulse responses to foreign shocks



economy are virtually identical under both models. [13]

4.5 *Robustness*

Whenever estimating unobservable natural rates, there is a large amount of uncertainty related to model and data specification. To address these issues I perform a number of robustness checks to the model. Specifically, I test how robust the model is to: (i) changes in priors; (ii) changes in annual inflation expectations; and (iii) an

[13] See the Discussion Paper version of this paper for the impulse graphs of the foreign economy.

alternative output gap measure.

Overall, the qualitative results are fairly robust to the three tests I perform. However, they still highlight the large uncertainty we face when estimating natural rates.

4.5.1 Priors

I double the initial priors on the standard deviation of shocks to the natural real rate (σ_{r*}) and to the standard deviation of shocks to the inflation target ($\sigma_{\pi T}$). This gives us priors for the mean standard deviations of $\sigma_{\pi T} = 0.3$, and $\sigma_{r*} = 0.4$. These larger priors will be less restrictive on the track of the natural real rate of interest and the inflation target, allowing them to vary more with time.

4.5.2 Expectation data

Rational expectations are an important part of new Keynesian models. The assumption is made that agents within the model know the structure of the model, and believe it to be true. To test how robust the natural rate estimates are to the expectations of the model, I take the Reserve Bank's survey measure of expected annual inflation in one years time (π_t^S), and impose the restriction that the model's rational expectations over the next four quarters must be consistent with the survey measure:

$$\pi_t^S = E_t \pi_{t+4}^A = E_t(\pi_{t+4} + \pi_{t+3} + \pi_{t+2} + \pi_{t+1})/4$$

4.5.3 *Alternative output gap*

To test how sensitive the other natural rate estimates are to the estimate of potential output, I replace the model's endogenously estimated output gap with the output gap used in the Reserve Bank's FPS model (equations 4 and 7, and the output series y_t become redundent). The use of an observable output gap series should also assist the model in the identification of other parameters in other parts of the model.

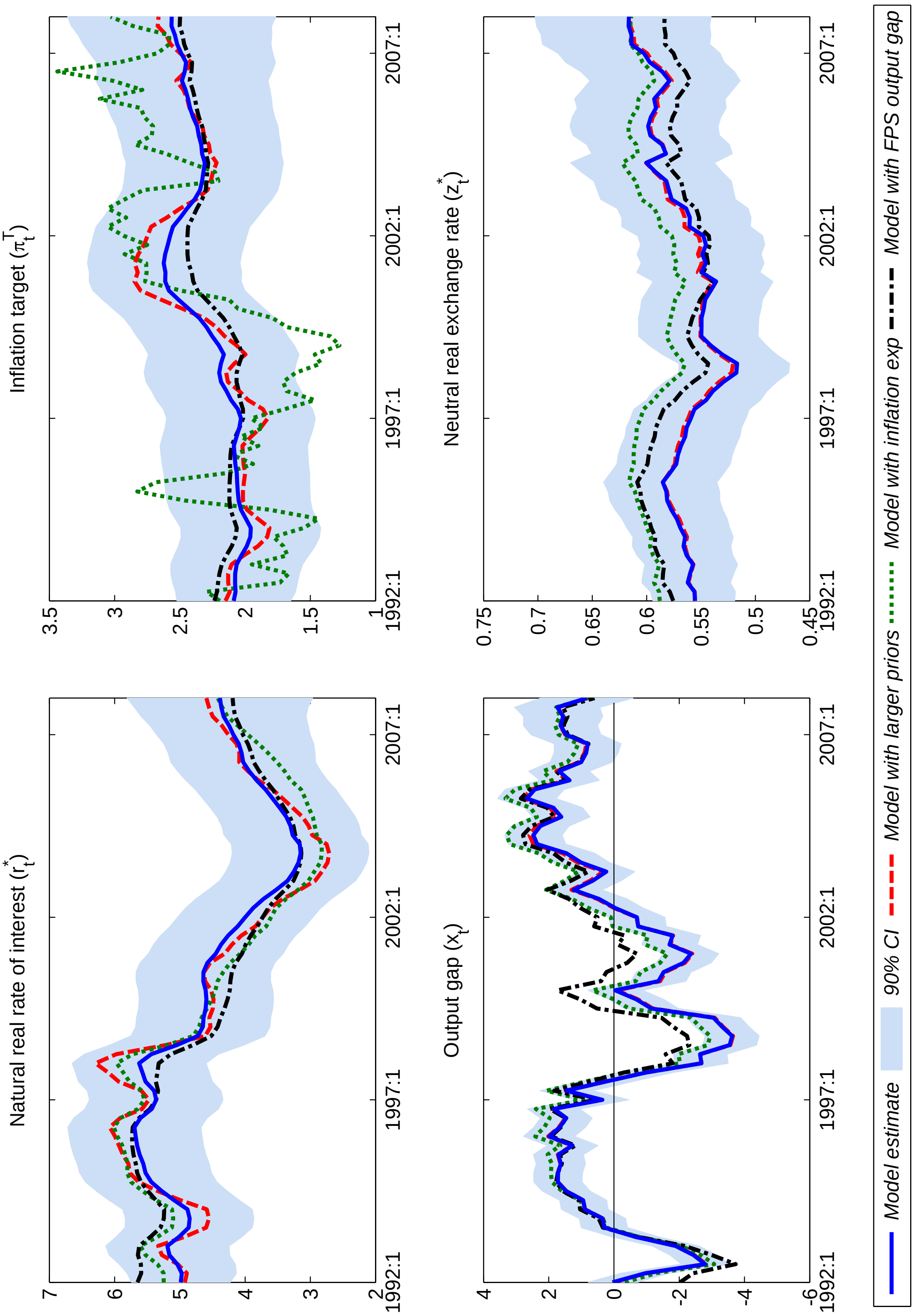
4.5.4 *Results of the robustness tests*

Figure 5 shows the natural rate estimates under the three robustness tests, alongside the original estimates from the time-varying model.

The plots of the natural real rate of interest in figure 5 shows that the model's estimate is relatively robust to the three tests. The robustness tests suggest some upside risk between 1992 and 1998, and some downside risk between 1998 and 2006 for the original estimate. The natural real rate estimate shows the largest sensitivity to the use of larger priors. However, all three robustness tests fall within the 90 percent confidence intervals, and are generally close to the original estimate.

The robustness test results for the inflation target (figure 5) suggest the model's inflation target track (π_t^T) is fairly robust to the use of more disperse priors and the alternative output gap. On the other hand, the inflation target is rather sensitive to the expectations of inflation within the model. The inflation target found under this robustness tests shows significantly more volatility than the original model estimate.

Fig. 5. Robustness of the natural rate estimates



The model's estimate of the output gap is robust to use of large priors for the standard deviation of shocks to the natural real rate and inflation target (figure 5). Using the survey data of inflation expectations has a noticeable impact on the output gap estimate, producing an output gap very close to the boundary of the 90 percent confidence interval of the original estimate between 1998 to 2002.

Figure 5 also shows the robustness of the neutral real exchange rate (z_t^*) to the various tests. From the graph we can see that using larger priors for the standard deviation of shock to the natural real rate and inflation target has a negligible effect. However, using the survey of inflation expectations data produces a neutral real exchange rate that is the higher than the original estimate between 1992 and 2007 (most of the sample period). The FPS output gap robustness test suggests the neutral real exchange rate has actually declined over the sample range. During the whole sample period, all three robustness tests fall within the 90 percent confidence intervals of the original neutral real exchange rate track.

5 Conclusion

I estimated (using Bayesian techniques) a small open-economy new Keynesian model with time-varying parameters for the natural real rate of interest, inflation target, potential output, and neutral real exchange rate. This time-varying model was compared to a time-invariant model in which the natural rates are constant.

The posterior odds ratio shows that the time-varying model is a significantly better fit to the data than the time-invariant model. Using the Kalman smoother, it is possible to back out estimates of the time-varying natural rates. The time-varying model suggests there has been noticeable variation in the natural real rate, potential output, and neutral real exchange rate over the sample period.

I found that allowing for time variation in the natural rates only slightly decreases the persistence parameter for the nominal interest rate and inflation processes (but not the output gap). I also found the difference between the impulse responses of the time-varying and time-invariant model can be quite large, with the time-varying model generally displaying less persistence to shocks. However, the individual results vary greatly for the different shocks considered.

The robustness tests I performed on the model show that the estimates are fairly robust to the use of larger priors on the standard deviations of shocks to the natural real rate of interest and the inflation target, and the use of FPS's output gap. On the other hand, the estimated natural rates do show some sensitivity to the use of inflation expectations data.

Overall, the analysis suggests that assuming the natural rates are time invariant in a small open-economy new Keynesian model (such as when working with demeaned data) does bias the dynamics of the model and its fit to the data. Therefore, model designers and users should consider what implicit assumptions they are making about the natural rates when using or analysing new Keynesian models based on demeaned or detrended data.

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