Entry and Location Choice with Network Formation*

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Abstract

We examine a firm's entry and location decision when all firms, including incumbents and an entrant, have an opportunity to form a pairwise link with mutual consent. We firstly show that when an entrant is allowed to form a link, it has an incentive to distort its location from a welfare viewpoint in order to form a link with each of the incumbents. Second, the existing link formed by incumbents cannot be a device for entry deterrence, especially when the marginal change of cost-reducing effect generated by a link formation scarcely depends on the number of links.

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1 Introduction

We examine a firm's entry and location decision when all firms, including incumbents and an entrant, have an opportunity to form a network. In particular, the main issue of our paper is: how does the network formation affect the possibility of entry and the entrant's location?

Let us consider a simple problem based on a linear city model with a uniform distribution of consumers. Suppose two retail shops are already located at the two extreme ends of a line city, respectively. They have the same retail technology, compete in prices, and obtain some positive profits. Now, there is an outsider that has a plan to open a shop in the city. Then, the two incumbent retail shops can collaborate with each other in order to deter its entry. If they collaborate with each other, they can lower their resale cost by sharing some skills or knowledge about technologies and the market situation, etc. However, there is a possibility that once the outsider opens a shop, it is also allowed to collaborate with one of the two incumbent retail shops. In such a case, is the outsider able to open its shop in the city? If so, where would it like to be located in the city?

Using the standard setting of a Hotelling model, we provide a curious answer to these questions. Our answer is: the collaboration between the two incumbent retail shops cannot always deter entry. Furthermore, once the outsider opens the shop in the city, its location cannot be always centered in the city: its location is distorted towards the location of one of the two retail shops.

Let us formally restate the answer. We define a collaboration of firms by a *pairwise link* between firms. Then, we firstly show that when an entrant is allowed to form a link and the link formation needs mutual consent, it has an incentive to distort its location from a welfare viewpoint in order to form a link with each of the incumbents. Second, we show that the existing link formed by incumbents cannot be a device for entry deterrence, especially when the marginal change of cost-reducing effect generated by a link formation scarcely depends on the num-

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ber of links. The "when" condition for the second result implies that a competition authority should care about an information concerning the change of the collaboration effect in order to evaluate the incumbents' alleged entry-deterrence strategies.

Our results are obtained by two main effects. *The cost-reducing effect*, which is a direct effect generated by the network formation, and *the price competition effect*, which is an indirect effect generated by the competition in the retail market, are the two driving forces. Then, the results come from the two driving forces with the following three prerequisites. First, while the cost-reducing effect depends on the number of the pairwise collaborations, it does not depend on the distance between the two firms that form a pairwise collaboration. The improvement of information technology seems to justify this prerequisite. Second, two firms decide whether to collaborate or not with a myopic view. That is, each of them are unable to foresee how the third firm react if it agrees (or does not agree) on the collaboration. Third, all the firms cannot use price discrimination across consumers.

According to one of our result, an entrant's location is distorted towards the location of one of the incumbents with a prerequisite that the cost-reducing effect generated by firms' collaboration does not depend on the distance between the two firms that form a collaboration. As mentioned above, this prerequisite is justified by the improvement of information technology such as the prevalence of internet. Then, the result implies that the improvement of information technology tends to amplify the agglomeration of firms. This implication may be supported by an empirical work by Gasper and Glaeser (1996). Gasper and Glaeser (1996) shows that the improvement of information technology can be a complement to face-to-face interactions because of the increase in frequency of contacts. Their result suggests that the improvement of information technology can amplify the agglomeration of firms because they need more face-to-face interactions than ever.

Needless to say, there is a large body of literature on a firm's location choice in a spacial model, following Hotelling (1929) and d'Aspremont et al. (1979).[1] Most of the existing literature explain firms' location choices by relying on the elements such as transportation cost, the distribution of resources, scale economies, production externalities, etc. Our analysis does not rely on these elements but another important element in a real business world; the collaboration among firms.

In fact, only a few studies examine the relationship between the collaboration among firms and their location choices. (For example, using a repeated-game framework, Friedman and Thisse (1993) show that partial tacit collusion on price fosters firms' agglomeration (i.e., minimum product differentiation).) Our study differs from the existing literature in that we consider not only the threat of potential entry but also the possibility of collaborative networks that is endogenously formed in a spacial model. In particular, we allow all firms (i.e., not only incumbents but also an entrant) to have an opportunity to form a network through pairwise links with mutual consent. Then, we restrict our attention to the effect of network formation on an entrant's behavior, especially the feasibility of entry and an entrant's location.[2]

The endogenous formation of collaboration among firms can be formulated in several ways. For example, Bloch (1995) examines the coalition formation procedure where each firm sequentially offers the proposal of coalition to other firms with which it wants to collaborate. Our approach to collaboration among firms is a network structure through the formation of pairwise links between two firms. The procedure of network formation with pairwise links assumes that mutual consent is required to form a link between two firms.[3] Some studies already apply the network formation with pairwise links to collaboration among firms. Goyal and Joshi (2003) address the relationship between a firm's incentive to form a network and the nature of market competition. Song and Vannetelbosch (2007) also use a strategic network model to analyze how government policies affect the stability and efficiency of networks of R&D collaboration among firms in different countries. As far as we know, however, our paper is the first attempt to examine the effect of endogenous formation of networks upon the

[1] See Gabszewicz and Thisse (1992) for a survey on the location theory.

[2] None of studies deal with an endogenous formation of collaborative networks in a spacial model, except for Okumura (2009). While Okumura examines a formation of collaborative networks in the framework of a circular model, his main focus is the characterization of networks in equilibrium, given the number of firms and each firm's location. Contrary to his study, this paper deals with the relationship between endogenously formed networks and the opportunity of entry and a firm's location choice.

[3] For the basic notions and applications of network formation, see Goyal (2007) and Jackson (2008). Many studies also apply the network formation with pairwise links to several economic issues. For example, Ellison and Fudenberg (1995) examine the effect of communication structure on a consumer's purchasing behavior.

feasibility of entry and an entrant's location.

Next section describes the framework of the model. Section 3 characterizes the pairwise stable networks, taking a potential entrant's entry and its location as given. Section 4 then analyzes the effect of network formation on the feasibility of entry and an entrant's location. Concluding remarks are in Section 5.

2 The Model

We examine a linear market model à la Hotelling with two incumbent firms, firm 1 and firm 2, and one potential entrant firm, firm e. Consumers are uniformly distributed on the interval $[0,1]$ with mass one. Firm 1 is already located at the left end, 0, while firm 2 is at the right end, 1. On the other hand, once firm e decides to enter the market by incurring an entry sunk cost F, it can choose its location $a \in (0,1)$.[4]

All the firms sell the same physical good whose reservation value for each consumer is v.[5] Each consumer buys a single unit of good from one of the firms. Then, the utility of a consumer located at θ is represented by

$$u(p_i;\theta) = v - (\theta - s_i)^2 t - p_i,$$

where s_i is the location of firm i ($i = 1,\ 2,\ e$), t is the parameter of quadratic transportation cost incurred by the consumer, and p_i is the price set by firm i.

A departure of our model from the existing literature is that we allow the firms to be in collaboration with other firms in the market. That is, a firm has an opportunity to form a *pairwise* collaborative link with one of the other firms in the market. In particular, we assume that firm i's marginal cost c_i is represented by a function of the number of the collaborative links. That is,

$$c_i = c(\eta_i(g)),$$

where $\eta_i(g)$ is the number of pairwise links firm i has with the other firms in the *network* g. Here, a "network" represents a structure of the pairwise links in a given market. For example, if we see a network $\widetilde{g}$ where firm e forms a pairwise link with firm 1 after it enters the market and firm 1 also forms a pairwise link with firm 2, we state that $\eta_1(\widetilde{g}) = 2$, $\eta_2(\widetilde{g}) = 1$, and $\eta_e(\widetilde{g}) = 1$. We also assume that a pairwise link has an effect to lower marginal cost of production as follows:

$$c(0) > c(1) > c(2).$$

That is, a firm's marginal cost is strictly decreasing in the number of pairwise links. Note that the marginal cost depends only on the number of links, not on the distance between the two firms that form a link.[6]

The timing of the game is as follows. In the first stage, firm e decides whether or not to enter the market by incurring an entry sunk cost F. Once it decides to enter, it also determines its location a. In the second stage, all the firms in the market determine their pairwise links, which means that each firm's marginal cost is decided in this stage. In other words, this stage gives a network structure among firms in the market. In the next section, we describe the details of the procedure of pairwise link formation. Then, in the third stage, the firms compete in price (i.e., Bertrand competition) and consumers buy the goods from one of them in the market.

Finally, we denote two types of the difference of the marginal cost by $\Delta_0 \equiv c(0) - c(1)$ and $\Delta_1 \equiv c(1) - c(2)$. That is, Δ_0 (resp. Δ_1) represents the marginal effect of the link firstly (resp. secondly) formed. Then, we make an assumption on the degree of the differences of the marginal cost.

Assumption 1 *(i)* $\Delta_0 < \left(3/\left(4+\sqrt{17}\right)\right)t$, *(ii)* $\Delta_1 < (3/4)\,t$.

The two conditions of Assumption 1 guarantee an interior solution in which all three firms can obtain a positive demand and a positive profit after firm e's entry in equilibrium. In fact, we restrict our attention to the interior solution in the following analysis. This assumption is important, because we examine an oligopoly with firms that may have different technology through a link formation. In fact, when there exists a cost difference among firms, the monopolization of a firm that has a cost advantage is easy to occur in a Hotelling model.[7] That is, if a cost-disadvantaged firm locates too near to a

[4] As shown below, we can restrict our attention to an interior solution for firm e's location choice in the setting of our model.

[5] We assume that v is sufficiently high so that all consumers buy the good from one of the firms.

[6] Here, we also exlcude the possibility of spillovers. That is, the marginal cost does not depend on the number of links that a *neighbor* (i.e., a firm which consists of a pair to a particular firm) has. See Section 5 on this point.

[7] Matsumura and Matsushima (2009) and Meza and Tombak (2009) examine the effect of cost differentials among firms in a Hotelling model, sometimes called an *asymmetric duopoly* model.

cost-adavanatged firm, it cannot obtain a positive demand. The conditions of Assumption 1 guarantee a positive demand and profit for each of three firms in equilibrium, irrespective of whatever type of network is established.

3 Pairwise Stable Networks

3.1 Bertrand competition

We examine the outcome of Bertrand competition in the third stage, given firm e's entry and its location choice in a type of network structure. Suppose firm e enters and locates at a. Restricting our attention to an interior solution in which all three firms can obtain a positive demand and a positive profit in equilibrium, we denote a consumer who is indifferent between buying from firm 1 or firm e by $\theta_{1e} \in (0, a)$. (See Appendix A for the check of the existence of the interior solution under Assumption 1.) Then, we have

$$\begin{aligned} v - \theta_{1e}^2 t - p_1 &= v - (a - \theta_{1e})^2 t - p_e \\ \text{or } \theta_{1e} &= \frac{p_e - p_1}{2at} + \frac{a}{2}. \end{aligned}$$

Similarly, denoting a consumer who is indifferent between buying from firm 2 or firm e by $\theta_{e2} \in (a, 1)$, we have

$$\begin{aligned} v - (\theta_{e2} - a)^2 t - p_e &= v - (1 - \theta_{e2})^2 t - p_2 \\ \text{or } \theta_{e2} &= \frac{p_2 - p_e}{2(1-a)t} + \frac{1+a}{2}. \end{aligned}$$

Then, the firms' respective demands are

$$\begin{aligned} D_1(p_1, p_e) &= \theta_{1e} = \frac{p_e - p_1}{2at} + \frac{a}{2}, \\ D_e(p_1, p_e, p_2) &= \theta_{e2} - \theta_{1e} \\ &= \frac{p_2 - p_e}{2(1-a)t} - \frac{p_e - p_1}{2at} + \frac{1}{2}, \\ D_2(p_e, p_2) &= 1 - \theta_{e2} = \frac{p_e - p_2}{2(1-a)t} + \frac{1-a}{2}. \end{aligned}$$

Each firm maximizes its profit with respect to price. Deriving the first-order conditions and rearranging them, we have the equilibrium prices as follows:

$$\begin{aligned} p_1^* &= \frac{1}{6}(4-a)c_1 + \frac{1}{3}c_e + \frac{1}{6}ac_2 + \frac{1}{2}at, \\ p_e^* &= \frac{1}{3}(1-a)c_1 + \frac{2}{3}c_e + \frac{1}{3}ac_2 + a(1-a)t, \\ p_2^* &= \frac{1}{6}(1-a)c_1 + \frac{1}{3}c_e \\ &\quad + \frac{1}{6}(3+a)c_2 + \frac{1}{2}(1-a)t. \end{aligned}$$

The equilibrium profits are represented as follows:

$$\begin{aligned} \Pi_1 &= D_1(p_1^*, p_e^*)(p_1^* - c_1) = \frac{1}{2at}(p_1^* - c_1)^2, \\ \Pi_e &= D_e(p_1^*, p_e^*, p_2^*)(p_e^* - c_e) = \frac{1}{2a(1-a)t}(p_e^* - c_e)^2, \\ \Pi_2 &= D_1(p_1^*, p_e^*)(p_2^* - c_2) = \frac{1}{2(1-a)t}(p_2^* - c_2)^2. \end{aligned}$$

When firm e does not enter the market, the equilibrium prices and profits are easily derived in a similar way. Hence, we report them when we discuss the characteristics of firm e's entry decision and its location choice in Section 4.

3.2 The formation of pairwise stable networks

We now proceed to the analysis of network structure (i.e., the formation of pairwise links) in the second stage. In this subsection, we examine the case in which firm e enters the market and locates at some point of the line, a. Since our model has a symmetric structure, we can restrict our attention to the case where $a \in \left[\frac{1}{2}, 1\right)$. The case in which firm e does not enter the market will be mentioned in Section 4.

Suppose a network structure g is established. Then, given g, the equilibrium prices and profits are rewritten by

$$p_1^*(g,a) = \frac{1}{6}(4-a)c(\eta_1(g)) + \frac{1}{3}c(\eta_e(g)) + \frac{1}{6}ac(\eta_2(g)) + \frac{1}{2}at, \quad (1)$$

$$p_e^*(g,a) = \frac{1}{3}(1-a)c(\eta_1(g)) + \frac{2}{3}c(\eta_e(g)) + \frac{1}{3}ac(\eta_2(g)) + a(1-a)t, \quad (2)$$

$$p_2^*(g,a) = \frac{1}{6}(1-a)c(\eta_1(g)) + \frac{1}{3}c(\eta_e(g)) + \frac{1}{6}(3+a)c(\eta_2(g)) + \frac{1}{2}(1-a)t \quad (3)$$

$$\Pi_1(g,a) = \frac{1}{2at}(p_1^*(g,a) - c(\eta_1(g)))^2, \quad (4)$$

$$\Pi_e(g,a) = \frac{1}{2a(1-a)t}(p_e^*(g,a) - c(\eta_e(g)))^2 \quad (5)$$

$$\Pi_2(g,a) = \frac{1}{2(1-a)t}(p_2^*(g,a) - c(\eta_2(g)))^2 \quad (6)$$

We define the following function.

$$f_i(g,a,ij) \equiv p_i^*(g+ij,a) - c(\eta_i(g+ij)) - (p_i^*(g,a) - c(\eta_i(g))), \quad (7)$$

where ij represents a direct pairwise link between firm i and j ($i,j = 1$, 2, and e, $i \neq j$). Then, $g+ij$ represents the network obtained by adding the link ij to the network g. Hence, the function $f_i(g,a,ij)$ defines the difference of firm i's price-cost margin under the network $g+ij$ from that under the network g, given any location a. Apparently, the sign of $f_i(g,a,ij)$ is the same as that of $\Pi_i(g+ij,a) - \Pi_i(g,a)$. Hence, hereafter, we use $f_i(g,a,ij)$ in order to analyze firm i's incentive to form pairwise links. In fact, we have:

1. When $f_i(g,a,ij) > 0$, firm i has an incentive to add ij to the network g. Otherwise, it has no incentive to do so.

2. When $f_i(g-ij,a,ij) < 0$, firm i has an incentive to sever ij from the network g. Otherwise, it has no incentive to do so.

At first, we obtain the following lemma. (All the proofs of lemmas and propositions are relegated to Appendix.)

Lemma 1 *Suppose $\eta_i(g) = \eta_j(g)$ where $i,j = 1,2,e$ and $i \neq j$. Then, for any $a \in [\frac{1}{2}, 1)$, we have:*

For $ij \notin g$, $f_i(g,a,ij) > 0$ and $f_j(g,a,ij) > 0$.

For $ij \in g$, $f_i(g-ij,a,ij) > 0$ and $f_j(g-ij,a,ij) > 0$.

Lemma 1 states that, as long as the number of links is the same between firm i and firm j in a given network g, the link ij must be included in g (i.e., they both have an incentive to form a link between them), irrespective of the location of an entrant, a. (We should remember that this statement holds to all the firms, including firm e.)

In our analysis, we use a concept of pairwise stability due to Jackson and Wolinsky (1996). The pairwise stability in our model is defined as follows.

Definition 1 *A network g is* ***pairwise stable*** *if*

(i) for all $ij \in g$, $f_i(g-ij,a,ij) \geq 0$ and $f_j(g-ij,a,ij) \geq 0$.

(ii) for all $ij \notin g$, if $f_i(g,a,ij) > 0$, then $f_j(g,a,ij) < 0$.

As is well-known, the pairwise stabiltiy captures the idea of mutual consent. In other words, it supposes that pairs of players can communicate and agree to form a link.

[Insert Figure 1 around here.]

Now, we try to characterize the pairwise stable networks in our model. All the possible networks are drawn in Figure 1. From the result of Lemma 1, it is easy to find that the networks g^0, g^4, g^5, and g^6 are not pairwise stable. This is because they contain firms i and j such that $\eta_i(g) = \eta_j(g)$ and the link between them is not included in the associated network.

The remaining networks, g^1, g^2, g^3, and g^7 are the candidates of pairwise stable networks. In fact, the set of pairwise stable networks depends on the degree of the effect of pairwise link on a firm's marginal cost, as is shown in the following lemma. Before reporting the lemma, we define $\Delta \equiv \Delta_0/\Delta_1$ where $\Delta_0 \equiv c(0) - c(1)$ and $\Delta_1 \equiv c(1) - c(2)$. That is, Δ represents the ratio of the marginal change of cost-reducing effect of a pairwise link. In particular, we call the case where $\Delta > 1$ the case of *decreasing marginal change of cost-reducing effect.* That is, when $\Delta > 1$, the marginal effect of the link firstly formed is larger than that of the link secondly formed. Similarly, we call the case where $\Delta < 1$ the case of *increasing marginal change of cost-reducing effect.* Then, we report the lemma.

Lemma 2 *When $a \in [\frac{1}{2}, \sqrt{3}-1)$, we have:*

(i) If $\Delta > \frac{3+a}{1-a}$, g^1, g^2, g^3, and g^7 are pairwise stable.

(ii) If $\frac{3+a}{1-a} \geq \Delta > \frac{2+a}{a}$, g^1, g^3, and g^7 are pairwise stable.

(iii) If $\frac{2+a}{a} \geq \Delta > \frac{2+a}{2}$*,* g^3 *and* g^7 *are pairwise stable.*

(iv) If $\frac{2+a}{2} \geq \Delta \geq 1-a$*,* g^7 *is pairwise stable.*

(v) If $1-a > \Delta \geq \frac{a}{2+a}$*,* g^3 *and* g^7 *are pairwise stable.*

(vi) If $\frac{a}{2+a} > \Delta \geq \frac{1-a}{3-a}$*,* g^2*,* g^3 *and* g^7 *are pairwise stable.*

(vii) If $\frac{1-a}{3-a} > \Delta > 0$*,* g^1*,* g^2*,* g^3 *and* g^7 *are pairwise stable.*

When $a \in \left[\sqrt{3}-1, 1\right)$*, the above (i), (ii), (iii), (vii) hold, and (iv) to (vi) are replaced by the followings:*

(iv') If $\frac{2+a}{2} \geq \Delta \geq \frac{a}{2+a}$*,* g^7 *is pairwise stable.*

(v') If $\frac{a}{2+a} > \Delta \geq 1-a$*,* g^2 *and* g^7 *are pairwise stable.*

(vi') If $1-a > \Delta \geq \frac{1-a}{3-a}$*,* g^2*,* g^3 *and* g^7 *are pairwise stable.*

In Lemma 2, the location of an entrant affects not only the threshold values for the range of the ratio of the marginal change of cost-reducing effect of pairwise link, but also the set of pairwise stable networks (compare *(v)* and *(v')*). The crucial point of Lemma 2 is that the set of pairwise stable networks varies, depending on the ratio of the marginal change of cost-reducing effect of pairwise links.

According to the lemma, there are multiple types of pairwise stable network. To predict which networks are likely to emerge, we adopt a dynamic process for network formation introduced by Watts (2001). In Watts' dynamic process, a link ij, irrespective of whether it is in the existing network or not, is randomly identified to be updated with uniform probability. If the link is already in the existing network, then either firm i or j can decide to sever the link. If the link is not in the network, and at least one of the two firms involved would benefit from adding it and the other would be at least as well off given the existing network, the link is added. Then, if the process reaches a fixed configuration, it must be a stable network. We should note that this dynamic process assumes a firm's myopic behavior in the sense that a firm's decision depends only on its period t profit and it does not account for what might happen in the future.[8]

Furthermore, we use the following assumption for analytical simplicity.

Assumption 2 *Before firm e enters the market, firms* 1 *and* 2 *has already formed a link between them. That is,* $g_{(0)} = g^3$ *where* $g_{(0)}$ *represents a network at the initial state.*

Assumption 2 is natural in our setting. This is because the link between firms 1 and 2 is pairwise stable when there exists only the two incumbents in the market. In addition, it is easy to verify that, when there exists a threat of potential entry, this pairwise stable network is strictly better for the two incumbents than a network that does not include the link 12 (a network without any link)), even though they are indifferent when there exists only the two incumbents in the market.

Then, we obtain our first main finding.

Proposition 1 *When* $\Delta > \frac{2+a}{2}$ *or* $\Delta < 1-a$*, the network formation process converges to* g^3*. On the other hand, when* $1-a \leq \Delta \leq \frac{2+a}{2}$*, it converges to* g^7*.*

Proposition 1 states that when the ratio of the marginal change of cost-reducing effect of pairwise links is large, irrespective of whether the marginal effect is increasing or not, the existing link between the two incumbents remains and the entrant cannot form a pairwise link with any incumbent. On the other hand, when the ratio of the marginal change of cost-reducing effect is small, all the firms, including the entrant, can form two pairwise links with other two firms (a *complete* network).

The intuition of Proposition 1 is explained by two effects generated by the formation of pairwise links. One is *the cost-reducing effect*, which is a direct effect of link formation, while the other is *the price competition effect*, which is an indirect effect generated by competition in the retail market. Let us explain it in more detail.

First, consider the case where $\Delta > \frac{2+a}{2}$. Suppose firms 1 and e are chosen at the first period, and consider firm 1's incentive for link formation. The fact that $\Delta > \frac{2+a}{2}$ means that the (marginal) cost-reducing effect of the second link is pretty smaller than that of the first link. Then, when firm 1 does not form a link with firm e, it can enjoy a higher price (and obtain a larger market share) than that with the link, as long as firm e does not have a link with firm 2. This is because firm e has a cost-disadvantage over not only firm 1 but also firm 2. On the other hand, if firm 1 forms a link with firm e, firm e becomes more efficient with a link than without it, whereas it still has a cost-disadvantage over firm 1. Indeed, we make sure of this fact by

$$\begin{aligned} & p_1^*\left(g^3+12, a\right) - p_1^*\left(g^3, a\right) \\ = & -\frac{1}{6}\left(4-a\right)\Delta_1 - \frac{1}{3}\Delta_0 < 0. \end{aligned}$$

[8] See p.335 of Watts (2001).

Since the cost-reducing effect generated by the link with firm e is small, the price competition effect can be larger than the cost-reducing effect. Hence, firm 1 has no incentive to form a link with firm e. By the same reason, we can verify that firm 2 has no incentive to form a link with firm e if it is chosen at the first period.

Second, consider the case where $\Delta < 1 - a$. In this case, the cost-reducing effect of the second link is pretty larger than that of the first link. This makes firm e cost-disadvantageous over the two incumbents if it forms a link with one of them. Again, suppose firms 1 and e are chosen at the first period. Then we have

$$\begin{aligned} & p_e^*\left(g^3 + 12, a\right) - p_e^*\left(g^3, a\right) \\ = \quad & -\frac{1}{3}\left(1-a\right)\Delta_1 - \frac{2}{3}\Delta_0 < 0, \end{aligned}$$

and ensure that as for firm e, the cost-reducing effect of a link with firm 1 is smaller than the price competition effect. Hence, firm e has no incentive to form a link with firm 1. This argument can be applied to the case where firms e and 2 are chosen at the first period.

In sum, for $\Delta > \frac{2+a}{2}$ or $\Delta < 1 - a$, the network g^3 survives in the dynamic process, because as for a firm that tries to form a link, the cost-reducing effect is smaller than the price competition effect. On the contrary, for $1 - a \leq \Delta \leq \frac{2+a}{2}$, the cost-reducing effect is larger than the price competition effect, so that each of the three firms have an incentive to form a pairwise link with each other.

4 Entry and Location Choice of an Entrant

We turn to the first stage in which firm e makes its entry decision, and if it decides to enter, it also decides its location $a \left(\in \left[\frac{1}{2}, 1\right)\right)$. Needless to say, firm e enters the market as long as the entry sunk cost F is small. Hence, we firstly examine the case in which $F = 0$ for analytical simplicity, and we then characterize a threshold level of the entry sunk cost F^* such that firm e does (does not) enter the market if $F \leq (>) F^*$.

4.1 Firm e's location choice

Suppose $F = 0$. Then, firm e enters the market as long as it can obtain a nonnegative profit. Then, the following proposition characterizes firm e's location choice in equilibrium, a^*.

Proposition 2 *Suppose $F = 0$. Then, we have:*

(i) When $0 < \Delta \leq \frac{1}{2}$, $a^* = 1 - \Delta \left(> \frac{1}{2}\right)$ if $\left(3t - 4\Delta_0\right)^2 \leq 36t^2\Delta\left(1 - \Delta\right)$. Otherwise, $a^* = \frac{1}{2}$.

(ii) When $\frac{1}{2} < \Delta \leq \frac{5}{4}$, $a^* = \frac{1}{2}$.

(iii) When $\frac{5}{4} < \Delta < \frac{3}{2}$, $a^* = 2\left(\Delta - 1\right) \left(> \frac{1}{2}\right)$ if $\left(3t - 4\Delta_0\right)^2 \leq 72t^2\left(\Delta - 1\right)\left(3 - 2\Delta\right)$. Otherwise, $a^* = \frac{1}{2}$.

(iv) When $\Delta \geq \frac{3}{2}$, $a^* = \frac{1}{2}$.

The result of Proposition 2 is explained by a firm's incentive to form a link with another firm. We describe the details of this point below.

Consider the case where $0 < \Delta \leq 1/2$. In this case, according to Proposition 1, if firm e chooses the location a between $1/2$ and $1 - \Delta$, the incumbent network g^3 still stands in the following second stage. This is because as for a firm that tries to form a link, the cost-reducing effect of the link is smaller than the price competition effect. On the other hand, if it chooses a between $1 - \Delta$ and 1, the complete network g^7 is formed, since the cost-reducing effect is larger than the price competition effect. Then, how about firm e's incentive for its location? Notice that irrespective of a type of network, firm e would like to avoid the severe price competition in the retail market. That is, given a type of network, firm e chooses a location such that it can set a price as high as possible. Formally,

$$\begin{aligned} \frac{\partial \Pi_e\left(g_3, a\right)}{\partial a} &< 0 \text{ for any } a \in \left[\frac{1}{2}, 1 - \Delta\right) \\ \text{and } \frac{\partial \Pi_e\left(g_7, a\right)}{\partial a} &< 0 \text{ for any } a \in \left[1 - \Delta, 1\right) \end{aligned}$$

Hence, firm e chooses $a^* = 1/2$ in g^3, while it chooses $a^* = 1 - \Delta$ in g^7.

Similarly, in the case where $1/2 < \Delta \leq 5/4$, we ensure that the network g^7 is formed in the second stage, irrespective of firm e's location $a \left(\in \left[\frac{1}{2}, 1\right)\right)$, by the result of Proposition 1 and the condition that $1/2 < \Delta$ (see the proof of Proposition 1 in Appendix). Then, firm e chooses $a^* = 1/2$.

In the case where $5/4 < \Delta < 3/2$, the incumbent network g^3 survives if firm e chooses the location a between $1/2$ and $2\left(\Delta - 1\right)$, whereas the complete network g^7 is formed if it chooses a between $2\left(\Delta - 1\right)$ and 1. Hence, firm e chooses $a^* = 1/2$ in g^3, while it chooses $a^* = 2\left(\Delta - 1\right)$ in g^7. The condition that indicates firm e's choice is given by the statement in the proposition.

Lastly, in the case where $\Delta \geq 3/2$, we ensure that the network g^3 survives in the second stage, irrespective of firm e's location $a \left(\in \left[\frac{1}{2}, 1\right)\right)$, by the result of Proposition 1 and the condition that $2(\Delta - 1) > 1$. Then, firm e chooses $a^* = 1/2$.

In Proposition 2, it is difficult to interpret the condition that indicates the choice of location when there exists two candidates of them (i.e., the case where $0 < \Delta \le 1/2$ or $5/4 < \Delta < 3/2$). Let us try to give an interpretation to these conditions. Consider the case where $0 < \Delta \le 1/2$. When does firm e choose $a^* = 1/2$ rather than $a^* = 1 - \Delta$? Note that, given Assumption 1(i) (i.e., $\Delta_0 < \left(3/\left(4+\sqrt{17}\right)\right)t$), the larger Δ_0, the smaller firm e's profit in g^3 (i.e., $\Pi_e^*\left(g^3, 1/2\right)$). This is because the two incumbents who has one link with each other becomes more efficient as Δ_0 becomes larger. On the other hand, firm e's profit in g^7 (i.e., $\Pi_e^*\left(g^7, 1-\Delta\right)$) becomes larger as Δ_0 becomes larger, when $\Delta \le 1/2$. Hence, as the marginal cost-reducing effect of the first link becomes large, firm e has more incentive to choose $a^* = 1 - \Delta$, given any other parameters constant.

Next, examine the effect of the parameter Δ_1. It is easy to check that $\partial \Pi_e^*\left(g^7, 1-\Delta\right)/\partial \Delta_1 < 0$ when $\Delta \le 1/2$. This is because when the marginal production cost is the same among all the firms, the profit-maximizing location for firm e is $1/2$. Then, since $a^* = 1 - \Delta$ gets farther from $1/2$ as Δ_1 becomes large, firm e's profit under g^7 decreases. Hence, it is more likely for firm e to take $a^* = 1/2$ rather than $a^* = 1 - \Delta$ when Δ_1 is large.

Similarly, we can give an interpretation to the condition for the case where $5/4 < \Delta < 3/2$. In fact, since $\partial \Pi_e^*\left(g^7, 2(\Delta-1)\right)/\partial \Delta_1 > 0$ when $\Delta > 1$, it is more likely for firm e to take $a^* = 1/2$ rather than $a^* = 2(\Delta - 1)$ when Δ_1 becomes large.[9]

We summarize these interpretations as a corollary.

Corollary 1 *Consider the case where $0 < \Delta \le 1/2$. Then, when Δ_0 is large or Δ_1 is small, firm e chooses $a^* = 1 - \Delta$. On the other hand, for the case where $5/4 < \Delta < 3/2$, firm e chooses $a^* = 2(\Delta - 1)$ when Δ_1 is large.*

4.2 Does the incumbent network deter entry?

Next, we examine the case in which $F > 0$. As stated in the beginning of this section, when F is small, firm e enters the market and its location is characterized by Proposition 2. Hence, we restrict our attention to the characterization of the threshold level of the entry sunk cost F^* such that firm e does (does not) enter the market if $F \le (>) F^*$.

We prepare two benchmarks in order to obtain the insight of F^* in our model. As a first benchmark, we provide the threshold level of the entry sunk cost when there exists no opportunity for all the firms to form pairwise links. In that case, it is easy to verify that firm e locates at $a^{B*} = 1/2$, once it decides to enter the market. Then, firm e's profit is

$$\Pi_e^{B*}\left(\varnothing, a^{B*}\right)\left(= \Pi_e^*\left(g^0, a^{B*}\right)\right) = \frac{1}{8}t.$$

Hence, the threshold level of the entry sunk cost when there exists no pairwise link between the two incumbents is $F^{B*} = (1/8)\,t$. Note that $a^{B*} = 1/2$ and $\Pi_e^* = (1/8)\,t$ hold as long as the marginal cost is the same among all the three firms, including the entrant. Hence, a^{B*} and F^{B*} are also the first-best solution in the sense that they are realized when the marginal production costs of all the firms (firms 1, 2, and e) are $c(2)$.

Next, as a second benchmark, we derive the threshold level of the entry sunk cost when only the incumbents can form a pairwise link and the entrant is not allowed to do so. This benchmark comes from the idea that a newcomer may be more difficult to communicate with incumbents without the improvement of information technology than with it. Furthermore, this situation is justified because the link benefits the two incumbents when there exists a threat of potential entry, as mentioned after Assumption 1. In this second benchmark, firm e locates at $\widetilde{a}^{B*} = 1/2$, once it decides to enter the market. Then, firm e's profit is given by

$$\widetilde{\Pi}_e^{B*}\left(g^3, \widetilde{a}^{B*}\right) = \frac{1}{72t}\left(3t - 4\Delta_0\right)^2.$$

Then, the threshold level of the entry sunk cost when there exists a link between the two incumbents is $\widetilde{F}^{B*} = \left(3t - 4\Delta_0\right)^2/72t$. Comparing $\widetilde{F}^{B*}$ with F^{B*}, we ensure that the link between the two incumbents can actually have an entry-deterrence effect, because $\widetilde{F}^{B*} < F^{B*}$.

Let us turn to the model analyzed so far. Firm e's profit excluding the entry sunk cost in each case of the equilibrium is calculated as follows.

(i) When $0 < \Delta \le \frac{1}{2}$, $\Pi_e^*\left(g^3, \frac{1}{2}\right) = \frac{1}{72t}\left(3t - 4\Delta_0\right)^2$ or $\Pi_e^*\left(g^7, 1-\Delta\right) = \frac{1}{2}t\Delta\left(1-\Delta\right)$.

(ii) When $\frac{1}{2} < \Delta \le \frac{5}{4}$, $\Pi_e^*\left(g^7, \frac{1}{2}\right) = \frac{1}{8}t$.

(iii) When $\frac{5}{4} < \Delta < \frac{3}{2}$, $\Pi_e^*\left(g^3, \frac{1}{2}\right) = \frac{1}{72t}\left(3t - 4\Delta_0\right)^2$ or
$\Pi_e^*\left(g^7, 2(\Delta-1)\right) = t\left(\Delta - 1\right)\left(3 - 2\Delta\right)$.

(iv) When $\Delta \ge \frac{3}{2}$, $\Pi_e^*\left(g^3, \frac{1}{2}\right) = \frac{1}{72t}\left(3t - 4\Delta_0\right)^2$.

Then, we immediately obtain the following finding.

[9] In this case, the effect of Δ_0 is ambiguous, since both $\Pi_e^*\left(g^3, 1/2\right)$ and $\Pi_e^*\left(g^7, 2(\Delta-1)\right)$ are decreasing in Δ_0.

Proposition 3 *Suppose the two incumbents already have a pairwise link with each other, and an entrant is allowed to form a pairwise link with an incumbent. Then, if the marginal change of cost-reducing effect scarcely depends on the number of links, the incumbent link never has an entry-deterrence effect. Otherwise, the incumbent link has an entry-deterrence effect.*

According to Proposition 3, the incumbent link cannot be a device for entry deterrence when the ratio of the marginal change of cost-reducing effect of pairwise links is small (i.e., when $1/2 < \Delta \leq 5/4$). In other words, when the marginal change of cost-reducing effect scarcely depends on the number of links, the incumbent network formation has no effect on entry deterrence, because the incumbents are also willing to form a link with an entrant. This is because, in that case, the cost-reducing effect becomes larger than the price competition effect, which in turn benefits *not only an entrant but also two incumbents*. Then, the entrant is likely to be welcome so that the complete network g^7 is certainly formed. Hence, the incumbent network does not have an entry-deterrence effect in that case.

Except for the case mentioned in Proposition 3, the incumbent link has an entry-deterrence effect, even though an entrant also has an opportunity to form a link with an incumbent. However, we should also note that when the entrant chooses $a^* = 1 - \Delta$ (i.e., when $0 < \Delta \leq 1/2$ and Δ_0 is large or Δ_1 is small), the entry-deterrence effect of the incumbent link becomes weak. Similarly, its entry-deterrence effect becomes weak when it chooses $a^* = 2(\Delta - 1)$ (i.e., when $5/4 < \Delta < 3/2$ and Δ_1 is large).

The result of Proposition 3 provides us a crucial implication for competition policy. Indeed, when forming a collaboration among firms become easy through the development of information technology, a collaboration among incumbents does not always deter entry. This is because the incumbents can fully use the opportunity to form a collaboration with an entrant, so that they may easily allow the entry. This collaboration may in turn enhance social welfare. Then, according to Proposition 3, with a improvement of information technology, a competition authority should care about the information concerning the change of the collaboration effect in order to evaluate the incumbents' alleged entry-deterrence strategies.

5 Concluding Remarks

We have examined a firm's entry and location decision when all firms, including incumbents and an entrant, have an opportunity to form a pairwise link with mutual consent. We have firstly shown that when an entrant is allowed to form a link and the link formation needs mutual consent, it has an incentive to distort its location from a welfare viewpoint in order to form a link with each of the incumbents. Second, we have shown that the incumbent network formed by two incumbents cannot be a device for entry deterrence, especially when the marginal change of cost-reducing effect generated by a link formation scarcely depends on the number of links.

Our model presented in this paper is quite simple, so that several extensions deserves to be mentioned for future research. Here, we just mention three of them. First, introducing multiple entrants or more than two incumbents may affect the analytical results derived above. This is because the relative magnitude between the cost-reducing effect and the price competition effect changes according to the number of firms. Second, if the cost-reducing effect generated by a pairwise link formation depends not only on the number of links but also on the distance between the firms that forms a link, our analytical results become quite complicated. While the existence of spillovers makes the analysis complicated, our main qualitative result may still hold in that setting. Third, the change of spatial structure from a linear city to a circular city seems to provide a dramatic change in the above results qualitatively, as it comes about when examining other research issues.[10]

Appendix

A. The existence of the interior solution in equilibrium

As stated in the text, we claim that under Assumption 1, we can restrict our attention to the interior solution in which all three firms can obtain a positive demand and a positive profit after firm e's entry in equilibrium.

There are three steps for the procedure to derive a sufficient condition for the existence of the interior solution. In Step 1, given a network g, we derive a condition that guarantees a positive demand and a

[10] This is mainly because all the incumbents are symmetric in the sense that they are located with equal distance and they compete with both sides of rivals, even though we allow an entrant to choose its location between two incumbents.

positive profit (i.e., $p_i > c_i$). In Step 2, given a network g, we derive a condition under which firm e's profit-maximizing location is in the relevant range of [1/2, 1). In Step 3, we derive a sufficient condition for the exisitence of the interior condition in all possible networks by comparing all the conditions derived in the second step.

Step 1:

First, consider $g = g^3$. Note that $D_i(.) > 0$ is equivalent to $p_i^* > c_i$ $(i = 1, e, 2)$. Then, we have:

$$\begin{aligned}
p_1^*\left(g^3, a\right) &> c\left(\eta_1\left(g^3\right)\right) \text{ if and only if} \\
\frac{1}{3}\Delta_0 + \frac{1}{2}at &> 0, \\
p_e^*\left(g^3, a\right) &> c\left(\eta_e\left(g^3\right)\right) \text{ if and only if} \\
-\frac{1}{3}\Delta_0 + a(1-a)t &> 0, \\
p_2^*\left(g^3, a\right) &> c\left(\eta_2\left(g^3\right)\right) \text{ if and only if} \\
\frac{1}{3}\Delta_0 + \frac{1}{2}(1-a)t &> 0.
\end{aligned}$$

Then, comparing all the above conditions, we obtain the condition under which all the three firms have a positive demand as follows.

$$\Delta_0 < 3a(1-a)t. \tag{8}$$

The same procedure is applied to the other networks. In fact, we have:

$$\begin{aligned}
\text{For } g &= g^1,\ \Delta_0 < \frac{3(1-a)}{3-a}t, \\
\text{For } g &= g^2,\ \Delta_0 < \frac{3a}{2+a}t, \\
\text{For } g &= g^4,\ \Delta_1 < 3at, \\
\text{For } g &= g^5,\ \Delta_1 < \frac{3}{2}(1-a)t, \\
\text{For } g &= g^6,\ \Delta_1 < 3(1-a)t, \\
\text{For } g &= g^0 \text{ and } g = g^7, \text{ no restriction.}
\end{aligned}$$

Step 2

Consider $g = g^3$. Rearranging (8), we have

$$3ta^2 - 3ta + \Delta_0 < 0. \tag{9}$$

For a that satisfies (9) to exist in the range of $[1/2, 1)$, it is necessary to have the following condition:

$$\text{For } g = g^3,\ \Delta_0 < \frac{3}{4}t. \tag{10}$$

The same procedure is applied to the other networks. In fact, we have:

$$\begin{aligned}
\text{For } g &= g^1,\ \Delta_0 < \frac{3}{5}t, &(11)\\
\text{For } g &= g^2,\ \Delta_0 < \frac{3}{4+\sqrt{17}}t, &(12)\\
\text{For } g &= g^4,\ \Delta_1 < 3t, &(13)\\
\text{For } g &= g^5,\ \Delta_1 < \frac{3}{4}t, &(14)\\
\text{For } g &= g^6,\ \Delta_1 < \frac{3}{2}t, &(15)\\
\text{For } g &= g^0 \text{ and } g = g^7, \text{ no restriction.}
\end{aligned}$$

Step 3

Summarizing (11) to (15), we obtain a sufficient condition for the interior solution, which is stated in Assumption 1; $\Delta_0 < \left(3/\left(4+\sqrt{17}\right)\right)t$ and $\Delta_1 < (3/4)\,t$.

B. Proof of Lemma 1

Consider a network g in which $\eta_1 = \eta_e = \eta$ and $\eta_2 = \widetilde{\eta}\,(\neq \eta)$. Suppose the link $1e \notin g$. Then, from (1) and (2), we have:

$$\begin{aligned}
f_1(g, a, 1e) &\equiv p_1^*(g + 1e, a) - c(\eta + 1) \\
&\quad - \left(p_1^*(g, a) - c(\eta)\right) \\
&= \frac{1}{6}a\left(c(\eta) - c(\eta+1)\right) > 0. \\
f_e(g, a, 1e) &\equiv p_e^*(g + 1e, a) - c(\eta + 1) \\
&\quad - \left(p_e^*(g, a) - c(\eta)\right) \\
&= \frac{1}{3}a\left(c(\eta) - c(\eta+1)\right) > 0.
\end{aligned}$$

Next, suppose $1e \in g$. Again, from (1) and (2), we have:

$$\begin{aligned}
f_1(g - 1e, a, 1e) &\equiv p_1^*(g, a) - c(\eta) \\
&\quad - \left(p_1^*(g - 1e, a) - c(\eta - 1)\right) \\
&= \frac{1}{6}a\left(c(\eta-1) - c(\eta)\right) > 0. \\
f_e(g - 1e, a, 1e) &\equiv p_e^*(g, a) - c(\eta) \\
&\quad - \left(p_e^*(g - 1e, a) - c(\eta - 1)\right) \\
&= \frac{1}{3}a\left(c(\eta-1) - c(\eta)\right) > 0.
\end{aligned}$$

Notice that all the above inequalities holds for any $a \in \left[\frac{1}{2}, 1\right)$.

Likewise, we can ensure the claim for any other pairs by using (1) to (3). ■

Proof of Lemma 2

Using (1) to (3) and the definition of pairwise stability, we need to check when each of the networks g^1, g^2, g^3, and g^7 can be pairwise stable.

First, consider g^1. The link 12 satisfies the condition of pairwise stability from Lemma 1. By checking the definition of pairwise stability, we can ensure that the link 13 satisfies its condition when $\Delta > \frac{2+a}{a}$ or $\Delta < \frac{1-a}{3-a}$. Similarly, the link 23 satisfies its condition when $\Delta > \frac{1}{a}$ or $\Delta < \frac{2}{3-a}$. Therefore, g^1 is pairwise stable when $\Delta > \frac{2+a}{a}$ or $\Delta < \frac{1-a}{3-a}$.

The same procedure for the check of pairwise stability conditions can be applied to g^2, g^3, and g^7. In fact, we ensure that: (i) g^2 is pairwise stable when $\Delta > \frac{3+a}{1-a}$ or $\Delta < \frac{a}{2+a}$, (ii) g^3 is pairwise stable when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$, and (iii) g^7 is pairwise stable for any $\Delta > 0$.

Furthermore, we know that $1-a \geq (<) \frac{a}{2+a}$ if and only if $\frac{1}{2} \leq a \leq \sqrt{3}-1 \left(\sqrt{3}-1 < a < 1\right)$. Then, the results derived above are summarized as in the text. ■

C. Proof of Proposition 1

We need to give the proofs for the two cases; the case where $a \in \left[\frac{1}{2}, \sqrt{3}-1\right)$ and the case where $a \in \left[\sqrt{3}-1, 1\right)$. Hereafter, we denote the network at tth round by $g_{(t)}$.

(i) The case where $a \in \left[\frac{1}{2}, \sqrt{3}-1\right)$

First, we consider the case where $a \in \left[\frac{1}{2}, \sqrt{3}-1\right)$. From Lemma 2, we know that, given $g_{(0)} = g^3$, g^3 and g^7 are the candidates of pairwise stable networks to which the dynamic process converges when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$. On the other hand, when $1-a \leq \Delta \leq \frac{2+a}{2}$, only g^7 is its candidate, whereas there is a possibility that no network survives this process.

Consider the case where $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$. Suppose firms 1 and 2 are chosen with probability $\frac{1}{3}$ at the 1st round. We know that $f_1\left(g^3, a, 12\right) < 0$ when $\Delta > \frac{2+a}{2}$. Therefore, $g_{(1)} = g_{(0)} = g^3$. Next, suppose firms 1 and 3 are chosen with probability $\frac{1}{3}$. Since $13 \in g^3$, we have $g_{(1)} = g_{(0)} = g^3$ from Lemma 1. Lastly, suppose firms 2 and 3 are chosen with probability $\frac{1}{3}$. Since $\Delta < 1-a < a$ for $a \geq \frac{1}{2}$, $f_2\left(g^3, a, 23\right) < 0$ which implies that firm 2 has no incentive to form 23, so that $g_{(1)} = g_{(0)} = g^3$. Therefore, when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$, g^3 remains forever, i.e., the dynamic process converges to g^3.

Next, consider the case where $1-a \leq \Delta \leq \frac{2+a}{2}$. Suppose firms 1 and 2 are chosen with probability $\frac{1}{3}$ at the 1st round. Then, since $f_1\left(g^3, a, 12\right) \geq 0$ and $f_2\left(g^3, a, 12\right) \geq 0$ when $1-a \leq \Delta \leq \frac{2+a}{2}$, the two firms have an incentive to form the link 12. Hence, $g_{(1)} = g_{(0)} + 12 = g^4$. Then, from Lemma 1, we know that firms 2 and 3 have an incentive to form the link 23, because $\eta_2\left(g^4\right) = \eta_3\left(g^4\right) = 1$ and $23 \notin g^4$. This means that, once firms 2 and 3 are chosen with probability $\frac{1}{3}$, g^4 converges to g^7. Next, suppose firms 1 and 3 are chosen at the 1st round. Since $13 \in g^3$, we have $g_{(1)} = g_{(0)} = g^3$ from Lemma 1. Notice that, once firms 1 and 2 are chosen after this round, g^3 goes to g^4. Again, g^4 converges to g^7 as long as firms 2 and 3 are chosen.

Suppose firms 2 and 3 are chosen at the 2nd round after $g_{(1)} = g_{(0)} = g^3$. We need to divide the case where $1-a \leq \Delta \leq \frac{2+a}{2}$ into three cases: the case where $\frac{3-a}{2} < \Delta \leq \frac{2+a}{2}$, the case where $a < \Delta \leq \frac{3-a}{2}$, and the case where $1-a \leq \Delta \leq a$. When $\frac{3-a}{2} < \Delta \leq \frac{2+a}{2}$, g^3 converges to g^7 through g^4 as long as firms 1 and 2 are chosen. When $a < \Delta \leq \frac{3-a}{2}$, firms 2 and 3 have an incentive to form the link 23 under g^3, which means g^3 goes to g^6. Also, once firms 1 and 2 are chosen after this round, firms 1 and 2 have an incentive to form the link 12, which means g^6 goes to g^7. The process at the case where $1-a \leq \Delta \leq a$ is the same as the case where $\frac{3-a}{2} < \Delta \leq \frac{2+a}{2}$. Hence, we can summarize that when firms 1 and 3 are chosen at the 1st round, the dynamic process converges to g^7.

Lastly, suppose firms 2 and 3 are chosen with probability $\frac{1}{3}$ at the 1st round. The analysis is the same as the case where firms 2 and 3 are chosen at the 2nd round after $g_{(1)} = g_{(0)} = g^3$. Therefore, the process converges to g^7.

In sum, we ensure that when $1-a \leq \Delta \leq \frac{2+a}{2}$, the dynamic process with the initial state g^3 converges to g^7.

(ii) The case where $a \in \left[\sqrt{3}-1, 1\right)$

From Lemma 2, we know that, given $g_{(0)} = g^3$, g^3 and g^7 are the candidates of pairwise stable networks to which the dynamic process converges when $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$. On the other hand, when $1-a \leq \Delta \leq \frac{2+a}{2}$, only g^7 is its candidate. Hence, the procedure for the check of the convergence of the dynamic process is the same as in (i), except that when $1-a \leq \Delta \leq \frac{2+a}{2}$, we need to divide the case into two cases: the case where $\frac{a}{2+a} \leq \Delta \leq \frac{2+a}{2}$ and the case where $1-a \leq \Delta < \frac{2+a}{2}$. However, the same reasoning as in the case where $1-a \leq \Delta \leq \frac{2+a}{2}$ in (i) applies to both cases. Therefore, we derive the same result as in (i). ■

D. Proof of Proposition 2

Let us consider firm e's problem in the first stage. To do so, we first rewrite the conditions under which the network formation process converges to g^3 and g^7, respectively. In fact, the condition that $\Delta > \frac{2+a}{2}$ or $\Delta < 1-a$ is rewritten as

$$a < 2(\Delta - 1) \text{ or } a < 1 - \Delta.$$

We know that $2(\Delta-1) \geq (<)\, 1-\Delta$ if and only if $\Delta \geq (<)\, 1$. Hence, when $\Delta \geq 1$, we need to check only the condition that $a < 2(\Delta-1)$. Otherwise, we check the condition that $a < 1-\Delta$. Similarly, the condition that $1-a \leq \Delta \leq \frac{2+a}{2}$ is rewritten as

$$a \geq 1-\Delta \text{ and } a \geq 2(\Delta-1).$$

Again, we obtain that when $\Delta \geq 1$, we need to check only the condition that $a \geq 2(\Delta-1)$. Otherwise, we check the condition that $a \geq 1-\Delta$.

Then, using these conditions and the fact that $a \in \left[\frac{1}{2}, 1\right)$, we have five cases to be considered: (i) $0 < \Delta \leq \frac{1}{2}$, (ii) $\frac{1}{2} < \Delta \leq 1$, (iii) $1 < \Delta \leq \frac{5}{4}$, (iv) $\frac{5}{4} < \Delta \leq \frac{3}{2}$, and (v) $\Delta > \frac{3}{2}$. Let us examine each of the cases separately. After that, we check whether firm e can actually obtain a positive profit with positive demand in each case.

(i) The case where $0 < \Delta \leq \frac{1}{2}$.

In this case, since $\Delta < 1$, the condition that $a \geq (<)\, 1-\Delta$ is a relevant constraint for firm e's problem. Then, if $\frac{1}{2} \leq a < 1-\Delta$, g^3 is formed with probability 1 in the second stage. On the other hand, if $1-\Delta \leq a < 1$, g^7 is formed with probability 1 in the second stage. Hence, we need to solve each of two subproblems and then compare firm e's maximized profits in order to obtain firm e's profit-maximizing location.

The first subproblem is formulated as follows:

$$\begin{aligned} & \max_a \Pi_e\left(g^3, a\right) \\ = \; & \frac{1}{2a(1-a)t}\left(p_e^*\left(g^3, a\right) - c\left(\eta_e\left(g^3\right)\right)\right)^2 \\ s.t. \; \frac{1}{2} \; \leq \; & a < 1-\Delta. \end{aligned}$$

Then, we obtain the solution $a^{*(i),1} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^*\left(g^3, \frac{1}{2}\right) = \frac{1}{72}\left(3t - 4\Delta_0\right)^2.$$

Similarly, the second subproblem is formulated by:

$$\begin{aligned} & \max_a \Pi_e\left(g^7, a\right) \\ = \; & \frac{1}{2a(1-a)t}\left(p_e^*\left(g^7, a\right) - c\left(\eta_e\left(g^7\right)\right)\right)^2 \\ s.t. \; 1-\Delta \; \leq \; & a < 1. \end{aligned}$$

We obtain the solution $a^{*(i),2} = 1-\Delta$, and the associated profit is as follows:

$$\Pi_e^*\left(g^7, 1-\Delta\right) = \frac{1}{2}t\Delta(1-\Delta).$$

We can ensure that both $a^{*(i),1}$ and $a^{*(i),2}$ can be a solution when $0 < \Delta \leq \frac{1}{2}$, depending on the level of parameters. Then, comparing $\Pi_e^*\left(g^3, 1/2\right)$ with $\Pi_e^*\left(g^7, 1-\Delta\right)$ gives the condition in the text.

(ii) The case where $\frac{1}{2} < \Delta \leq 1$

Since $\Delta \leq 1$, the condition that $a \geq (<)\, 1-\Delta$ is a relevant constraint for firm e's problem. Furthermore, since $1-\Delta < \frac{1}{2}$, firm e's problem is formulated as

$$\max_a \Pi_e\left(g^7, a\right) \quad s.t. \; \frac{1}{2} \leq a < 1.$$

We obtain the solution $a^{*(ii)} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^*\left(g^7, \frac{1}{2}\right) = \frac{1}{8}t.$$

(iii) The case where $1 < \Delta \leq \frac{5}{4}$

Since $\Delta > 1$, the condition that $a \geq (<)\, 2(\Delta-1)$ is a relevant constraint for firm e's problem. Furthermore, since $2(\Delta-1) \leq \frac{1}{2}$, firm e's problem is formulated as

$$\max_a \Pi_e\left(g^7, a\right) \quad s.t. \; \frac{1}{2} \leq a < 1.$$

We obtain the solution $a^{*(iii)} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^*\left(g^7, \frac{1}{2}\right) = \frac{1}{8}t.$$

(iv) The case where $\frac{5}{4} < \Delta \leq \frac{3}{2}$

Since $\Delta > 1$, the condition that $a \geq (<)\, 2(\Delta-1)$ is relevant for firm e's problem. Then, if $\frac{1}{2} \leq a < 2(\Delta-1)$, g^3 is formed with probability 1 in the second stage. On the other hand, if $2(\Delta-1) \leq a < 1$, g^7 is formed with probability 1 in the second stage. Again, we need to solve each of two subproblems and then compare firm e's maximized profits in order to obtain firm e's profit-maximizing location.

The first subproblem is formulated as follows:

$$\max_a \Pi_e \left(g^3, a\right) \quad s.t.\ \frac{1}{2} \leq a < 2\left(\Delta - 1\right).$$

Then, we obtain the solution $a^{*(iv),1} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^* \left(g^3, \frac{1}{2}\right) = \frac{1}{72}\left(3t - 4\Delta_0\right)^2.$$

The second subproblem is formulated by:

$$\max_a \Pi_e \left(g^7, a\right) \quad s.t.\ 2\left(\Delta - 1\right) \leq a < 1.$$

We obtain the solution $a^{*(iv),2} = 2\left(\Delta - 1\right)$, and the associated profit is as follows:

$$\Pi_e^* \left(g^7, 2\left(\Delta - 1\right)\right) = t\left(\Delta - 1\right)\left(3 - 2\Delta\right).$$

We can ensure that both $a^{*(iv),1}$ and $a^{*(iv),2}$ can be a solution when $\frac{5}{4} < \Delta \leq \frac{3}{2}$, depending on the level of parameters. Then, comparing $\Pi_e^* \left(g^3, 1/2\right)$ with $\Pi_e^* \left(g^7, 2\left(\Delta - 1\right)\right)$ gives the claim in the text.

(v) The case where $\Delta > \frac{3}{2}$

Since $\Delta > 1$, the condition that $a \geq (<)\, 2\left(\Delta - 1\right)$ is relevant for firm e's problem. Furthermore, since $2\left(\Delta - 1\right) \geq 1$, firm e's problem is formulated as

$$\max_a \Pi_e \left(g^3, a\right) \quad s.t.\ \frac{1}{2} \leq a < 1.$$

We obtain the solution $a^{*(v)} = \frac{1}{2}$, and the associated profit is as follows:

$$\Pi_e^* \left(g^3, \frac{1}{2}\right) = \frac{1}{72}\left(3t - 4\Delta_0\right)^2.$$

Summarizing the above results gives the claim in the text. ■

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Figure 1 All Networks