Steepest Ascent Tariff Reforms

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Abstract: This paper introduces the concept of a steepest ascent tariff reform for a small open economy. By construction, it is locally optimal in that it yields the highest gain in utility of any feasible tariff reform vector. Accordingly, it provides a convenient benchmark for the evaluation of the welfare effectiveness of other well known tariff reform rules, such as the proportional and the concertina rules. We develop the properties of this tariff reform in detail and provide geometric illustrations of our method. Overall, the paper's contribution lies in developing a theoretical concept where the focus is upon the size of welfare gains accruing from tariff reforms rather than simply with the direction of welfare effects that has been the concern of the existing literature.

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1 Introduction

The present paper is concerned with the measurement of the welfare effects of policy reforms. Our starting point is an economy at a distortionary initial equilibrium. Removing the existing distortions at once is assumed not possible. The question then becomes one of which piecemeal policy reforms should be applied. The literature on these policy reform issues has concentrated on finding simple rules that deliver a welfare gain, i.e., finding a reform of distortions in the right direction. However, the literature has contributed nothing about the size of these welfare gains - we do not know which reforms are more effective in raising welfare, nor how well they perform relative to the best attainable gain. The present paper addresses this issue and provides a methodology for evaluating and comparing the sizes of welfare gains.

Our application is on the theory of trade policy reforms, but it will become clear that our methodology applies to any other policy dimension. Within the literature of piecemeal tariff reform there is a well-established set of results that provide conditions under which welfare will rise in response to particular tariff reforms. The most general result is that a proportional reduction in tariff rates will raise welfare in a single-household small open economy provided only that a weak normality condition holds on preferences. Within the same framework another result, known as the concertina theorem, is that the reduction of the tariff rate on the imports of a single commodity will raise welfare if that good has the highest ad valorem tariff rate and it is a net substitute for every other good. Such results fall into the class of problems of the second best.[1]

While this is an extensive body of literature, it is a general feature of the literature that each potential reform is treated separately and the main concern is with whether welfare rises or falls. Within this context, we propose a new tariff reform concept, and we use it in order to provide a framework within which all particular tariff reforms can be compared in terms of their effectiveness in generating welfare gains. In this sense, while the literature has been focused on the sign of the welfare effect of a tariff reform, we focus

[1]The early literature on tariff reform in open economies includes Meade (1955), followed by Lloyd (1974), Hatta (1977a, 1977b) and Fukushima (1979). More recent contributions to this literature include Abe (1992), Diewert, Turunen-Red and Woodland (1989, 1991), Turunen-Red and Woodland (1991), Anderson and Neary (1992, 1996, 2007) and Neary (1995).

on the relative sizes of these welfare effects.

We propose a tariff reform that is locally optimal amongst all feasible tariff reforms. We refer to this reform as the Steepest Ascent Tariff Reform (SATR). Starting at an equilibrium established under an initial tariff setting, we formulate the differential tariff reform problem as one of choosing a direction of tariff changes that yields a positive directional derivative, that is, a positive slope to the utility function. There will generally be a set of such tariff reform directions. We formulate the locally optimal tariff reform problem as one of choosing the best direction, as measured by the directional derivative. We show that the locally optimal, or steepest ascent, tariff reform is one that has a direction of change proportional to the gradient of the indirect utility function with respect to the tariff vector.

Having derived the best possible differential reform, we can then proceed by ranking other well known reform rules in terms of the welfare gain that they yield - a feature that has not hitherto been exploited in the literature.[2] Thus, while previously reforms have been chosen on the basis of, for example, their informational requirements (undoubtedly, an important element), it is now possible to also base the choice of a reform rule on its efficiency properties. Accordingly, this provides an improvement in the information set that policy makers have before choosing a particular policy reform.

While the concept of a steepest ascent tariff reform has not previously appeared in the literature (to our knowledge), it is closely related to, and inspired by, the concept of an optimal tax perturbation introduced into the tax reform literature by Diewert (1978, p.152). In a similar vein to our reform concept, Tirole and Guesnerie (1981) make use of a related gradient projection tax reform in a closed economy with many consumers and a social welfare function, but they do not compare reforms as is done here. The idea of locally optimal tax changes is also behind Weymark's (1981) concept of undominated directions of tax reform but he does not make use of a steepest ascent tax reform.

Our steepest ascent tariff reform concept is used in the paper to develop new results in the theory of tariff reform. First, we characterize the steepest ascent tariff reform and develop some of its properties. We show that the reform can be made operational

[2] This ranking is of course undertaken with respect to the best practice, viz. our SATR.

provided information is available on the net substitution matrix for the economy at the initial equilibrium as well as the initial tariffs. Increasingly, detailed empirical estimations and computable general equilibrium models are coming available for many countries thus making the concept of practical as well as theoretical interest.

Second, we provide a characterization of the sources of the potential welfare gains in terms of measures of the level and dispersion of initial tariff rates, viz. the generalized average tariff and the generalized variance of tariffs. The greater is the generalized average tariff rate, or the greater is the generalized variance of tariffs, the greater is the potential increase in welfare from tariff reforms. These generalized mean and variance measures of the distortions differ from, but are inspired by, measures of the same name recently proposed by Anderson and Neary (2007).

Third, by comparing the results of applying SATR to the cases where all tariff rates are subject to the reform and where only the tariffs on non-numeraire goods are permitted to be reformed, we establish that the latter involves a welfare loss compared to the former reform. That is, if we restrict attention to differential tariff reforms, it matters a great deal as to whether the tariff reform is applied to the tariff of all goods or only to those of non-numeraire goods. This may appear at first glance to contradict the well-known result that homogeneity considerations allow the equivalent analyses of tariffs on either all or non-numeraire goods, but this is not the case as will be demonstrated below.

Fourth, we undertake a comparison of the proportional, univariate and steepest ascent tariff reforms. We establish several results concerning this comparison. In particular, we characterize the conditions under which the proportional tariff reform and the univariate tariff reform are locally optimal. We also provide a geometric illustration of these reforms and show that the more acute the angle between a reform and the steepest ascent tariff reform, the greater will be its welfare efficiency. In doing this, we develop a new index that measures the welfare effectiveness of any tariff reform. A numerical example provides concrete measures of the relative efficiency of the proportional and univariate reforms in raising welfare.

In our concluding remarks, we provide a discussion on the role of our steepest ascent reform concept in a broader policy context and on potential extensions. In doing so, we argue that our methodology can be used with any objective function and any policy instruments, and we provide several examples for future work.

2 Small Open Economy Tariff Reform

We consider a perfectly competitive general equilibrium model of a small open economy that trades in n internationally tradeable commodities. The model may be expressed as

$$p'S_\pi(\pi, u) = b, \tag{1}$$

in terms of the world price vector p (p' denotes the transpose of a vector), the domestic price vector π, the representative agent's utility level u and the transfers abroad b. The world and domestic price vectors are related by $\pi = P(\iota + t)$, where t is a vector of ad valorem tariff rates, ι is the unit vector with all elements equal to unity and $P = diag(p)$ is a diagonal matrix with the world price vector along the diagonal.[3] In this specification, $S(\pi, u) \equiv E(\pi, u) - G(\pi)$ is the net expenditure function, being the difference between the consumer expenditure function, E, and the producer revenue function, G. Also, $S_\pi(\pi, u) \equiv \nabla_\pi S(\pi, u)$ denotes the gradient of the net expenditure function with respect to prices and represents the vector of compensated net import functions. Equation (1) is the country's budget constraint, which requires that the value at world prices, p, of the net import vector, S_π, be equal to the net transfers from abroad, b. If $b = 0$, then the budget constraint simply requires that there is a zero balance of trade.

Let $S_{\pi\pi} \equiv \nabla^2_\pi S(\pi, u) = \nabla_\pi S_\pi(\pi, u)$ be the substitution matrix, measuring the response of compensated net imports to changes in prices, and let $S_{\pi u} \equiv \nabla_{\pi u} S(\pi, u) = \nabla_u S_\pi(\pi, u)$ be a vector of "income" effects, measuring the response of compensated net imports to changes in utility. It is assumed that the Hatta normality condition, $p'S_{\pi u} > 0$, holds. It is well known that the substitution matrix, $S_{\pi\pi}$, is a symmetric, negative semi-definite matrix satisfying the homogeneity identity that $\pi' S_{\pi\pi}(\pi, u) \equiv 0$ for all domestic price

[3] As is standard convention in the literature, t defines the wedge between domestic and world prices. If a good i is imported and $t_i > 0$ then it represents an import duty, but if $t_i < 0$ then it represents an import subsidy. Similarly, if a good i is exported and $t_i > 0$ then it represents an export subsidy, but if $t_i < 0$ then it represents an export tax. For simplicity, we nevertheless refer to t as a 'tariff' vector.

vectors, π.

The budget constraint (1) may be solved for utility, u, as a function of the world price vector, p, and the tariff vector, t. This yields the indirect utility function $U(t;p)$, which may be written more simply as $U(t)$ since the world price vector is assumed to remain fixed and so may be subsumed.

2.1 Steepest Ascent Tariff Reforms

We consider piecemeal reforms of tariffs. However, rather than simply consider all such reforms or some special cases, we wish to characterize piecemeal reforms that are locally optimal. That is, we want to find the direction vector, δ, that maximizes the differential change in utility. Such a direction of reform is then said to be locally optimal.

Suppose that the initial tariff vector is t^0. The gradient of the indirect utility function at this initial point is $\nabla U(t^0)$ and the directional derivative (in direction δ) at this initial point is expressed as $D(t^0,\delta) = \nabla U(t^0)'\delta$.[4] This directional derivative indicates the slope of the indirect utility function in the direction δ. Specifically, if the differential of the tariff vector is expressed as $dt = \delta d\alpha$, where α is the distance in direction δ, the differential change in utility is $dU(t^0)/d\alpha = \nabla U(t^0)'\delta = D(t^0,\delta)$.

We wish to find a direction that maximizes the slope of the indirect utility function, since this is the direction of a differential (piecemeal) tariff reform that yields the greatest improvement in utility; reforms of tariffs in all other directions will yield lower increases (or decreases) in utility. The locally optimal tariff reform problem may be expressed formally as

$$\max_{\delta} \left\{ \nabla U(t^0)'\delta \right\}, \tag{2}$$

[4]Let f be a numerical function defined on an open set X in R^n and let $x \in X$. Let $\delta \in R^n$. The directional derivative of f at x^0 in direction δ is

$$D_\delta f(x^0) \equiv lim_{h\to 0} \frac{f(x^0+h\delta)\ - f(x^0)}{h},$$

when the limit exists. See, for example, Apostol (1957, 104-105). By convention, the direction vector is assumed to have unit length ($\|\delta\| = 1$) thus yielding the expression $D_\delta f(x^0) = \nabla f(x^0)'\delta$ in terms of the gradient vector $\nabla f(x^0)$.

which is to choose a direction of tariff reform that maximizes the value of the directional derivative $D(t^0, \delta) = \nabla U(t^0)'\delta$. Accordingly, we define the solution for δ to the problem defined by (2) as the *steepest ascent tariff reform (SATR) direction.* The solution for δ is given by

$$\delta^S = \theta \nabla U(t^0), \quad \theta = \left\| \nabla U(t^0) \right\|^{-1} > 0, \tag{3}$$

where $\nabla U(t^0)$ is the gradient vector, whose elements are the partial derivatives $\partial U(t^0)/\partial t_i$, $i = 1, 2, ..., n$, and θ is the Euclidean length of the gradient vector.[5]

In the present context, it can be shown that the gradient of the indirect utility function with respect to the vector of ad valorem tariffs, t, is

$$\nabla U(t^0) = P' S_{\pi\pi}(\pi^0, u^0) P t^0 / p' S_{\pi u}(\pi^0, u^0), \tag{4}$$

recalling that P is a matrix with the world price vector along the diagonal, and where $\pi^0 = P(\iota + t^0)$ is the domestic price vector and u^0 is the utility level at the initial tariff vector t^0.[6] This expression for the gradient vector may be more compactly written as

$$\nabla U(t^0) = \mathcal{S} t^0 / H, \tag{5}$$

where $H \equiv p' S_{\pi u}(\pi^0, u^0) > 0$ is the Hatta normality term and $\mathcal{S} \equiv P' S_{\pi\pi}(\pi^0, u^0) P$ is the 'normalized' substitution matrix at the initial equilibrium. This normalized substitution matrix is negative semi-definite and satisfies the homogeneity condition $\mathcal{S} P^{-1} \pi^0 \equiv 0$, both properties being inherited from the substitution matrix $S_{\pi\pi}$.

Accordingly, we can use this result to obtain the steepest ascent tariff reform as in the following proposition.

[5] The steepest ascent algorithm for the maximization of a function provides the motivation for our *locally optimal* tariff reform problem. For a description and properties of the steepest ascent algorithm see, for example, Luenberger (1984, 214-220), Press et al. (1986, ch. 10) and Bhatti (2000, ch. 5).

[6] To obtain this result, totally differentiate (1) to get $p' S_{\pi u}(\pi^0, u^0) du + p' S_{\pi\pi}(\pi^0, u^0) d\pi = 0$. Because the net expenditure function is homogeneous of degree zero in domestic prices, it follows that the substitution matrix $S_{\pi\pi}$ must obey the identity $\pi' S_{\pi\pi}(\pi, u^0) \equiv 0$ for all domestic price vectors π. Noting that $\pi^0 = P(\iota + t^0)$, this identity may be expressed as $p' S_{\pi\pi}(\pi, u^0) + t^{0\prime} P' S_{\pi\pi}(\pi, u^0) \equiv 0$. The above total derivative may then be written as $p' S_{\pi u}(\pi^0, u^0) du - t^{0\prime} P' S_{\pi\pi}(\pi^0, u^0) P dt = 0$. Thus, we obtain the effect of a change in tariffs upon utility as $du = t^{0\prime} P' S_{\pi\pi}(\pi^0, u^0) P dt / p' S_{\pi u}(\pi^0, u^0)$. This provides the formula for the gradient of the indirect utility function with respect to the tariff vector as expressed in the text.

Proposition 1 *The steepest ascent tariff reform is given by* $dt = \delta^S d\alpha, d\alpha > 0$*, where*

$$\delta^S = \lambda \mathcal{S} t^0, \quad \lambda = \left\| \mathcal{S} t^0 \right\|^{-1}, \tag{6}$$

and the steepest ascent slope (directional derivative) of the utility function is $D(t^0, \delta^S) = H^{-1} \left\| \mathcal{S} t^0 \right\|$*, where* $H \equiv p' S_{\pi u}(\pi^0, u^0) > 0$ *is the Hatta normality term.*

According to this proposition, the locally optimal tariff reform requires the direction vector to be proportional to the vector $\mathcal{S}t^0$. This vector depends upon the initial tariff vector, t^0, and the 'normalized' substitution matrix, $\mathcal{S}$, evaluated at the initial equilibrium. Thus, local information about the substitution possibilities amongst all goods in the economy is required.

The proposition also shows that the directional derivative (slope of the utility function) in the locally optimal direction is positive provided that $\|\mathcal{S}t^0\| > 0$, which is the case if $\mathcal{S}t^0 \neq 0$.[7] Clearly, the increase in utility will be zero if, and only if, $\mathcal{S}t^0 = 0$, which means that there are no tariff distortions. In general, the condition $\mathcal{S}t^0 = 0$ requires a special configuration of tariff rates and substitution terms.[8]

Turunen-Red and Woodland (2001) define the vector $d \equiv S_{\pi\pi}(\pi^0, u^0)Pt^0$ as a "local measure of tariff distortion", each element measuring the distortion for each good. Accordingly, we see that the steepest ascent tariff reform may be expressed in terms of this distortion vector as $\delta^S = \lambda P' S_{\pi\pi}(\pi^0, u^0)Pt^0 = \lambda P'd$. Thus, the optimal direction vector of tariff reform has the same sign structure as the tariff distortion vector, as defined by Turunen-Red and Woodland.

The interpretation of the steepest ascent tariff reform may be assisted by reference to Figures 1 and 2. Figure 1 shows iso-utility contours in tariff space with just two

[7] In general, this is given by $dU^S/d\alpha = D(t^0, \delta^S) = \nabla U(t^0)'\delta^S = \theta \nabla U(t^0)' \nabla U(t^0) > 0$, where the inequality follows because the inner product of any vector with itself is positive (unless the vector is the null vector), being the sum of squares of its elements. This inequality establishes that the steepest ascent tariff reform always raises welfare, irrespective of the initial tariff vector or the nature of the economy, provided that the gradient of the utility function does not vanish.

[8] There are no distortions if $t^0 = 0$, of course, but this is not a necessary requirement. For example, there will be no distortions if there are non-zero tariffs but the substitution matrix vanishes ($\mathcal{S} = 0$). This is the case if equilibrium occurs where the production possibilities frontier and the indifference curve have a "corner". As another example, if the tariff vector is uniform in the sense that $t^0 = \kappa\iota$, where ι is a vector of ones, then it is readily shown that $\mathcal{S}t^0 = 0$.

commodities. These indifference curves are rays emanating from the point $-\iota = (-1,\ -1)'$ due to the fact that the net expenditure function is homogeneous of degree one and, hence, the indirect utility function is homogeneous of degree zero, in domestic prices, $\pi = P(\iota + t)$, and in tariff factors, $(\iota + t)$. Point A is the highest utility point corresponding to free trade (with the domestic price vector equalling the world price vector, p), but all points (such as C) along the ray through A have the same utility since they all imply the same relative domestic prices. Point B denotes the initial tariff vector t^0, which involves an import tariff on good 1 and free trade in good 2. The tangent to the indifference curve passing through point B is the indifference curve itself and orthogonal (at right angle) to the tangent is the gradient vector, $\nabla U(t^0)$, as depicted. This gradient vector indicates the direction of steepest ascent at the point B, and so is the direction of the steepest ascent tariff reform. This reform requires a decrease in the tariff rate on good 1 but an increase in the tariff rate on good 2 (which is initially zero). If good 1 is imported and good 2 exported, this means that the import tariff on good 1 is increased while an export subsidy is imposed on good 2. This reform achieves the greatest slope (directional derivative) of the utility function of any direction of tariff reform and, hence, the greatest differential increase in utility.

Figures 1 and 2: (about here)

Figure 2 illustrates the steepest ascent tariff reform for an economy with three traded goods. To facilitate this illustration in two dimensions, we assume that the tariff rate for good 1 is zero and show a slice of the indifference curve map at $t_1 = 0$ in $\tau = (t_2, t_3)'$ space. Point A is the highest utility point corresponding to free trade. Point B denotes the initial tariff vector for goods 2 and 3, $\tau^0 = (t_2^0, t_3^0)'$. The tangent to the indifference curve passing through point B is depicted in the figure, as is the portion of the gradient vector corresponding to goods 2 and 3, denoted $\nabla_\tau U(t^0)$, which is orthogonal to the tangent of the indifference curve at point B. As in the previous figure, this vector indicates the direction of steepest ascent at the point B. This is the direction of the steepest ascent tariff reform for goods 2 and 3. If goods 2 and 3 are both imported, this reform requires a reduction in the tariff on good 3 but an increase in the tariff on good 2.

2.2 Properties of Steepest Ascent Tariff Reforms

The steepest ascent tariff reforms have several interesting and useful features that we now record and discuss.

The first property is not obvious and has important implications for locally optimal tariff reforms.

Corollary 1.1 *The steepest ascent (locally optimal) tariff reform on all goods, δ^S, satisfies the condition $\pi^{0\prime}P^{-1}\delta^S = 0$ and therefore has both positive and negative elements.*

Proof. Pre-multiply the steepest ascent tariff reform by the inverse of the world price matrix and then by the domestic price vector to get

$$\pi^{0\prime}P^{-1}\delta^S = \lambda\pi^{0\prime}P^{-1}PS_{\pi\pi}(\pi^0, u^0)Pt^0 = \lambda\pi^{0\prime}S_{\pi\pi}(\pi^0, u^0)Pt^0 = 0,$$

since $\pi^{0\prime}S_{\pi\pi}(\pi^0, u^0) = 0$ from the homogeneity properties of the net expenditure function. Since $\pi^0 P^{-1}$ (the vector of ratios of domestic to world prices) is positive, by the assumption of positive prices, it is clear that δ^S has to have both positive and negative elements. ■

According to this corollary to Proposition 1, locally optimal reforms require that the tariff rate on at least one product be increased along with a reduction in the tariff on at least one other product.[9] Of course, as usual, we have to be careful here in the interpretation of the corollary as the 'tariff' is really a trade tax - a tariff on an imported good, but a subsidy on an exported good.

In the case illustrated in Figure 1, if good 1 is imported and good 2 is exported the steepest ascent tariff reform requires a decrease on the duty on the imports of good 1 ($\delta_1 < 0$) and the introduction of a subsidy on the exports of good 2 ($\delta_2 > 0$). In more general situations with more than two goods, the steepest ascent tariff reform may well require that some import duties increase. This is, of course, entirely possible as a result of the well-known theorem of the second best; welfare improvements may require some tax rates to rise.

Second, it is evident from the expression for the steepest ascent tariff reform that the

[9]The proposition does not apply to the case where the tariff reform is restricted to non-numeraire goods, which is dealt with further below.

sign structure of δ^S depends upon the initial tariff vector, t^0, and the initial substitution matrix, $S_{\pi\pi}(\pi^0, u^0)$, and upon how these combine. To get some understanding of this relationship, we write out the elements of δ^S in component form and express them in a form that lends itself to interpretation. Specifically, we examine the steepest ascent direction of reform to determine situations when a tariff on a good will be reduced and situations when the tariff will be raised.

The locally optimal direction for tariff reform may be expressed as follows. In this expression, the reform for product i is written as:

$$\begin{aligned} \delta_i^S &= \lambda p_i \sum_{j=1}^{n} S_{ij}(\pi^0, u^0) p_j t_j^0, \quad \lambda > 0, \\ &= -\lambda p_i \sum_{j \neq i}^{n} S_{ij}(\pi^0, u^0)(\sigma_i^0 - \sigma_j^0)\pi_j^0, \qquad i = 1, 2, ..., n. \end{aligned} \tag{7}$$

This expression gives an indication of how the locally optimal direction of tariff change relates to the (cross-product) substitution terms S_{ij} and the ad valorem (with domestic price bases) tariff rates $\sigma_j^0 \equiv p_j t_j^0 / \pi_j^0$.

In general, expression (7) indicates the requirements for a locally optimal reform to involve an increase ($\delta_i^S > 0$) or a decrease ($\delta_i^S < 0$) in the tariff rate on a good i. The higher is the tariff on good i relative to other tariff rates and the more substitutable good i is with other goods, the more likely it is that a locally optimal tariff reform involves a reduction in the tariff on good i. Conversely, if good i is complementary with a good j ($S_{ij} < 0$) and it has a lower tariff than j then that combination contributes to a reduction in the tariff rate on good i. That is about as much that (7) allows us to say in general.

However, more precise statements may be made if we are prepared to make assumptions about the initial tariffs and the sign structure of the initial substitution matrix. To illustrate this point and to thereby get a better understanding of this expression, consider a concertina commodity. We say that good i is a *concertina commodity* if (a) $\sigma_i^0 - \sigma_j^0 > 0$ for all $j \neq i$ and (b) $S_{ij}(\pi^0, u^0) > 0$ for all $j \neq i$. Thus, good i is a concertina commodity if it has the highest ad valorem tariff rate and it is a net substitute for all other goods. Under this definition, we see that the right hand side of (7) is negative. Thus, the locally optimal tariff reform involves a reduction in the tariff rate on a concertina good. Accordingly, we get a very precise result from (7) if good i has concertina good properties: if a concertina good exists, the steepest ascent reform demands that its tariff be reduced as part of the reform.[10]

Corollary 1.2 *If a concertina good exists, the steepest ascent reform demands that its tariff be reduced as part of the reform.*

Of course, there can be at most a single concertina good (excluding the case where there are several goods with the same highest ad valorem tariff rates) so there always remains the issue of whether the tariffs on the non-concertina goods rise or fall as part of the steepest ascent reform. As a final observation, it is important to distinguish Corollary 1.2 from the well-known concertina theorem. This theorem states that a *unilateral* reduction of the tariff on a concertina commodity (as defined above) is guaranteed to raise welfare. By contrast, our steepest ascent tariff reform involves the reform of all tariffs. What the above corollary establishes is that, as part of that reform, the tariff on a concertina good is to be reduced.

2.3 Sources of Potential Welfare Gains

It was shown above that the steepest ascent tariff reform is welfare improving and is locally optimal in that it provides the highest level of welfare increase of any tariff reform. Here we show that the increase in welfare may be expressed in terms of two "sufficient statistics" that fully describe the distortions in the tariff structure. We call these the generalized mean and generalized variance, but it must be emphasized that these are different from (but inspired by) the concepts of the same name introduced by Anderson and Neary (2007).

To simplify notation, let t be the initial tariff vector (without the 0 superscript). Thus, the welfare change arising from the steepest ascent tariff reform applied to all goods may

[10] On the other hand, if good i satisfies property (b) but has the *lowest* ad valorem tariff rate (the opposite of property (a)), then the right hand side of (7) is positive and so the tariff rate on such a good is to be raised. Again, we get a precise result.

be expressed as

$$\eta du^S/d\alpha = t'\mathcal{S}'\mathcal{S}t. \tag{8}$$

where $\eta = H\left\|\mathcal{S}t\right\|$ is a positive scalar. This expression may be rewritten in terms of new concepts describing the level and dispersion of the initial tariff rates, namely the generalized mean tariff rates and the generalized variances and covariances of the tariff rates, as demonstrated in the following proposition.

Proposition 2 *The change in utility for the steepest ascent tariff reform may be expressed as*

$$\eta du^S/d\alpha = t'\mathcal{S}'\mathcal{S}t = V + s\bar{t}^2, \tag{9}$$

where $\eta = H\left\|\mathcal{S}t\right\| > 0$*,* $s \equiv \iota'\mathcal{S}'\mathcal{S}\iota$*,* $\bar{t} \equiv \iota'\mathcal{S}'\mathcal{S}t/s$ *and* $V \equiv (t-\iota\bar{t})'\mathcal{S}'\mathcal{S}(t-\iota\bar{t})$.

Proof. Consider any scalar $\bar{t}$ and let ι be a vector of ones. Write the change in utility as $\eta du^S/d\alpha = t'\mathcal{S}'\mathcal{S}t = (t-\iota\bar{t}+\iota\bar{t})'\mathcal{S}'\mathcal{S}(t-\iota\bar{t}+\iota\bar{t}) = (t-\iota\bar{t})'\mathcal{S}'\mathcal{S}(t-\iota\bar{t})+\bar{t}^2\iota'\mathcal{S}'\mathcal{S}\iota+2(t-\iota\bar{t})'\mathcal{S}'\mathcal{S}\iota\bar{t}$. Defining $V \equiv (t-\iota\bar{t})'\mathcal{S}'\mathcal{S}(t-\iota\bar{t})$, the first term in the previous expression for the welfare gain is V. Defining $s \equiv \iota'\mathcal{S}'\mathcal{S}\iota$, the second term becomes $s\bar{t}^2$. Finally, the third term becomes zero if we define $\bar{t}$ as $\bar{t} \equiv \iota'\mathcal{S}'\mathcal{S}t/s$. ■

In this expression, $\bar{t}$ is defined as the *generalized mean tariff rate*. If $\mathcal{S}'\mathcal{S}$ were to be the identity matrix, then $\bar{t}$ would coincide exactly with the arithmetic mean of the tariff rates. The transformation of the tariff rates by the matrix $\mathcal{S}'\mathcal{S}$ leads to the use of the qualifier "generalized" in the name of $\bar{t}$. In a similar fashion, we denote V to be the *generalized variance of the tariff structure*. V would be the conventional variance of the tariff rates if $\mathcal{S}'\mathcal{S}$ had been the identity matrix. The generalized variance will generally be positive unless all tariff rates are equal, in which case it becomes zero. Clearly, therefore, $\bar{t}$ and V respectively measure the level and the dispersion of the tariff rates.

Proposition 2 is useful in that it provides a description of the sources of welfare gain from the steepest ascent tariff reform expressed in terms of the level and dispersion of the initial tariff rates. This formula has several interesting features and properties. First, both the mean and variance enter the formula positively, implying that the welfare gain is higher the greater is the generalized mean and the greater is the generalized variance.

Second, this implies that the welfare gain from a steepest ascent tariff reform is greater the higher is the overall level of tariffs as measured by the generalized mean, $\bar{t}$. This makes sense, since large distortions suggest that tariff reform will be very effective. Third, it also implies that the welfare gain is greater the greater is the overall dispersion of the tariff rates, as measured by the generalized variance, V. This indicates that it is not just levels, but dispersion of tariff rates that characterize distortions.

Fourth, the generalized mean and variance measure different aspects of the tariff distortion. The generalized mean tariff rate measures the distortion due to the level of the tariff structure. In the special case where all tariffs equal the scalar β ($t = \beta\iota$), it is readily shown that $\bar{t} = \beta$, $s = 0$ and $V = 0$. Thus, as expected, the generalized mean tariff rate coincides with the uniform rate on every good, β. Because $s = 0$ and $V = 0$, the formula in (9) shows a zero welfare gain. This reflects the well-known observation that an equilibrium with no distortions in the tariff structure (uniform tariffs on all goods imply that relative domestic and world prices are the same) is equivalent to free trade. Accordingly, the best (steepest ascent) tariff reform is no reform and no welfare gain is possible. When the tariff rates are not all equal, s and V will be non-zero. In this case, even if the mean tariff rate were zero, there would still be a distortion captured by the generalized variance being positive due to a dispersion of the tariff rates around the mean. In general, both $\bar{t}$ and V will be non-zero, each capturing (exhaustively) different aspects of the distortions created by the tariff structure.[11]

2.4 Reforms Restricted to Non-Numeraire Goods

It is customary in the literature on tariff analysis to choose the first good as the numeraire (with unit price) and to assume that its tariff is zero and remains such in the tariff reform process. In this sub-section, we consider the steepest ascent tariff reform applied to only the tariffs on non-numeraire goods and demonstrate that the resulting welfare gain is less

[11] Below, we consider tariff reforms restricted to tariffs on non-numeraire goods. An analogous decomposition of the welfare gain from the steepest ascent reform of this restricted set of tariffs may be readily obtained and interpreted. This decomposition, expressed in terms of the generalized mean and generalized variance for non-numeraire tariffs, is derived in the Appendix for the case of a zero tariff on the numeraire good.

than can be achieved by reforming all tariffs.

To develop these results, we choose good 1 as the numeraire whose tariff is set at zero and therefore decompose the domestic price, world price and tariff vectors as $\pi' = (1 \quad \rho')$, $p' = (1 \quad q')$ and $t' = (0 \quad \tau')$. Thus, q denotes the world price vector for non-numeraire goods, ρ denotes the domestic price vector for non-numeraire goods and τ denotes the vector of tariffs on non-numeraire goods. The substitution matrix may be similarly decomposed. To simplify notation, we write Σ as the block of the 'normalized' substitution matrix at the equilibrium, $\mathcal{S} \equiv P' S_{\pi\pi}(\pi^0, u^0) P$, corresponding to the non-numeraire goods. That is,

$$\mathcal{S} = \begin{bmatrix} \mathcal{S}_{11} & \mathcal{S}_{1\cdot} \\ \mathcal{S}_{\cdot 1} & \Sigma \equiv \mathcal{S}_{\cdot\cdot} \end{bmatrix}.$$

It is evident from the previous discussion of the steepest ascent reform concept that the steepest ascent reform of the tariffs on non-numeraire goods requires us to only use the gradient vector without the first element. Thus, the steepest ascent direction for *non-numeraire* goods and the corresponding directional derivative may be expressed as in the following proposition.

Proposition 3 *The steepest ascent tariff reform on non-numeraire goods alone is given by* $d\tau^0 = \delta_{\cdot}^N d\alpha, d\alpha > 0$, *where*

$$\delta_{\cdot}^N = \lambda^N \Sigma \tau^0, \quad \lambda^N = \left\| \Sigma \tau^0 \right\|^{-1}, \tag{10}$$

and the corresponding steepest ascent slope of the utility function is $D(t^0, \delta^N) = H^{-1} \left\| \Sigma \tau^0 \right\|$, *where* $H \equiv p' S_{\pi u}(\pi^0, u^0) > 0$ *is the Hatta normality term and the full direction vector is* $\delta^{N\prime} = (0, \ \delta_{\cdot}^{N\prime})$.

Proof. The proof follows directly from the observation that only the non-numeraire portion of the gradient vector of the utility function is relevant. Using the assumption that the first tariff is zero, the gradient for non-numeraire goods is $\Sigma \tau^0 / H$. Normalizing it to be of unit length yields the optimal direction vector $\lambda^N \Sigma \tau^0$, where $\lambda^N = \left\| \Sigma \tau^0 \right\|^{-1}$, and the directional derivative is $D(t^0, \delta^N) = H^{-1} \left\| \Sigma \tau^0 \right\|$. ■

These two expressions, for the direction of tariff reform and the resulting change in utility, depend only upon the portion of the substitution matrix relating to non-numeraire goods, $\Sigma = \mathcal{S}_{\cdot\cdot}$, and the non-numeraire tariff vector, τ^0. It should be evident that the steepest ascent direction vector, $\delta_{\cdot}^{N}$, is the non-numeraire portion, $\delta_{\cdot}^{S}$, of the vector δ^S, apart from a factor of proportionality. Thus, signs of the directions of tariff reform for non-numeraire goods are the same as when the reform is applied to all goods.

As noted above, it is customary in the literature to assume that the numeraire good is not subject to tariffs and that the tariff reform is therefore restricted to non-numeraire goods. This can always be done without loss of generality, as is well known. Specifically, the tariff vector can always be chosen with the tariff rate on any one good being zero and it is customary (but not necessary) to choose this good to be the numeraire. The consequence is that we can undertake our analysis of tariffs in a model where there is a numeraire that is not taxed and not subject to the tariff reform or, equivalently, undertake the analysis using all goods. The equilibrium for all quantities will be unaffected by this choice.

However, when undertaking differential tariff reforms, the reform of the tariffs of non-numeraire goods only may not be equivalent to the reforms of the tariffs of all goods. There may be a welfare loss to restricting attention to non-numeraire goods. This possibility is established in the following proposition.

Proposition 4 *The steepest ascent tariff reform on only non-numeraire goods generally yields a smaller directional derivative of the utility function at the initial tariff vector than the steepest ascent reform of the tariffs on all goods. The two directional derivatives are equal if, and only if,* $\partial U(t^0)/\partial t_1 \equiv H^{-1}\mathcal{S}_{1\cdot}\tau^0 = 0$.

Proof. The directional derivative for the case of a steepest ascent reform of all tariffs is

$D(t^0, \delta^S) = H^{-1} \|\mathcal{S}t^0\|$. This expression may be decomposed as

$$\begin{aligned} D(t^0, \delta^S)^2 &= H^{-2} \left\|\mathcal{S}t^0\right\|^2 \\ &= H^{-2} t^{0\prime} \mathcal{S}' \mathcal{S} t^0 \\ &= H^{-2} \tau^{0\prime} \mathcal{S}_{1\cdot}' \mathcal{S}_{1\cdot} \tau^0 + H^{-2} \tau^{0\prime} \Sigma' \Sigma \tau^0 \\ &= H^{-2} \left\|\mathcal{S}_{1\cdot} \tau^0\right\|^2 + H^{-2} \left\|\Sigma \tau^0\right\|^2 \\ &= H^{-2} \left\|\mathcal{S}_{1\cdot} \tau^0\right\|^2 + D(t^0, \delta^N)^2. \end{aligned}$$

Thus, $D(t^0, \delta^S) = D(t^0, \delta^N)$ if, and only if, $\mathcal{S}_{1\cdot}\tau^0 = 0$. ■

This proposition has interesting implications for the comparison of the welfare effectiveness of different tariff reforms. It must be carefully interpreted, however. This result applies to differential tariff reforms and compares slopes of the utility function in different directions. It states that the slope of the utility function in the direction of the steepest ascent reform of all tariffs is generally, except in very special circumstances, greater than the slope in the direction of the steepest ascent reform of tariffs on only non-numeraire goods.[12] That is, restricting the tariff reform space restricts the welfare gain from differential tariff reforms.

While this proposition is of novel theoretical interest, it may be of lesser practical importance. In practice, the steepest ascent (or any) reform would be to take a finite step in the steepest ascent direction. If the two reforms (of all goods and of non-numeraire goods only) were restricted to be of equal length then the proposition would imply that the welfare gain achieved under the reform of only non-numeraire goods is less than that achieved by reforming all tariffs.[13] In this case, the proposition has important practical implications.

The implications of the proposition may be illuminated by looking at a comparison of the two reforms in another way. Since the reform of tariffs on only non-numeraire goods

[12] As indicated in the statement of the proposition, the special circumstance occurs only when $\partial U(t^0)/\partial t_1 = 0$. That is, the gradient with respect to the first tariff is zero meaning that the tariff on that good is already optimally set. Removing this good from the tariff reform therefore is of no welfare consequence. Thus, in this case, the directional derivatives for the two tariff reforms (on all goods and on non-numeraire goods) are identical.

[13] Formally, $dU^S/d\alpha = D(t^0, \delta^S) > D(t^0, \delta^N) = dU^N/d\alpha$, where $d\alpha$ is the step length.

has a smaller directional derivative than the reform of the tariffs on all goods and hence a smaller welfare gain when the reforms are of equal step length, equal welfare gains can only be obtained if the non-numeraire reform is permitted to have a larger step length.

This point may be illustrated in Figure 3. In this figure, good 2 is the numeraire and point B is the tariff vector that corresponds to a tariff on good 1 and free trade in good 2. A steepest ascent reform of both tariffs, with step size given by the radius of the circle illustrated, moves the domestic price vector to point C. If the reform is restricted to non-numeraire goods then the best such reform of the same step length takes the domestic price vector to point E. It is evident that the point E is on a lower indifference curve than point C, illustrating the welfare loss involved in restricting the reform to a subset of goods. This welfare loss can only be recovered by choosing to increase the reform step size sufficiently to move the domestic price vector to point D. Since points D and C correspond to the same ratios of domestic prices and are therefore on the same indifference curve, the welfare gains from the two reforms are now equal.

Figure 3: (about here)

The upshot of this discussion is that comparisons of the welfare gains from different reforms need to be carefully made. The steepest ascent reform on all goods has, by construction, the largest directional derivative. Other reforms, including steepest ascent reforms restricted to non-numeraire goods or to other subsets of goods, necessarily (except in special circumstances) have smaller directional derivatives. They therefore have smaller welfare gains when the step lengths of the reforms are equal (as is implicitly the case when one compares expressions $dU/d\alpha$ for reforms $dt = \delta d\alpha$). Conversely, these reforms can only achieve the same welfare gains as the steepest ascent reform on all goods by having larger reforms with larger step sizes.

3 Welfare Efficiency of Existing Tariff Reforms

Having examined the properties of the steepest ascent tariff reform, we now compare it with the well-known reforms examined in detail in the existing literature. We start with some theoretical results concerning the optimality of some existing tariff reforms, then compare these reforms and finally move on to some numerical simulations that exemplify the main points of our analysis.

Since the steepest ascent tariff reform is locally optimal, it forms a benchmark by which any other tariff reform formula may be evaluated. The two most familiar tariff reforms for a small open economy are (a) proportional reductions in all tariffs and (b) the concertina tariff reform, whereby the highest ad valorem rate is reduced (or the lowest ad valorem rate is increased). The concertina reform is simply a univariate reform, where the good is chosen with special characteristics. Formally the proportional and univariate tariff reforms are defined in terms of the reform direction vectors, which are given by $\delta^P = -t^0/\left\|t^0\right\|$ and $\delta^i = -e_i, \quad (i = 1, 2, ..., n)$, where e_i is the ith unit vector.

The welfare changes arising from the steepest ascent, proportional and univariate reforms may be expressed as

$$H\ du^S/d\alpha = H\ D(t^0, \delta^S) = \left\|\mathcal{S}t^0\right\|, \tag{11}$$

$$H\ du^P/d\alpha = H\ D(t^0, \delta^P) = -\left\|t^0\right\|^{-1}\ t^{0\prime}\mathcal{S}t^0, \tag{12}$$

$$H\ du^i/d\alpha = H\ D(t^0, \delta^i) = -t^{0\prime}\mathcal{S}e_i, \tag{13}$$

where $H > 0$ is the previously defined Hatta normality term. It is well known that the proportional tariff reform is welfare improving under the weak normality condition that $H > 0$ and the assumption that the initial tariff rates are distortionary. The proof follows from (12) by noting that $t^{0\prime}\mathcal{S}t^0 < 0$ due to S being negative semi-definite and the assumption that $\mathcal{S}t^0 \neq 0$. Similarly, it is also well known that the reduction of a tariff for a concertina good (which has the highest tariff rate and is a net substitute for all other goods) will be welfare increasing under the same normality assumption.

Although it is clear from the definition of the steepest ascent tariff reform that any other tariff reform cannot do better than the steepest ascent tariff reform in terms of raising welfare, the interesting question arises as to whether a particular tariff reform can do as well as the steepest ascent tariff reform and, hence, be locally optimal. If it can achieve this outcome, it is also interesting to establish the conditions required for local optimality. This is a question that does not appear to have been previously addressed in the literature.

3.1 Proportional Tariff Reduction Reform

First, we consider the proportional tariff reform. We ask whether and, if so, under what circumstances the proportional tariff reduction reform will be locally optimal. The answer is provided in the following proposition.

Proposition 5 *The proportional tariff reduction reform is locally optimal if, and only if, the initial tariff vector, t^0, is an eigenvector of the substitution matrix $\mathcal{S}$.*

Proof. This result is proved using the Cauchy-Schwarz inequality (Fulks, 1969, p.176), which states that $\|x\| \cdot \|y\| \geq |x'y|$ for any two n-dimensional vectors x and y with equality holding if, and only if, one of the vectors is a scalar multiple of the other. In the following, we let $x = \mathcal{S}t^0$ and $y = t^0$ and apply the inequality to get that $\|\mathcal{S}t^0\| \cdot \|t^0\| \geq |t^{0\prime}\mathcal{S}t^0|$ (recalling that $\mathcal{S}$ is symmetric). Using this inequality, the negative semi-definiteness of $\mathcal{S}$ and equations (11) and (12), we obtain that $Hdu^P/d\alpha = D(t^0, \delta^P) = -\|t^0\|^{-1}\ t^{0\prime}\mathcal{S}t^0 = \|t^0\|^{-1} |t^{0\prime}\mathcal{S}t^0| \leq \|t^0\|^{-1}\ \ \|\mathcal{S}t^0\| \cdot \|t^0\| = \|\mathcal{S}t^0\| = Hdu^P/d\alpha$, with equality holding if, and only if, one of the vectors $x = \mathcal{S}t^0$ and $y = t^0$ is a scalar multiple of the other. That is, the utility gain from the proportional tariff reform will equal the maximal possible utility gain obtained from the steepest ascent tariff reform if, and only if, the vectors are proportional to one another, which means that $\mathcal{S}t^0 = \kappa t^0$ for some scalar κ. This, in turn, means that κ is an eigenvalue for matrix $\mathcal{S}$ and that t^0 is the corresponding eigenvector. The substitution matrix, $\mathcal{S}$, is a symmetric, negative semi-definite matrix and so has n real eigenvalues and corresponding eigenvectors. Thus, there exist n real valued eigenvectors of the substitution matrix $\mathcal{S}$. If the initial tariff vector coincides with any one of these vectors, then $\|\mathcal{S}t^0\| \cdot \|t^0\| = |t^{0\prime}\mathcal{S}t^0|$ (by the the Cauchy-Schwarz inequality) and so $du^P/d\alpha = du^S/d\alpha$. If the initial tariff is not equal to one of these eigenvectors, then the strict inequality holds in the Cauchy-Schwarz inequality and so $du^P/d\alpha < du^S/d\alpha$. ■

This proposition characterizes the necessary and sufficient conditions under which the proportional reduction reform is locally optimal. Of course, the condition holds trivially when $t^0 = 0$ (free trade) and when t^0 is uniform (effectively free trade), but it may also hold at a non-trivial tariff vector that is an eigenvector for the substitution matrix. If the initial tariff vector is an eigenvector, local optimality of the proportional reduction reform is assured.

While this proposition relates to reforms of the tariffs on all goods, it may be readily extended to reforms on the tariffs of non-numeraire goods only. In the above statement and proof, we simply restrict attention to non-numeraire goods and so replace δ^S by $\delta^N_\cdot$, t^0 by τ^0 and $\mathcal{S}$ by Σ. Using a similar proof, it follows that the proportional tariff reduction policy for all non-numeraire goods is optimal if, and only if, the initial non-numeraire tariff vector, τ^0, is an eigenvector for the non-numeraire substitution matrix, Σ. Figure 4 illustrates such a case for two non-numeraire goods, where the steepest ascent direction is proportional to the tariff vector and points directly at the world price vector.

Figure 4: (about here)

3.2 Univariate Tariff Reduction Reform

We now consider the conditions under which a univariate tariff reform, in which the tariff on a single good is reduced, is locally optimal.

First, it should be immediately obvious that a univariate tariff reform cannot be welfare equivalent to the steepest ascent tariff reform applied to all goods. This follows because Corollary 1.1 states that the steepest ascent tariff reform applied to all goods must a direction vector having both positive and negative elements, while the univariate reform has only one non-zero element that is negative. The univariate reform must, therefore, be sub-optimal.

Second, it was previously noted that Corollary 1.1 does not apply to a steepest ascent tariff reform applied to only non-numeraire goods. Accordingly, it is possible that a univariate reduction in a single tariff rate could be locally optimal when considered in the context of tariff reforms restricted to the non-numeraire subset of goods. This is indeed the case, as the following proposition establishes.

Proposition 6 *Let the normalized substitution matrix Σ for non-numeraire goods be of maximal rank, $n-1$, where n is the number of goods. Then a necessary condition for the univariate tariff reform, in which the tariff on good n (for example) is reduced, to be locally optimal for the class of tariff reforms on only non-numeraire goods is that the initial tariff vector for non-numeraire goods, τ^0, satisfies the condition $\left\|\Sigma\tau^0\right\|^{-1}\cdot\Sigma\tau^0=-e_n$. Equivalently, the necessary and sufficient conditions are that the initial tariff vector, τ^0, satisfies (i) $\tau^0_{\cdot}=-\Sigma_{\cdot\cdot}^{-1}\Sigma_{\cdot n}\tau^0_n$ and (ii) $\tau^0_n=-\left[\Sigma_{nn}-\Sigma_{n\cdot}\Sigma_{\cdot\cdot}^{-1}\Sigma_{\cdot n}\right]^{-1}\left\|\Sigma\tau^0\right\|>0$.*

Proof. To prove this result, note that the univariate reform direction is locally optimal if, and only if, it coincides with the steepest ascent tariff reform direction for non-numeraire goods. Using equations (10) and (13), this means that $\delta^N_{\cdot}=\left\|\Sigma\tau^0\right\|^{-1}\Sigma\tau^0=-e_n$, as required. This can be split into two separate equation systems - the nth equation and the remainder - as follows:

$$\left\|\Sigma\tau^0\right\|^{-1}\begin{bmatrix}\Sigma_{\cdot\cdot} & \Sigma_{\cdot n}\\ \Sigma_{n\cdot} & \Sigma_{nn}\end{bmatrix}\begin{bmatrix}\tau^0_{\cdot}\\ \tau^0_n\end{bmatrix}=\begin{bmatrix}-1\\ 0\end{bmatrix}.$$

Solving each block sequentially, the conditions (i) and (ii) in the statement of the proposition are obtained. Positivity of τ^0_n follows from the negative semi-definiteness property of Σ. ■

While the proposition contains conditions on the initial tariff vector and the substitution matrix that are complex, they simply require that (i) $\partial U(t^0)/\partial t_n=-1$ and (ii) $\partial U(t^0)/\partial t_i=0$ for all $i\neq 1,n$. That is, the optimal direction of tariff reform must be such that there is no welfare gain from changing any non-numeraire tariff except that on good n, and its tariff is to be reduced. In the special case of just two goods, with good 1 being the untaxed numeraire, the reduction of the tariff on good 2 (the only tariff) is identical to the proportional tax reform and both are locally optimal. More generally, the above proposition indicates the conditions under which the univariate reform is locally optimal. Clearly the conditions are rather strict.

Figure 5 illustrates this proposition for the case of two non-numeraire goods by showing a case where the steepest ascent direction is proportional to the ith unit vector and points directly downwards, indicating that the optimal policy is to reduce one tariff (on good 3) and leave the other (on good 2) unchanged.

Figure 5: (about here)

3.3 Comparisons of Reform Directions

Using the fact that the gradient vector and steepest ascent tariff reform vector are the same except for length and related by equation (3), the welfare changes for the steepest ascent tariff reform may be expressed as

$$du^S/d\alpha = D(t^0, \delta^S) = \theta \quad \delta^{S\prime}\delta^S,$$

where $\theta = H^{-1} \left\| \mathcal{S}t^0 \right\| > 0$. The welfare gain from an arbitrary reform in direction δ may be expressed as

$$du/d\alpha = D(t^0, \delta) = \theta \quad \delta'\delta^S.$$

These two expressions suggest a proposed *index of the welfare effectiveness* of the reform defined as

$$I(\delta) \equiv \delta'\delta^S/\delta^{S\prime}\delta^S = D(t^0, \delta)/D(t^0, \delta^S) \leq 1. \tag{14}$$

This index of welfare effectiveness measures how well any given reform performs relative to the best possible local reform, given by the steepest ascent reform. The index is equal to unity if the direction of tariff reform given by δ is equal to the steepest ascent reform direction, δ^S, meaning that the reform is locally optimal. Conversely, if these reform directions are orthogonal, the index is zero and the reform does not yield a gain in utility. If the angle between the direction vectors δ and δ^S is acute then the index is positive but less than unity; the smaller the angle the higher the index. Finally, if the reform direction δ is at an obtuse angle to δ^S (the inner product of the reform directions is negative) then there will be a welfare loss associated with the reform.

Figure 6 illustrates these observations for several tariff reforms in the case of two non-numeraire goods. In this figure, which illustrates one possibility and not a general result, the reform that yields the welfare gain closest to that attained by the steepest ascent reform is the reduction in the tariff on good 2 (δ^2). Next comes the proportional tariff reduction reform (δ^P), followed in last place by the reduction in the tariff on good 3 (δ^3), a reform that yields a welfare loss. The proportional tariff reduction reform yields modest welfare gains since it reduces both tariffs, while the locally optimal reform calls for an increase in the tariff on good 3. The reform reducing the tariff on good 2 alone does much better and its welfare gain is closest to that of the steepest ascent reform. The reduction of the tariff on good 3 is a bad choice of policy in this second best framework.

Figure 6: (about here)

3.4 Numerical Example

In this sub-section, we undertake some calculations of welfare gains from the steepest ascent, proportional and univariate tariff reforms using a numerical example. By undertaking these numerical simulations, we are able to gain some further insights into the welfare effectiveness of these tariff reforms.

Our illustrative example is for a model that has $n = 9$ goods. There is a zero tariff on good 1 and positive tariffs on the remaining goods so it is convenient (though not required) to interpret the first good as being exported and the remaining goods as imported.[14] Table 1 provides the results for the indices of welfare gains, relative to the steepest ascent reform, arising from the application of the various tariff reforms in this example.

The results in Table 1 starkly illustrate the observation that there can be wide variations in the effectiveness of the proportional and univariate (single good) tariff reforms relative to the (locally optimal) steepest ascent reform even when the reforms are restricted to tariffs of non-numeraire goods.[15] In this example, the proportional tariff reform is only about 29% effective in raising welfare, despite the fact that the gradient vector calls for

[14] Details of the initial tariffs, prices and substitution matrix used in this example are available from the authors.

[15] Many other examples, with different numbers of goods and substitution matrices, provided similar support for this observation.

Table 1: Indices of Welfare Gain from Various Tariff Reforms: Example with Nine Goods

	Goods Subject	**to Reform**
Tariff Reform	Non-numeraire	All goods
Steepest Ascent	1.0	1.0
Proportional	.289	.287
Good 2	.263	.260
Good 3	.159	.158
Good 4	.095	.094
Good 5	.179	.177
Good 6	.213	.211
Good 7	.015	.014
Good 8	.114	.113
Good 9	−.898	−.889

a reduction in the tariff of every good except one. The distortion on this good (good 9) is very large, however. Moreover, good 9 has the lowest tariff and is strongly net-substitutable with every good, except the numeraire. Consequently, it is a concertina good in reverse - welfare gains arise from raising its tariff, not reducing it as the proportional reform requires. This reduces the effectiveness of the proportional reduction tariff reform.

The univariate tariff reforms vary substantially in their effectiveness at raising welfare. For those univariate reforms that raise welfare, the indices of welfare gain vary from a high of 0.26 when the tariff on good 2 is reduced down to a gain of just 0.01 when the tariff on good 7 is reduced. Of course, the reduction of the tariff on good 9, which is a concertina good in reverse, yields a large reduction in the index of welfare gain. Conversely, if the tariff on this good were to be raised, the index of welfare gain would be large.

As a final observation on Table 1, we note that the efficiencies of the various reforms relative to the steepest ascent reform applied to all goods are only marginally less than their efficiencies relative to the steepest ascent reform applied to only non-numeraire goods. This result for the 9-good example is in contrast with results that we have obtained with a 3-good example, in which there is a substantial loss in welfare gain if the reform is restricted to the non-numeraire goods (illustrating Proposition 4). This difference in results suggests that the welfare loss associated with restricting the steepest ascent reform to non-numeraire goods is smaller, the larger the number of goods. As might be expected, the restriction on the single numeraire good takes lower importance due to the larger number of goods whose tariffs are reformed.

Before concluding, we note that, while specific examples cannot provide general conclusions, they do provide valuable illustrations of the relative merits of alternative tariff reforms in a range of contexts. Our example illustrates several main observations. First, the restriction of reforms (of the same length) to only non-numeraire goods can result in a substantial loss of welfare gain compared to allowing all tariffs to be reformed, but this loss is likely to be smaller the greater the number of traded goods. Second, the effectiveness of different reforms depends crucially upon the initial distortions captured by the tariff rates and the substitution matrix. Finally, there can be substantial variation in the welfare effectiveness of the various reforms such as the proportional and univariate reforms.

4 Concluding Remarks

The steepest ascent tariff reform concept proposed in this paper provides a standard or benchmark by which other reforms may be compared in terms of their effectiveness in generating welfare gains. Differential tariff reforms can, at best, attain the welfare gain that is achieved by the steepest ascent tariff reform, since it is locally optimal. Hitherto, the tariff reform literature has been focused on establishing reforms that yield a welfare improvement and has therefore been concerned only with the sign of the welfare effect. The motivation for the present paper has been two-fold. The first has been to characterize and establish the nature of the tariff reform that is locally optimal and this has yielded our steepest ascent tariff reform. The second motivation was to provide a benchmark tariff reform with which all other existing and potential reforms can be compared and to provide a means of determining their relative efficiency in yielding welfare improvements. The steepest ascent tariff reform also provides this outcome. In other words, our paper provides a different focus from that which has characterized the literature - one which is upon the size of the welfare effects rather than simply the sign.

We have established several properties of the steepest ascent tariff reform, characterized the sources of potential gains from tariff reforms and compared the welfare effectiveness of the proportional and univariate tariff reforms.

Not surprisingly, application of the steepest ascent tariff reform formula to an actual economy requires knowledge of the net substitution matrix at the initial equilibrium as well as the initial tariff rates and is therefore demanding in terms of information requirements. While the econometric estimation of the substitution matrix for a reasonably disaggregated set of commodities is not feasible, there are many computable general equilibrium models for various countries that have very detailed information about the production and consumption sectors that enable the computation of this net substitution matrix.[16] Accordingly, the calculation of the steepest ascent tariff reforms for such economies is already feasible for many countries thus making the concept of practical policy relevance as well as being a valuable theoretical benchmark for various reform proposals.

It should be emphasized that our analysis of tariff reform is local in that it is only concerned with differential reforms that are based completely on information (especially the substitution matrix) at the initial distorted equilibrium. In a broader policy context, policy makers may be interested in a series of discrete policy changes leading eventually to free trade. Our steepest ascent method can be used to undertake continuous or discrete policy changes in accordance with what is known as the steepest ascent algorithm for maximizing a function, here the utility function in tariff space. The steepest ascent direction is used to move discretely to a new equilibrium, and the process repeated until convergence. However, in this broader context, the steepest ascent algorithm may not necessarily be the best in reaching the first best. Other algorithms, such as the Newton algorithm (that maximizes a quadratic approximation of the initial equilibrium), for example, may well be superior. Additionally, policy makers may be concerned with the complete path to the first best policy solution, raising issues to do with constraints on policy change and the possibility of choosing the policy change algorithm to maximize an

[16] A similar issue could be raised with respect to the Anderson and Neary (1996) index of trade restrictiveness. However, and as it has been shown by the work of Anderson and Neary (2005) and by recent work of Kee et al. (2006), such indexes can indeed be measured with relative ease.

inter-temporal welfare function subject to these constraints. Again, the steepest ascent algorithm may not be the ideal method in a more general inter-temporal context. Having said that, the steepest ascent method that we propose here is particularly appropriate in our differential context, where only the information at the initial equilibrium is necessary and where the focus is on choose a policy direction to yield the greatest local welfare gain.

While our application of the steepest ascent tariff reform concept was undertaken in a model with no constraints on the nature of the reform other than being differential, it can also be applied in more general contexts. One possibility is to include costs of making tariff reforms into the model and to restrict the reform vectors to be of equal cost. Although we have followed the international trade literature in which revenue implication of reforms are typically not of primary concern, another potentially important possibility is to require that feasible tariff reforms have the same tariff revenue implications. The concept can also be applied when there are institutional or political constraints on directions of reforms. For example, some subset of goods may be excluded from the reforms. Compliance with World Trade Organization rules may require no tariff to increase and this restriction can be incorporated into the steepest ascent tariff reform concept.

Finally, it is noteworthy that the concept of a steepest ascent or locally optimal policy reform is not restricted to tariff reform but may be applied to any set of policy instruments. It could be applied, for example, to quotas on trade. Similarly it could be applied to the reform of domestic taxes. Also, the concept does not have to be restricted to measuring the gains in utility of a reform but can be applied to any objective function. As an example, the steepest ascent tariff reform may be applied where the objective function is a measure of market access. An alternative objective function might be the value of production in some domestic industry or in the income of a factor of production, to give just two additional examples. In short, the concept is applicable whenever we wish to determine the best possible piecemeal reform in terms of some objective function or to compare the effectiveness of alternative policy reforms in attaining that objective.

5 Appendix

An alternative decomposition of the potential welfare gain, expressed in alternative definitions of the generalized mean and generalized variance, is possible if one of the tariff rates has been normalized to zero. In particular, the welfare decomposition may alternatively be expressed as in the following amended statement of Proposition 2.

Non-numeraire Tariff Version of Proposition 2 *If the tariff on good 1 is zero, the change in utility for the steepest ascent tariff reform may be expressed as*

$$\eta du^S/d\alpha = \tau'\Sigma'\Sigma\tau = V^N + s^N\overline{\tau}^2,$$

where $\eta = H\,\|\mathcal{S}t\| > 0$*,* $s^N \equiv \iota'\Sigma'\Sigma\iota > 0$*,* $\overline{\tau} \equiv \iota'\Sigma'\Sigma\tau/s^N$ *and* $V^N \equiv (\tau - \iota\overline{\tau})'\Sigma'\Sigma(\tau - \iota\overline{\tau})$*.*

Proof. From Proposition 2, we have that $\eta du^S/d\alpha = t'\mathcal{S}'\mathcal{S}t$. Since the first tariff rate is zero by assumption and τ is the remaining sub-vector of t, this expression for the change in utility can be written as $\eta du^S/d\alpha = t'\mathcal{S}'\mathcal{S}t = \tau'\Sigma'\Sigma\tau$. The decomposition shown above follows by analogy with the proof in Proposition 2. ■

The interpretation is as previously provided for Proposition 2, except that now the mean and variance apply to the non-numeraire goods. To facilitate the exposition, suppose that good 1 is exported (all exported goods that have a zero tariff can always be aggregated into good 1) and that all other goods are imported. Then $\overline{\tau}$ is the generalized mean tariff rate for all imports and V^N is the generalized variance of these tariff rates for non-numeraire goods. The latter measures the dispersion of all import duty rates amongst all imported goods, as measured by the deviations from the generalized mean import duty rate. If all such rates are equal ($\tau = \gamma\iota$), it is readily shown that $\overline{\tau} = \gamma$ and $V^N = 0$. In this case of equal import duty rates, the higher is the common rate the greater is the optimal welfare gain. Also, even if the generalized mean rate is zero (but rates vary) there will be a potential welfare gain due to import duty rate dispersion as measured by the generalized variance.

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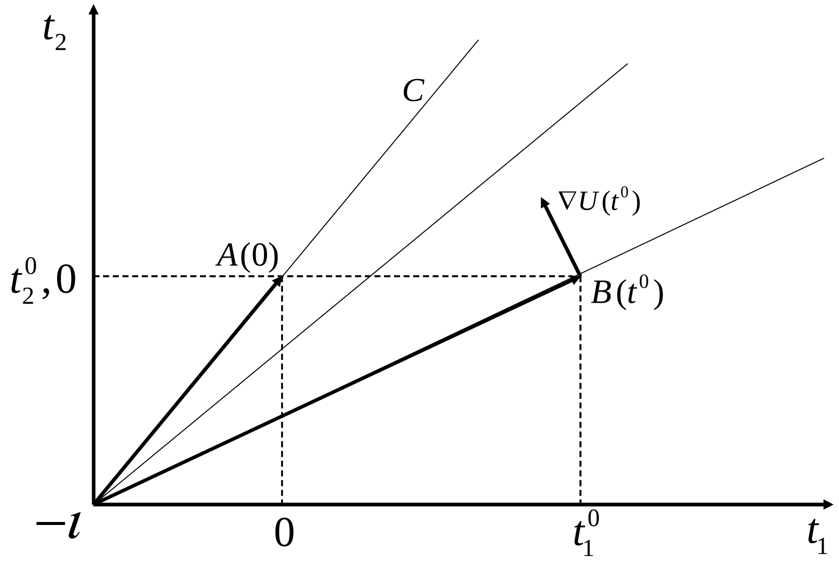

Figure 1: Steepest Ascent Tariff Reform with Two Goods

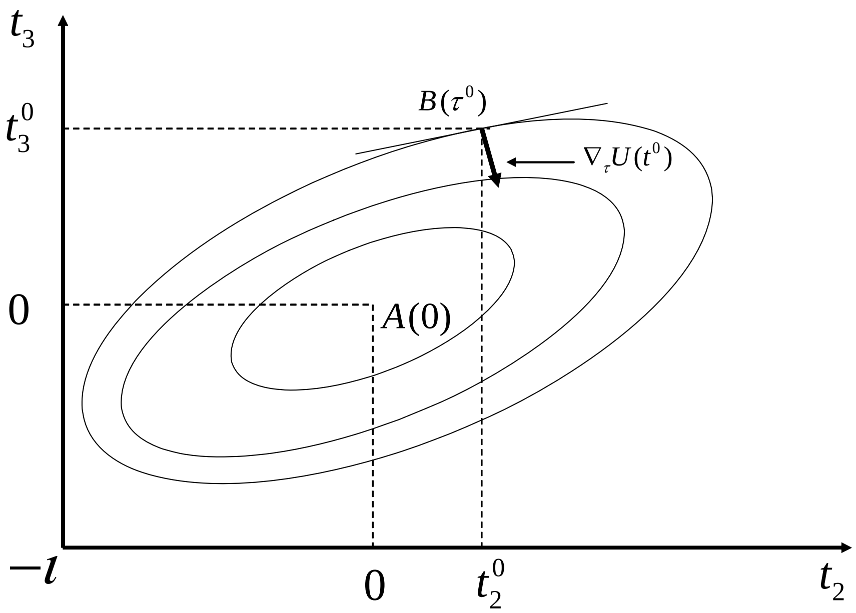

Figure 2: Steepest Ascent Tariff Reform with Three Goods

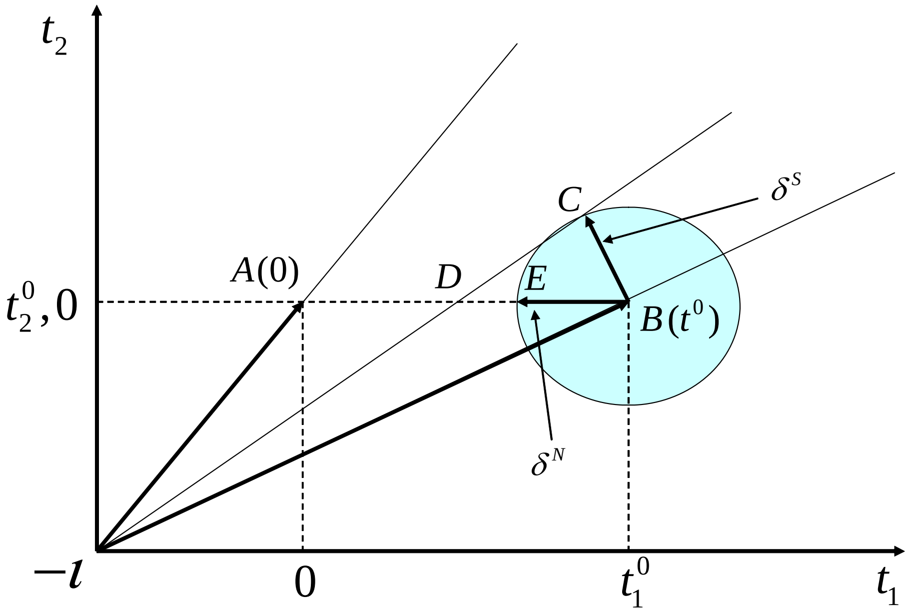

Figure 3: Complete and Restricted Tariff Reforms

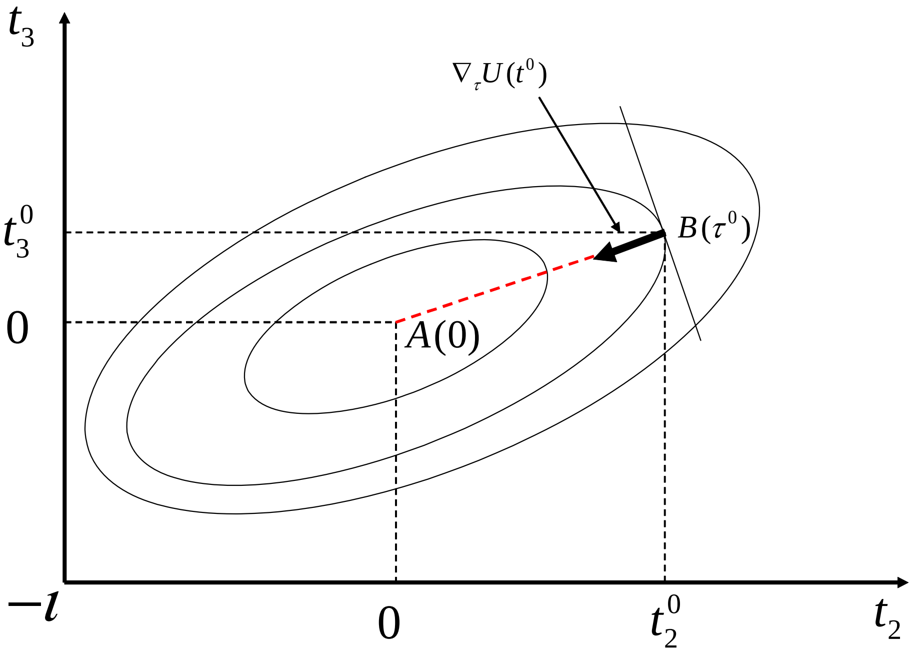

Figure 4: Proportonal Tariff Reform is Locally Optimal

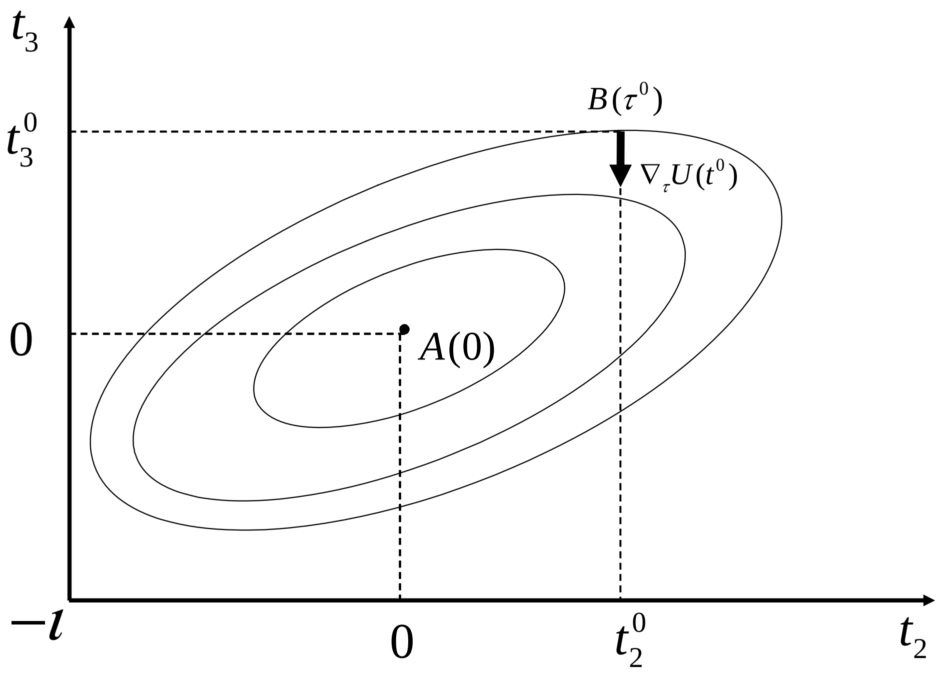

Figure 5: Univariate Tariff Reform is Locally Optimal

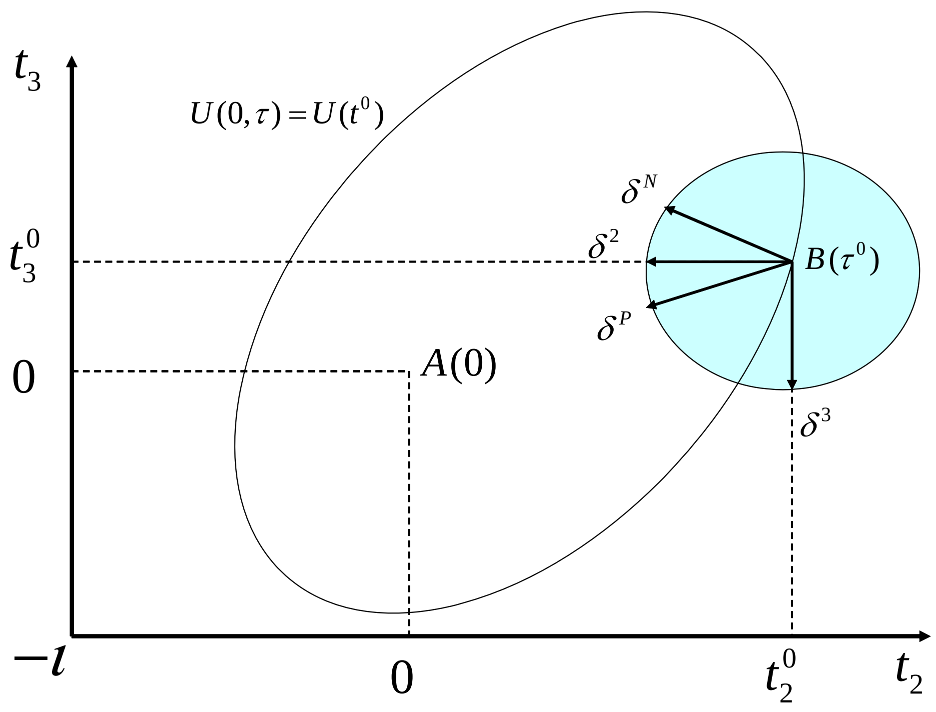

Figure 6: Directions of Tariff Reform and Welfare Gains