Gains from Migration in a New-Keynesian Framework

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Abstract

This paper presents a simple New-Keynesian small open economy model allowing for labour to be supplied both domestically and abroad with a locational preference to work at home rather than abroad. From this small change in the otherwise standard setup follows an important implication for the Phillips-curve: The opening of the "labour account" reduces the output-inflation trade-off, i.e. the Phillips-curve becomes flatter. The theoretic intuition is simple: Any given boost to output is followed by increasing real wages, marginal costs and thereby inflation if labour is the only input to production because workers need to be compensated for a reduction in leisure. However, to the extent to which this additional labour is mobilized from a substitution away from labour formerly supplied abroad, due to an improving real wage differential, there is no need for such a compensation. Hence real wages, marginal costs and inflation are less affected by the expansion.

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1 Introduction

In recent years researchers became increasingly aware of the fact that labour migration is not only a once-and-for-all move from one country to another:

> "[.] it has now become a reality that circular, repeat, recurrent, revolving door, multiple, frequent, repetitive, intermittent, seasonal, sojourning, cyclical, recycling, chronic or shuttling migration is a salient trait of migration." (Constant and Zimmermann, 2003a)

Despite lack of data for a broad evaluation of the phenomenon[1] some evidence has been brought forward in support of it. For Germany, Constant and Zimmermann (2003a,b) found that more than 60% of immigrants have exited Germany in the sample from 1984-1997 at least once and stayed in the country of origin for at least one year using a representative GSOEP data set. [2] As major reasons for an increased importance of non-permanent forms of migration relative to permanent migration improved communication technologies, allowing intensified ties of migrants with source countries, and cheap transportation making frequent return visits or circular migration patterns easier (O'Neil, 2003).

What could be the consequences for the macroeconomy of a country of origin when a significant share of the labour force emigrates temporarily? Below I will argue that to the extent that workers' labor supply decisions are affected significantly through working opportunities inside *and* beyond national borders because they will allocate their labour supply both abroad and domestically, firms' supply conditions are different than in a purely closed-labour-market economy[3]. Changing labour market conditions abroad are then likely to have spillover effects in the domestic labour market and changing domestic labour market conditions will affect the labour supplied in the rest of the world by emigrants. In particular, firms' marginal costs are affected if workers compare foreign and domestic wages and allocate hours accordingly. In New-Keynesian macroeconomic models, marginal costs play a crucial role in determining firms'

[1] Current data on migration flows are mainly based on census and administrative sources not able to capture the repetitive nature of a great proportion of current migration flows (O'Neil, 2003). Therefore compilation of more longitudinal data sets will be necessary for an appropriate assessment.

[2] Furthermore, they found that the frenquency of returns depends on the degree to which moving back and forth is restricted or not. Migrants more frequently returned to those countries of origin from which re-entry is easier. This is interpreted as being evidence in favor of a lock-in argument: The risk of not being able to continue to benefit from the higher wages abroad results in less frequent visits and a potentially reduced attachment to the host country. This could be the reason why *"Gastarbeiter"* from Turkey and former Yugoslavia had a much lower return rate than EU-nationals.

[3] This indicates that I focus on purely economically driven migration and abstract from other motives.

price-setting, inflation and the Phillips-curve trade-off. Hence all these macroeconomic variables can be expected to change in an open labour-market setting. I will present a model with the basic set-up of Galí and Monacelli (2005) for a small open economy, incorporating specific open labour-market characteristics.

Loungani et al. (2001) observed that Phillips curves tend to be the flatter the more open an economy is. Razin and Yuen(2002) showed this phenomenon in an open-economy version of the model in Woodford (2003, ch.3) in which the opening of the trade balance and the capital account both flattened the Phillips-curve. Whether the additional opening of the "labour account" works in the same direction, has not been considered in this literature yet. I will show that the Phillips curve will be flattened by the particular labour supply behaviour assumed in the model because of the return migration caused by economic growth beyond the natural level.

In this framework I will analyse the differential effects of productivity and demand shocks due to the open labour market setting. The increase in domestic output due to a demand shock is substantially higher when workers are allowed to migrate. Monetary policy plays an important role in this process because it can respond much more accomodative to the shock. This is because firms can tap the pool of returning workers rather than having to compensate workers for reduced leisure. Hence there are significant gains from migration from the perspective of the sending country.

How does this relate to existing research on temporary migration which does not tackle macroeconomic implications? In this literature it is assumed that people typically migrate to benefit from higher wages abroad while at the same time not intending to move permanently but rather support their families back home. This would indicate location specific preferences, an assumption used in Hill (1987), Djajic and Milbourne (1988) and Raffelhüschen (1992). Furthermore, the usually much higher purchasing power of the host country's currency in the home country's economy is another driving force for emigration, remittances and return migration (Dustmann, 1995,1997)[4].

With temporary migration the duration of a stay will constitute an important factor explaining the size of a diaspora and the flows of migrants which will be crucial for labour market conditions. So what determines durations? In a life-cycle analysis the real wage differential, expressed in terms of the sending country's consumption basket, is the crucial driving force. However, different shocks affecting the wage differential can have different implications for the duration of a stay abroad. Dustmann (2003) finds that from a theoretical point of view an increase in the domestic real wage unambiguously decreases the duration because substitution and income effects work in the same direction while a foreign real wage increase has substitution and income effects working in opposite directions. The substitution effect increases the stay while the income effect reduces it. The author finds evidence in favour of a domination of the income effect in an analysis using German GSOEP-data.

[4] Further motives put forward but unrelated to the present analysis are credit market rationing in sending countries (Mesnard, 2000), higher returns to human capital, acquired in the host country, in the sending country (Dustmann, 1995, 1997).

In contrast, Constant and Massey (2002) find no significant relationship between migrant earnings and returns in the same dataset. Borjas (1989) finds that higher earnings are associated with less returns among foreign-born in the US. In a more recent study Yang (2004) analyses the exogenous shock of the currency depreciation during the Asian financial crisis in 1997 on Phillipino migrants' foreign earnings and their effect on return decisions. His finding is that more favorable income shocks lead to fewer migrant returns[5]. Hence the empirical evidence on durations of stays abroad is mixed.

What these analyses do not take into account, however, is a possible link between foreign and domestic wages which migrants ultimately compare. If migrants consider the wage differential and allocate labour accordingly, for a country with a large share of migrants moving back and forth across borders, foreign and domestic real wages are no longer likely to be independent from one another. Increasing foreign wages, to the extent that they increase the absence of workers from a domestic employer's point of view, will drive up domestic wages if labour is scarce[6]. Then the analysis of Dustmann (2003) would have to be reconsidered and positive foreign income shocks are likely to increase the duration of a stay abroad.

Hence this paper tries to fill the gap in the literature introducing the migration decision into a macroeconomic general equilibrium framework and the consequences for the Phillips-curve and the dynamics of an otherwise standard model. In section 2 I will present the basic model setup, followed by a description of the key equilibrium properties in section 3 and a discussion of the results in section 4. Section 5 concludes.

2 A small open economy model

The model presented in this section is based on the open economy model by Galí and Monacelli (2005) who model the perfectly symmetric case of a small open economy and the rest of the world which is a continuum of identical small economies. Here, however, I incorporate differing steady-state output levels in that the small economy is poorer than the rest of the world on average. This introduces the incentive to migrate and supply labour in two labour markets, those of the domestic and the world economy. It is assumed that the migrant consumes only domestically, i.e. that he remits all his earnings from abroad back home indicating his preference to consume at home, a standard assumption in the return migration literature. The representative agent's Euler equation is the basis of a dynamic IS-equation.

[5] A drawback of his analysis is that he does not control for incomes in the Phillipines which does not allow him to differentiate between foreign and domestic developments.

[6] The latter assumption is certainly not what one observes in most countries with significant diasporas abroad, but with more liberal migration regimes in receiving countries allowing more migrants to immigrate temporarily this could change at least for smaller countries. Temporary migration schemes were exactly what the Global Commission on International Migration (2005) proposed as reforms of migration regimes.

His preference to work at home is reflected in a wedge between the contribution to his disutility of labour of an hour worked for a domestic firm relative to an hour worked for a firm in the world economy. He "suffers" more when working abroad than at home. Hence he can be described as being "homesick" when away from home.

Domestic firms employ domestic labour and set prices in a forward-looking way with price staggering à la Calvo (1983), allowing the construction of a New-Keynesian Phillips-curve. The model is closed by a rule which is followed by the Central Bank when setting the rate of interest.

The rest of the world is modeled in an analogous manner with the exception that the steady state output is higher in the rest of the world than in the domestic economy and that workers in the rest of the world do not emigrate.

2.1 The representative household

The representative agent maximizes the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{e^{d_t} C_t^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^H + \phi N_t^M\right)^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where e^{d_t} is a domestic demand shock, assumed to follow and AR-process in logs, i.e. $d_t = \rho_d d_{t-1} + \varepsilon_t^d$ with $E_t \left\{ \varepsilon_{t+1}^d \right\} = 0$, and where

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

is a domestic consumption index with $C_{H,t}$ as an index for domestic consumption of domestically produced goods $C_{H,t}(j)$ with $j \in [0,1]$:

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

$C_{F,t}$ is an index for domestic consumption of foreign goods produced in country $i \in [0,1]$, $C_{i,t}$:

$$C_{F,t} \equiv \left(\int_0^1 \left(C_{i,t}\right)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} .$$

$C_{i,t}$ in turn is an index of goods produced in country i, $C_{i,t}(j)$ with $j \in [0,1]$:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon^*-1}{\varepsilon^*}} dj \right)^{\frac{\varepsilon^*}{\varepsilon^*-1}}$$

α is an indicator of the degree of openness of the domestic economy and indicates (inversely) the degree of home-bias in consumption preferences, ε, ε^*, η and γ

are the elasticities of substitution within the respective indices.

Two items enter the disutility of labour function: The labour supplied domestically, N_t^H, and abroad, N_t^M, where the later of the two is multiplied by a factor $\phi > 1$, indicating his relative preference for working at home, or "home sickness". N_t^H and N_t^M are indexes explained in more detail in section 2.2 which are constrained to be nonnegative.

The consumer faces the period budget constraint

$$\begin{aligned} W_t N_t^H + \epsilon_t W_t^M N_t^M + D_t \quad \geq \quad & \int_0^1 P_{H,t}(j) C_{H,t}(j) dj \qquad (2) \\ & + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t \{Q_{t,t+1} D_{t+1}\} \end{aligned}$$

where W_t is the domestic nominal hourly wage, W_t^M the world average of the

wages the migrant faces in the rest of the world and ϵ_t the nominal effective exchange rate. D_t is the nominal pay-off of the portfolio in period t, $Q_{t,t+1}$ a stochastic discount factor, $P_{H,t}(j)$ the price of domestically produced good j, and $P_{i,t}(j)$ the price of good j produced in country i.

From the consumption indexes the demand functions for the individual goods can be derived. They are as follows:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \quad \text{and} \qquad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon^*} C_{i,t} \qquad (3)$$

where

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \qquad \text{and} \qquad P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon^*} dj\right)^{\frac{1}{1-\varepsilon^*}}$$

It follows from the above that $\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t}$ and $\int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t}$. Furthermore, expressed as functions of the domestic and foreign indexes, the demand functions are the following:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \qquad (4)$$

$$\text{with } P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$$

From this we get $\int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t}$. Finally, because

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \qquad (5)$$

$$\text{with } P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

one has $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$ allowing us to re-write the period budget constraints as

$$W_tN_t^H + \epsilon_tW_t^MN_t^M + D_t \geq P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\}. \tag{6}$$

In section 3.1 the demand functions for individual goods expressed in terms of total consumption will be needed. They can be derived by combining (3), (4) and (5):

$$C_{H,t}(j) = (1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t \tag{7}$$

$$C_{i,t}(j) = \alpha\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon^*}\left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t \tag{8}$$

Maximizing (1) subject to (6) w.r.t C, N^H and N^M one gets the marginal rates of substitution equated to the respective real wages and, in case of the foreign wage in terms of the domestic goods prices, adjusted for the home-sickness coefficient ϕ:

$$e^{-d_t}C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \tag{9}$$

$$e^{-d_t}C_t^\sigma N_t^\varphi\phi = \frac{\epsilon_tW_t^M}{P_t} \tag{10}$$

where $N_t = N_t^H + \phi N_t^F$ is the argument of the disutility of labour function. From this follows that $\frac{W_t}{P_t} \leq \frac{\epsilon_tW_t^M}{P_t}$, i.e. that the purchasing power of the foreign wages in the domestic economy is larger than the one of domestic wages. To be more precise, the wedge in domestic earnings and those from abroad is determined endogenously and equals $\phi = \frac{\epsilon_tW_t^M}{W_t}$. With the world nominal wage assumed to be exogenous for the domestic economy, the exchange rate and the domestic nominal wage rate are endogenously determined to keep the wedge constant for a given value of ϕ.

From the first order condition w.r.t. D_t one obtains the intertemporal optimality condition

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)e^{d_{t+1}-d_t} = Q_{t+1}, \tag{11}$$

which, when taking conditional expectations on both sides, yields the standard stochastic Euler equation:

$$\beta R_tE_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}e^{d_{t+1}-d_t}\right\} = 1$$

where $R_t^{-1} = E_t \{Q_{t,t+1}\}$ is the domestic currency price of a one-period riskless bond. In summary, the optimality conditions (apart from the budget constraints)

take the following log-linearized form:

$$
\begin{aligned}
w_t - p_t &= \sigma c_t + \varphi n_t - d_t && (12)\\
w_t^M - p_t + e_t &= \sigma c_t + \varphi n_t + \log \phi - d_t && (13)\\
c_t &= E_t \{c_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\} - \rho) + (1-\rho_d) d_t && (14)
\end{aligned}
$$

where $e_t \equiv \log \epsilon_t$, $\rho \equiv \beta^{-1} - 1$ and where $\pi_t \equiv p_t - p_{t-1}$ is defined as the CPI inflation. Lower case letters indicate logs of the respective variables.

2.1.1 Domestic Inflation, CPI inflation, the real exchange rate and the terms of trade

The *bilateral terms of trade* $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$ is defined as the relative price of country i´s and the domestic economy´s goods. The *effective terms of trade* S_t are given by

$$
\begin{aligned}
S_t &\equiv \frac{P_{F,t}}{P_{H,t}} && (15)\\
&= \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}
\end{aligned}
$$

which, in the symmetric case, is approximately

$$
\widehat{s}_t = \int_0^1 \widehat{s}_{i,t} di
$$

where hats indicate percent deviations from the respective steady state values. With rich and poor countries, however, symmetry is a flawed assumption. For simplicity I will assume two different types of countries, rich and poor, with each group having a common average steady state level for the terms of trade. If, in addition, the weight of the poor group approaches zero in the calculation of the efective terms of trade, then the above approximation remains valid. This may be a reasonable approach for poor countries trading mainly with rich and little with other poor countries. It will be shown in the appendix that the terms of trade are not necessarily equal to one in the steady state, the precise value rather depending on the choice of parameters.

The price index can be approximated by

$$
\begin{aligned}
\widehat{p}_t &= \frac{1-\alpha}{1-\alpha+\alpha S^{1-\eta}} \widehat{p}_{H,t} + \frac{\alpha}{(1-\alpha)S^{\eta-1}+\alpha} \widehat{p}_{F,t}\\
\widehat{p}_t &= \widehat{p}_{H,t} + \alpha_S \widehat{s}_t && (16)
\end{aligned}
$$

where $\alpha_S \equiv \frac{\alpha}{(1-\alpha)S^{\eta-1}+\alpha}$. Note that $\alpha_S = \alpha$ for $S = 1$. From (16) follows that *domestic inflation*, defined as the rate of change of the domestic goods prices, $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$, is related to *CPI inflation* according to

$$\pi_t = \pi_{H,t} + \alpha_S \Delta s_t. \tag{17}$$

The *law of one price* (LOOP) holds at all times, i.e. $P_{i,t}(j) = \epsilon_{i,t} P^i_{i,t}(j)$ for all $i, j \in [0,1]$ where $\epsilon_{i,t}$ is the bilateral nominal exchange rate between country i and the domestic country, defined as the price of country i´s currency in terms of the domestic currency, and $P^i_{i,t}(j)$ is the price of country i´s good j expressed in country i´s, the producer´s currency. With this assumption and the definition of $P_{i,t}$ one obtains $P_{i,t} = \epsilon_{i,t} P^i_{i,t}$ where $P^i_{i,t} \equiv \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$. Substituting the definition of the LOOP into the definition of $P_{F,t}$, assuming again two types of countries (rich and poor) with a common average value of P_i in the steady state and neglecting the values of the poor countries one obtains through log-linearization:

$$\begin{aligned} p_{F,t} &= \int_0^1 p_{i,t} di \\ &= \int_0^1 (e_{i,t} + p^i_{i,t}) di \\ &= e_t + p^*_t \end{aligned} \tag{18}$$

where $e_t \equiv \int_0^1 e_{i,t} di$ is the log *nominal effective exchange rate,* $p^i_{i,t} \equiv \int_0^1 p^i_{i,t}(j) dj$ is the log domestic price index of country i in its own currency, and $p^*_t \equiv \int_0^1 p^i_{i,t} di$ is the log *world price index.* Note that for the rest of the world as a whole the distinction between the domestic and the consumer price index fades because $\alpha \to 0$.

From (15) and (18) we have

$$s_t = e_t + p^*_t - p_{H,t}. \tag{19}$$

Defining the *bilateral real exchange rate with country* i as $RER_{i,t} \equiv \frac{\epsilon_{i,t} P^i_t}{P_t}$ and the (log) real effective exchange rate $reer_t \equiv \log REER_{i,t} \equiv \int_0^1 rer_{i,t} di$, and using (19) and (16) we get a relationship between the real effective exchange rate and the terms of trade:

$$\begin{aligned} reer_t &= \int_0^1 (e_{i,t} + p^i_{i,t} - p_t) di \\ &= e_t + p^*_t - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1 - \alpha_S) s_t. \end{aligned} \tag{20}$$

2.1.2 International risk sharing

Assuming perfect securities markets, for country i an intertemporal equlibrium condition analogous to equation (11) of the form

$$\beta E_t \left\{ \left(\frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \frac{P^i_t}{P^i_{t+1}} \frac{\epsilon^i_t}{\epsilon^i_{t+1}} e^{d^*_{t+1}-d^*_t} \right\} = Q_{t,t+1} \tag{21}$$

has to hold where ϵ^i_t is the nominal effective exchange rate of country i. Equating (11) and (21) one obtains a relationship linking domestic and country i's consumption,

$$C_t = \vartheta_i C^i_t \left(RER_{i,t}\right)^{\frac{1}{\sigma}} \left(e^{d_t-d^*_t}\right)^{\frac{1}{\sigma}}, \tag{22}$$

where $\vartheta_i \equiv \frac{C_0}{C^i_0} RER^{-\frac{1}{\sigma}}_{i,0}$ when I assume $d_t = d^*_t = 0$. Assuming symmetric initial conditions across countries, $\vartheta_i = 1$, Galí and Monacelli (2005) showed that this would lead to a symmetric steady state where $C = C^i = C^*$, where C^* is an index of world consumption, and $REER_t = S_t = 1$. Here, however, we deviate from this assumption and assume instead that on average $\vartheta_i < 1$, i.e. that the domestic economy started with below world average consumption. For simplicity I assume that $\vartheta_i = \vartheta$ for all i.

Taking logs on both sides of (22) and integrating over i we obtain

$$\begin{aligned} c_t &= \log\vartheta + c^i_t + \frac{1}{\sigma} rer_{i,t} + \frac{1}{\sigma}(d_t - d^*_t) \\ &= \log\vartheta + c^*_t + \frac{1}{\sigma} reer_t + \frac{1}{\sigma}(d_t - d^*_t) \\ &= \log\vartheta + c^*_t + \frac{1-\alpha_s}{\sigma} s_t + \frac{1}{\sigma}(d_t - d^*_t) \end{aligned} \tag{23}$$

where $c^*_t = \int_0^1 c_{i,t} di$.

Because in any country i an analogous relationship to $R^{-1}_t = E_t\left\{Q_{t,t+1}\right\}$ has to hold and because of complete international securities markets we also need to have $\epsilon_{i,t}\left(R^i_t\right)^{-1} = E_t\left\{Q_{t,t+1}\epsilon_{i,t+1}\right\}$. Combining these two equations, log-linearizing and aggregating over all i, we obtain the *uncovered interest parity condition:*

$$r_t - r^*_t = E_t\left\{\Delta e_{t+1}\right\}.$$

2.2 Firms

The domestic firm $j \epsilon [0,1]$ produces with the linear production function

$$Y_t(j) = A_t N^H_t(j)$$

where $a_t \equiv \log A_t$ follows an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t$ with $E_t\{\varepsilon_{t+1}\} = 0$. From this and the definitions $Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ and $N_t^H \equiv \int_0^1 N_t^H(j)dj$ the following first order approximation to an aggregate relationship can be derived:

$$N_t^H = \frac{Y_t Z_t}{A_t} \tag{24}$$

where $Z_t \equiv \left(\int_0^1 \frac{Y_t(j)}{Y_t} dj\right)$. Galí and Monacelli (2005) further show that equilibrium variations in $z_t \equiv \log Z_t$ around the perfect foresight are of second order. Hence up to a first order approximation we can write

$$y_t = a_t + n_t^H \tag{25}$$

for the aggregate production relationship. When employed abroad, the migrant's labour input enters the following production function

$$y_t^M = a_t^M + n_t^M$$

where y_t^M is the output and a_t^M the productivity of the migrant abroad.

Variable costs in terms of domestic prices are common across domestic firms and given by $w_t - p_{H,t} + y_t - a_t$. Hence domestic real marginal costs are given by

$$mc_t = w_t - p_{H,t} - a_t. \tag{26}$$

With Calvo-type price-setting (Calvo, 1983) a measure $1-\theta$ of randomly-selected firms sets new prices every period with the probability of being selected independent of the time elapsed since prices were adjusted last. As shown by Galí and Monacelli (2005) and elsewhere, the optimal price-setting rule can be approximated by

$$\overline{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(mc_{t+k} + p_{H,t}\right)$$

where $\overline{p}_{H,t}$ denotes the optimal newly-adjusted log price and $\mu \equiv \log(\varepsilon/(\varepsilon-1))$ denoting the optimal mark-up in the steady state. Hence, as typical in New-Keynesian models, firms set their prices in a forward-looking manner equal to a weighted average of the expected discounted marginal costs plus a mark-up. An according equation applies for the migrant sector.

In an analog way world output is produced: Individual country i produces with production function $y_t^i = a_t^i + n_t^i$ where $a_t^i = \rho_{a^i} a_{t-1}^i + \varepsilon_t^i$ with $E_t\left\{\varepsilon_{t+1}^i\right\} = 0$ and $E_t\left\{\varepsilon_{t+1}^i \varepsilon_{t+1}^j\right\} = 0 \;\forall\; i, j$, implying marginal costs of $mc_t^i = w_t^i - p_{H,t}^i - a_t^i$. Integrating these relationships over all countries results in the world production and marginal cost function:

$$
\begin{aligned}
y_t^* &\equiv \int_0^1 y_t^i di = \int_0^1 a_t^i di + \int_0^1 n_t^i di \equiv a_t^* + n_t^* \\
mc^* &\equiv \int_0^1 mc_t^i di = \int_0^1 w_t^i di - \int_0^1 p_{H,t}^i di - \int_0^1 a_t^i di \equiv w_t^* - p_t^* - a_t^*
\end{aligned}
$$

where stars indicate world averages.

By assumption the migrant's labour input does not appear in the world production function which is reasonable for the small country/rest-of-the-world setting. This means that changes in N^M will not result in measurable changes of world output. However, I assume that world output increases and changes in world productivity have non-negligible effects on N^M through an assumed correlation betwen output and productivity of the world economy and the sectors employing migrant workers. Because N^M does not appear explicitly in the world production function I simply assume that the first effect increases the migrant´s labour input abroad while the second decreases it. Furthermore, we assume that migrant labour hours are a negative function of domestic output, i.e. domestic growth induces migrants to return, while they are a positive function of domestic productivity shocks, i.e. indicating that when set free domestically, labour partly moves abroad. Hence we can describe N^M as a function of these four factors:

$$
N^M = N^M(\overset{-}{Y}, \overset{+}{A}, \overset{+}{Y^*}, \overset{-}{A^*}). \tag{27}
$$

The plus and minus signs indicate the signs of the first derivatives of N^M with respect to the respective variables. With these assumptions and equation (25) it is possible to write the approximation of the the argument of the disutility of labour function around the steady state, i.e.

$$
\widehat{n}_t = \nu \widehat{n}_t^H + (1-\nu)\widehat{n}_t^M
$$

where $\nu = \frac{N^H}{N} < 1$, as follows:

$$
\begin{aligned}
\widehat{n}_t &= \nu \widehat{y}_t - \nu a_t \\
&\quad +(1-\nu)\left[\frac{\partial N_t^M}{\partial Y_t}\frac{Y}{N^M}\widehat{y}_t + \frac{\partial N_t^M}{\partial A_t}\frac{A}{N^M}a_t + \frac{\partial N_t^M}{\partial Y_t^*}\frac{Y^*}{N^M}\widehat{y}_t^* + \frac{\partial N_t^M}{\partial A_t^*}\frac{A^*}{N^M}a_t^*\right] \\
\widehat{n}_t &= (\nu - \zeta_Y)\,\widehat{y}_t - (\nu - \zeta_A)\,a_t + \zeta_{Y^*}\widehat{y}_t^* - \zeta_{A^*}a_t^*
\end{aligned}
$$

where $\zeta_X \equiv \left|\frac{\partial N_t^M}{\partial X_t}\frac{X}{N^M}\frac{\phi N^M}{N}\right|$ is defined as the elasticity of the argument of the disutility of labour function, N_t, with respect to changes in variable X. It is obvious that purely domestic variations of economic activity and productivity only partially affect the disutility of labour when the "labour account" is open because $(\nu - \zeta_Y) < 1$ and $(\nu - \zeta_A) < 1$. In the closed labour market setting these coefficients both equal one. At the same time world output and productivity affect the disutility of labour because of their impact on the migrant's hours worked.

What may appear a little bit strange in this set-up is that W^M is exogenous for the migrant worker. However, this assumption is straightforward for host countries from which migrants from a large number of different countries work implying that "our" migrant's labour supply decision does not affect the wage rate.

3 Equilibrium

3.1 Aggregate demand and output

3.1.1 Domestic equilibrium

In a goods market equilibrium domestic supply has to equal domestic and foreign demand. The domestic demand was derived above and given in equation (7):

$$C_{H,t}(j) = (1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{28}$$

For the derivation of foreign demand a demand function for domestic good j analogous to equation (8) needs to be derived. Because of the LOOP we have

$$P_{H,t}(j) = \epsilon_{i,t} P^i_{H,t}(j),$$

where $P^i_{H,t}(j)$ is the price of the domestically produced good j expressed in terms of country i's currency units. Furthermore, defining $P^i_{H,t} = \left(\int_0^1 P^i_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ as the index of domestically produced goods in terms of country i's currency units, one can easily check that

$$P_{H,t} = \epsilon_{i,t} P^i_{H,t}.$$

Country i's demand for good i, $C^i_{H,t}(j)$, is then:

$$C^i_{H,t}(j) = \alpha\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{\epsilon_{i,t} P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t$$

with P^i_t and C^i_t defined as country i's consumer price and consumption indexes, the former expressed in its own currency. Integrating this over all countries one obtains total foreign demand:

$$\int_0^1 C^i_{H,t}(j)di = \alpha\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\int_0^1\left(\frac{P_{H,t}}{\epsilon_{i,t} P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di$$

Hence total demand is

$$\begin{aligned}
Y_t(j) &= C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di \\
&= (1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \\
&\quad +\alpha\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\int_0^1 \left(\frac{P_{H,t}}{\epsilon_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di
\end{aligned}$$

Plugging this into the aggregate output relation, making use of the international risk sharing condition (22) and the definition of $RER_{i,t}$, we get

$$\begin{aligned}
Y_t &\equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \\
&= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{\epsilon_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1 \left(\frac{\epsilon_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} RER^{\eta}_{i,t}C^i_t di\right] \qquad (29) \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\left[1-\alpha+\alpha\vartheta^{-1}\left(e^{d_t-d^*_t}\right)^{-\frac{1}{\sigma}}\int_0^1 \left(S^i_t S_{i,t}\right)^{\gamma-\eta} RER^{\eta-\frac{1}{\sigma}}_{i,t} di\right]
\end{aligned}$$

with S^i_t defined as country i‘s effective terms of trade. This goods market clearing condition, as shown in more detail in the appendix, when log-linearized around the steady state, is

$$\widehat{y}_t = \widehat{c}_t + \frac{\alpha\varpi_S}{\sigma}\widehat{s}_t - \frac{\alpha}{\sigma}l(S)\left[d_t - d^*_t\right] \qquad (30)$$

where we made use of the substitutions

$$\begin{aligned}
l(S) &\equiv \left[(1-\alpha)\vartheta S^{\eta-\gamma}reer(S)^{\frac{1}{\sigma}-\eta}+\alpha\right]^{-1} \\
reer(S) &\equiv REER \\
\varpi_S &\equiv \left[\sigma\gamma + (1-\alpha_S)(\sigma\eta-1)\right]l(S) + \sigma\eta\left[\frac{\alpha_s}{\alpha} - l(S)\right]
\end{aligned}$$

and the fact that $\int_0^1 s^i_t di = 0$. Note that $\varpi_S = \sigma\gamma+(1-\alpha)(\sigma\eta-1)$ for $S = \vartheta = 1$ which is the solution for the symmetric case.

This condition holds for every country i, hence $\widehat{y}^i_t = \widehat{c}^i_t + \frac{\alpha\varpi_S}{\sigma}\widehat{s}^i_t$. Aggregation over all countries results in the world market clearing condition:

$$y^*_t \equiv \int_0^1 y^i_t di = \int_0^1 c^i_t di \equiv c^*_t \qquad (31)$$

where y_t^* and c_t^* are indexes for log world output and consumption. Combining this with (30) and (23) we obtain

$$\widehat{y}_t = \widehat{y}_t^* + \sigma_{\alpha,s}^{-1}\widehat{s}_t + \frac{1-\alpha l(S)}{\sigma}(d_t - d_t^*). \tag{32}$$

where $\sigma_{\alpha,s}^{-1} = \frac{(1-\alpha_S)+\alpha\varpi_S}{\sigma}$.

Finally, combining (30) with the domestic Euler equation (14) and the conditional expectations of both sides of equation (17) we obtain the Euler equation in terms of domestic output,

$$\begin{aligned}
\widehat{y}_t \quad = \quad & E_t\left\{\widehat{y}_{t+1}\right\} - \frac{1}{\sigma_{\alpha,s}}(\widehat{r}_t - E_t\left\{\pi_{H,t+1}\right\}) + \alpha\Theta E_t\left\{\Delta\widehat{y}_{t+1}^*\right\} \\
& - \left[\frac{\alpha\Theta}{\sigma}\sigma_{\alpha,s}\left(1-\alpha l(S)\right) + \alpha l(S) - \sigma\right]\frac{1-\rho_c}{\sigma_{\alpha,s}}d_t \\
& + \left[\frac{\alpha\Theta}{\sigma}\sigma_{\alpha,s}\left(1-\alpha l(S)\right) + \alpha l(S)\right]\frac{1-\rho_c^*}{\sigma_{\alpha,s}}d_t^*
\end{aligned}$$

where $\Theta \equiv (\omega_S - \frac{\alpha_S}{\alpha})$ which nests the symmetric benchmark for $S = 1$ where $\Theta \equiv \omega_S - 1$. Hence, the inflation rate which matters for the Euler-equation, expressed in terms of output, is the domestic inflation rate.

Before continuing the exposition of the model, we will discuss the steady state and its influence on the dynamics of the model. In the appendix we show that output and terms of trade are uniquely determined by two functions, given parameters and the steady state values of productivity and hours worked abroad. The later, in turn, will not be determined here explicitly, but rather assumed to be determined by exogenous demand and supply condition abroad. One function, derived from the steady state goods market condition condition, is a strictly positive function of S while the second function, derived from the household‘s optimality condition, is strictly decreasing in S.

Figure 1 displays what happens to the steady state when the initial condition ϑ is no longer assumed to be one as in the symmetric case. This shift clearly increases S while Y may fall or increase. Figure 2, on the other hand, shows the change of the steady state when labour is allowed to emigrate. This clearly reduces domestic output and improves the terms of trade. Hence in the analysis of comparing the model´s characteristics in the migration case relative to the no-migration benchmark, that we conduct below, output is clearly smaller while the terms of trade improved in the steady state in the first case.

Having said that, it would now be interesting to analyse the impact of a fall in S (the improvement in the terms of trade implied by opening the labour market) on the elasticity of substitution $\sigma_{\alpha,s}^{-1}$. In figure 4, the change in $\sigma_{\alpha,s}$ is shown when the terms of trade fall from two to one for a range of values for η and γ while setting $\alpha = 0.4$, $\vartheta = 0.1$ and $\sigma = 0.95$. It is obvious, that $\sigma_{\alpha,s}$ almost does not change at all with an exception being extremely low values of γ. This observation is confirmed by a much broader range of parameters not shown here. Hence for most of the reasonable parameter space, the change in the terms of trade only slightly changes the elasticity of substitution $\sigma_{\alpha,s}$.

3.1.2 External equilibrium

Net exports, in terms of domestic output in the steady state, are

$$nx_t \equiv \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right)$$

and, up to a first order approximation around the steady state

$$\widehat{nx}_t = \frac{1}{1-\frac{PC}{P_H Y}}\left(\widehat{y}_t + \widehat{c}_t + \alpha_S \widehat{s}_t\right)$$

3.2 Aggregate supply, marginal costs and inflation

As shown by Galí and Monacelli (2005), the *domestic* inflation dynamics in this model are analogous to CPI-inflation dynamics in a closed-economy:

$$\pi_{H,t} = \beta E_t\left\{\pi_{H,t+1}\right\} + \lambda \widehat{mc}_t \tag{33}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. However, in the set-up developed here, the deviations of the marginal cost function differ from the benchmark model as will be summarized in the following proposition:

Proposition 1 *The open labour market structure reduces the effects of output expansions on domestic marginal cost fluctuations $\widehat{mc}_t$ relative to the closed labour market setup for reasonable parameterizations.*
Proof. *From (26) and (12) we get*

$$\begin{aligned} mc_t &= w_t - p_{H,t} - a_t \\ &= (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= \sigma c_t + \varphi n_t + \alpha_S s_t - a_t - d_t \end{aligned} \tag{34}$$

and because of (16), when evaluated in the neighbourhood of the steady state is

$$\begin{aligned} \widehat{mc}_t &= \sigma\widehat{c}_t + \varphi\widehat{n}_t + \alpha_S\widehat{s}_t - a_t - d_t \\ &= \sigma\widehat{y}_t^* + (1-\alpha_S)\widehat{s}_t + d_t - d_t^* + \varphi\widehat{n}_t + \alpha_S\widehat{s}_t - a_t - d_t \\ &= \sigma\widehat{y}_t^* + \widehat{s}_t + \varphi\left[\left(\nu - \zeta_Y\right)\widehat{y}_t - \left(\nu - \zeta_A\right)a_t + \zeta_{Y^*}\widehat{y}_t^* - \zeta_{A^*}a_t^*\right] - a_t - d_t^* \\ &= \left(\sigma_{\alpha,s} + \varphi\left(\nu - \zeta_Y\right)\right)\widehat{y}_t - \left(1 + \varphi\left(\nu - \zeta_A\right)\right)a_t + \left(\sigma - \sigma_{\alpha,s} + \varphi\zeta_{Y^*}\right)\widehat{y}_t^* \\ &\quad -\varphi\zeta_{A^*}a_t^* - d_t^* - (1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}\left(d_t - d_t^*\right) \\ &= \left(\sigma_{\alpha,s} + \varphi\left(\nu - \zeta_Y\right)\right)\widehat{y}_t - \left(1 + \varphi\left(\nu - \zeta_A\right)\right)a_t + \left(\sigma - \sigma_{\alpha,s} + \varphi\zeta_{Y^*}\right)\widehat{y}_t^* \\ &\quad -\varphi\zeta_{A^*}a_t^* - (1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}d_t + \left((1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma} - 1\right)d_t^* \end{aligned} \tag{35}$$

where I inserted (31), (23) and (32). Comparing this with the closed labour market setting we see that in this case

$$\begin{aligned}\widehat{mc}_t &= (\sigma_{\alpha,s}+\varphi)\,\widehat{y}_t-(1+\varphi)a_t+(\sigma-\sigma_{\alpha,s})\widehat{y}_t^* \\ &\quad -(1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}d_t+\Big((1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}-1\Big)\,d_t^*.\end{aligned} \tag{36}$$

Hence the impact of output expansions on marginal costs in the open labour market setting is smaller because $\nu-\zeta_Y<1$ and because for reasonable parameterizations $\sigma_{\alpha,s}$ changes only by an insignificant amount. ■

Figures 5-7 show by how much the open labour market setting reduces the impact of output changes on marginal costs. Parameters and variables are chosen such that the impact can be shown for different steady state shares of migrant labour hours in total hours, for differnet elasticities by which migrants return when domestic output moves and for different wage differentials ϕ. The larger the return elasticity, the larger the wage differential and the larger the share of migrant labour hour in total hours, the lower is the impact of output fluctuations around the steady state on marginal costs. The impact even changes sign for some parameterizations implying that in these cases, marginal costs may even be reduced during a boom, due to strong re-migration.

In the next section we will derive the New-Keynesian Phillips-curve implied by the aggregate supply relation, consisting of (33) and (35), and discuss the implications of the open labour market structure.

3.3 Equilibrium dynamics

3.3.1 The New-Keynesian Phillips-curve

The New-Keynesian Phillips-curve is usually writtten in terms of the output gap x_t, defined as the difference between (log) domestic output y_t and its natural level y_t^n, i.e. the equilibrium output in the absence of nominal rigidities y_t^n. The latter is derived by setting marginal costs equal to its flexible price value $-\mu$ so that $\widehat{mc}_t=0$ in (35) and solving for output:

$$\begin{aligned}\widehat{y}_t^n &= \left(\frac{1+\varphi\,(\nu-\zeta_A)}{\sigma_{\alpha,s}+\varphi\,(\nu-\zeta_Y)}\right)a_t-\left(\frac{\sigma-\sigma_{\alpha,s}+\varphi\zeta_{Y^*}}{\sigma_{\alpha,s}+\varphi\,(\nu-\zeta_Y)}\right)\widehat{y}_t^* \\ &\quad +\left(\frac{\varphi\zeta_{A^*}}{\sigma_{\alpha,s}+\varphi\,(\nu-\zeta_Y)}\right)a_t^* \\ &\quad +\left(\frac{(1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}}{\sigma_{\alpha,s}+\varphi\,(\nu-\zeta_Y)}\right)d_t-\left(\frac{(1-\alpha l(S))\frac{\sigma_{\alpha,s}}{\sigma}-1}{\sigma_{\alpha,s}+\varphi\,(\nu-\zeta_Y)}\right)d_t^*\end{aligned} \tag{37}$$

With this and because

$$\widehat{y}_t=\left(\frac{1}{\sigma_{\alpha,s}+\varphi\,(\nu-\zeta_Y)}\right)\widehat{mc}_t+\widehat{y}_t^n$$

the relationship between marginal costs and the output gap is approximated by:

$$\widehat{mc}_t = (\sigma_{\alpha,s} + \varphi(\nu - \zeta_Y))\, x_t \tag{38}$$

The following Proposition 2 extends the reasoning of Proposition 1 to the Phillips-curve:

Proposition 2 *The open labour market structure reduces the effect of output gap changes on increases of domestic inflation relative to a closed labour market structure. In other words, the Phillips-curve becomes flatter.*
Proof. *Combining (38) with (33) one obtains the open economy New Keynesian Phillips-curve in terms of the output gap:*

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + \kappa^{open} x_t. \tag{39}$$

where

$$\kappa^{open} = \lambda (\sigma_{\alpha,s} + \varphi(\nu - \zeta_Y))$$

is the slope factor of the Phillips-curve in the open labour market setting. In contrast, in the closed labour-market, this same calculation yields

$$\kappa^{closed} = \lambda (\sigma_{\alpha,s} + \varphi)$$

Hence, because $(\nu - \zeta_Y) < 1$ and $\sigma_{\alpha,s}$ basically unchanged for reasonable parameterizations, the effect of output gap variations on the domestic inflation rate is smaller when the labour market is open, i.e. the Phillips-curve becomes flatter. ■

As pointed out by Loungani et al. (2002), more open economies tend to have flatter Phillips-curves. Yuen and Razin (2002) showed in a setting similar to the one in Woodford (2003, ch. 3) that opening the capital and trade accounts reduces the effect from output increases on marginal costs and thereby inflation. As shown above, the opening of the "labour account" obviously works in the same direction.

What's the mechanism behind this phenomenon? When output expands in the domestic economy, workers return from abroad and thereby serve as an extra, "cheap" pool for the additional labour which is needed for the expansion as an input to production. This pool is "cheap" in the sense that the alternative in the closed labour market setting is a reduction of leisure while returning emigrants substitute labour at home for labour abroad. To the extent that output expansions at home are fed by this substitution effect, the disutility of labour does not increase and hence the real wage, marginal costs, prices and inflation increase less as well.

3.3.2 The dynamic IS equation

In order to fully describe the dynamics of the model, we furthe need an Euler equation, written in terms of the output gap. This can be shown to be

$$x_t = E_t \{x_{t+1}\} - \frac{1}{\sigma_{\alpha,s}} (\widehat{r}_t - E_t \{\pi_{H,t+1}\} - \widehat{r}_t^n) \tag{40}$$

with the natural rate of interest

$$\widehat{r}_t^n = -\Gamma_a a_t + \Gamma_{\Delta y^*} E\left\{\Delta \widehat{y}_{t+1}^*\right\} - \Gamma_{a^*} a_t^* + \Gamma_d d_t + \Gamma_{d^*} d_t^*$$

and where

$$\begin{aligned}
\Gamma_a &= \left(\frac{1+\varphi(\nu-\zeta_A)}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\sigma_{\alpha,s}\right)(1-\rho_a) \\
\Gamma_{\Delta y^*} &= \left(\alpha\Theta\sigma_{\alpha,s} - \frac{\sigma-\sigma_{\alpha,s}+\varphi\zeta_{Y^*}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\sigma_{\alpha,s}\right) \\
\Gamma_{a^*} &= \left(\frac{\varphi\zeta_{A^*}\sigma_{\alpha,s}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)(1-\rho_{a^*}) \\
\Gamma_d &= \left[\sigma + \frac{\sigma_{\alpha,s}}{\sigma}(1-\alpha l(S))\left(1+\frac{\sigma_{\alpha,s}}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)-1\right](1-\rho_c) \\
\Gamma_{d^*} &= \left[\left(1-\frac{\sigma_{\alpha,s}}{\sigma}(1-\alpha l(S))\right)\left(\frac{\varphi(\nu-\zeta_Y)}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}\right)\right](1-\rho_{d^*})
\end{aligned}$$

It is obvious that the open labour market structure affects the real rate of interest through multiple channels. Demand, productivity shocks and fluctuations in world output all affect that variable and thereby, as will be shown below, the dynamics of the model.

3.3.3 Simulation

In order to illustrate the dynamics of the model I will present a simulation exercise for the differential impact of various shocks in the closed and the open labour market setting. In order to close the model, the central bank is assumed to follow the rule

$$r_t = r_t^n + \phi_\pi \pi_{H,t} + \phi_x x_t$$

which, as shown in Galí and Monacelli (2005), is optimal from a welfare perspective because it perfectly stabilizes the output gap and domestic inflation, i.e. $x_t = \pi_{H,t} = 0$ (and consequently $\widehat{mc}_t = 0$) at all times, and delivers a unique and stable equilibrium as long as

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_x > 0$$

for non-negative values of $\phi_\pi$and ϕ_x as shown by Bullard and Mitra (2002).

First we will analyse the effects of a domestic demand shock. Figure (8) shows the impulse resonses of key model variables to a unit demand shock d_t. Parameters are chosen to mimic the structure of the Polish economy which has several characteristics that make it a candidate country for which the mechanics underlying this model may apply, in particular after joining the European Union and the opening of the British, Irish and Swedish labour markets for Poles. α is set to 0.4, roughly in line with the country´s imports to GDP ratio. ϕ is assumed to be 10, proxying the wage diffenential between Poland and the EU15 while ϑ is assumed to be 0.1, somewhat below the ratio of Poland´s real GDP per capita to the EU15's at the beginning of the transition period in 1989. The share of emigrant hours in total hours is assumed to be 10%, a conservative estimate of the large Polish diaspora. The elasticity by which N^M reacts to changes in domestic output is set to 0.1, hence in figure (5) this amounts to a value of 1.26 for $\varphi(\nu - \zeta_Y)$ when φ is assumed to be 3. Finally, η and γ are set to 1 and 2 respectively.

The impulse responses show that output and domestic employment move about twice as much compared to the benchmark in which migration is not allowed. Hence the output effect is remarkably large, even though the elasticity of re-migration is assumed to be reasonably low. The re-migrating labour reduces the pressure on marginal costs, prices and thereby domestic inflation which ceteris paribus allows a greater output expansion at the zero domestic inflation rate prevailing throughout. However, the terms of trade improve less when migration is allowed, almost completely offsetting the positive impact on consumption of the output increase.

Secondly, the impulse responses due to a productivity shock are analysed. As can be seen from figure (9), the effect on output is only slightly larger in the migration set-up for which $\frac{\partial N_t^M}{\partial A_t}\frac{A}{N^M}$ was set to zero. Had this elasticity been set to 0.1 like the value assumed for $\frac{\partial N_t^M}{\partial Y_t}\frac{Y}{N^M}$, the impact would have been entirely negligible which can be seen from the output reaction coefficient in (37), $\frac{1+\varphi(\nu-\zeta_A)}{\sigma_{\alpha,s}+\varphi(\nu-\zeta_Y)}$, which in that case would be changed only slightly when $\sigma_{\alpha,s}$ is close to unity. Hence the labour saving (due to the increased productivity) and the re-migration (due to the implied output increase) effects on marginal costs offset each other.

4 Discussion

An obvious drawback for this line of reasoning is certainly that in countries with high levels of emigrated labour, involuntary unemployment is a severe problem and hence inflationary pressures due to quickly increasing real wages a minor problem. Therefore an analysis allowing for unemployment might be a valuable extension. However, with less restricted immigration policies than currently observed in most industrial countries, the problem of mass unemployment could be alleviated at least in small developing countries.

Another aspect may be even more important. It is very likely that it is very restrictive immigration regimes which prevent a smooth movement back and forth across borders. When emigrants, once back in their country of origin, have to fear they will have difficulties to return to the rich country labour market, they will tend to stay longer and potentially lose a close contact with their home country. This is exactly what Constant and Zimermann (2003a) found for German "Gastarbeiter", of whom those migrants facing less restrictions when crossing borders had a much higher probability to frequently return to their countries of origin and stay for extended periods. Bohning (1981) has estimated that the rates of return of foreign workers admitted to the Federal Republic of Germany between 1961 and 1976 were, on average, two thirds, while those figures where particularly high among Italians (90%), Spanish (80%) and Greeks (70%) but low for Yugoslavs (50%) and Turks (30%). Hence there is a positive correlation between low border controls and high rates of return. A similar argument has been made by Dustmann (1996) who argued that return rates were higher in the US than Europe because of a more liberal immigration policy.

For the recent EU enlargement, with very liberal immigration policies applied at least in three countries among the EU15, this might imply that circular migration patterns are likely to be observed and the mechanisms underlying the model presented above might be reasonable descriptions of reality.

The flip side of this paper´s analysis is the host country perspective. While here we focused on the implications of migration on the sending country, the implications for the receiving country have recently been analysed in Binyamini and Razin (2007). A potentially fruitful area of further research might be to bring these two perspectives into a common framework in order to analyse possible feedback effects.

5 Conclusion

The impact of migration from a poor small country to the rest of the world with the migrants' preference to live in the country of origin was analysed in a New-Keynesian framework. It was shown that the assumed labour market structure in which the representative worker supplies his labour both domestically and abroad flattens the Phillips-curve. Furthermore, the impact of demand and productivity shocks were analysed with large positive differential output effects found in the first one when migration is allowed relative to a no-migration benchmark.

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6 Appendix

6.1 The steady state

In this model, with its additional first order condition, the determination of the steady state variables deviates from the benchmark model. The steady state and the terms of trade are uniquely pinned down but with values determined by the relative labour market conditions facing the representative worker. This is in contrast to the benchmark model for which Galí and Monacelli (2005) showed that $S = 1$ and $Y = Y^*$.

The goods market clearing condition in the steady state is

$$\begin{aligned} Y &= (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \alpha\int_0^1 \left(\frac{P_H}{E_i P_F^i}\right)^{-\gamma}\left(\frac{P_F^i}{P^i}\right)^{-\eta} C^i di \\ &= h(S)^\eta \left[(1-\alpha)C + \alpha\int_0^1 \left(\frac{E_i P_F^i}{P_H}\right)^{\gamma-\eta} RER_i^\eta C^i di\right] \\ &= h(S)^\eta C\left[(1-\alpha) + \alpha\vartheta^{-1}\int_0^1 \left(S^i S_i\right)^{\gamma-\eta} RER_i^{\eta-\frac{1}{\sigma}} di\right] \\ &= h(S)^\eta C\left[(1-\alpha) + \alpha\vartheta^{-1} S^{\gamma-\eta} reer(S)^{\eta-\frac{1}{\sigma}}\right] \end{aligned}$$

where I made use of the risk sharing condition (36), the fact that $S^i = S^* = 1$, $S_i = S$ and $RER_i = REER$ $\forall i$ in the staedy state and the substitutions

$$\frac{P}{P_H} = \left[(1-\alpha) + \alpha S^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv h(S)$$

and $REER = \frac{S}{h(S)} \equiv reer(S)$. Note that $h(S) > 0$ and $reer(S) > 0$, $h'(S) > 0$ and $reer'(S) > 0$ and $h(1) = reer(1) = 1$.

Furthermore, in the steads state, the risk sharing condition, taking account of international goods market clearing, $C^* = Y^*$, is

$$C = \vartheta Y^* reer(S)^{\frac{1}{\sigma}}.$$

Combining this with the goods market clearing condition we have

$$\begin{aligned}
Y &= h(S)^\eta \vartheta Y^* reer(S)^{\frac{1}{\sigma}} \left[(1-\alpha) + \alpha \vartheta^{-1} S^{\gamma-\eta} reer(S)^{\eta - \frac{1}{\sigma}} \right] \\
&= Y^* \left[(1-\alpha)\vartheta h(S)^\eta reer(S)^{\frac{1}{\sigma}} + \alpha S^{\gamma-\eta} h(S)^\eta reer(S)^{\frac{1}{\sigma}} reer(S)^{\eta-\frac{1}{\sigma}} \right] \\
&= Y^* \left[(1-\alpha)\vartheta S^\eta reer(S)^{\frac{1}{\sigma}-\eta} + \alpha S^{\gamma-\eta} h(S)^\eta reer(S)^\eta \right] \\
&= Y^* \left[(1-\alpha)\vartheta S^\eta reer(S)^{\frac{1}{\sigma}-\eta} + \alpha S^\gamma \right] \\
&\equiv Y^* v(S) \qquad (41)
\end{aligned}$$

with $v(S) > 0$, $v'(S) > 0$ and $v(1) < 1$ implying $Y < Y^*$ for $S = 1$ when $\vartheta < 1$, i.e. when the country was initially poorer than the rest of the world. Moreover, output is uniquely determined when the steady state terms of trade are known. This means that up to some upper limit, values of $S > 1$ are possible which would be in line with typical approaches and observations of developing countries' terms of trade.

The domestic labour market clearing condition can be re-written as

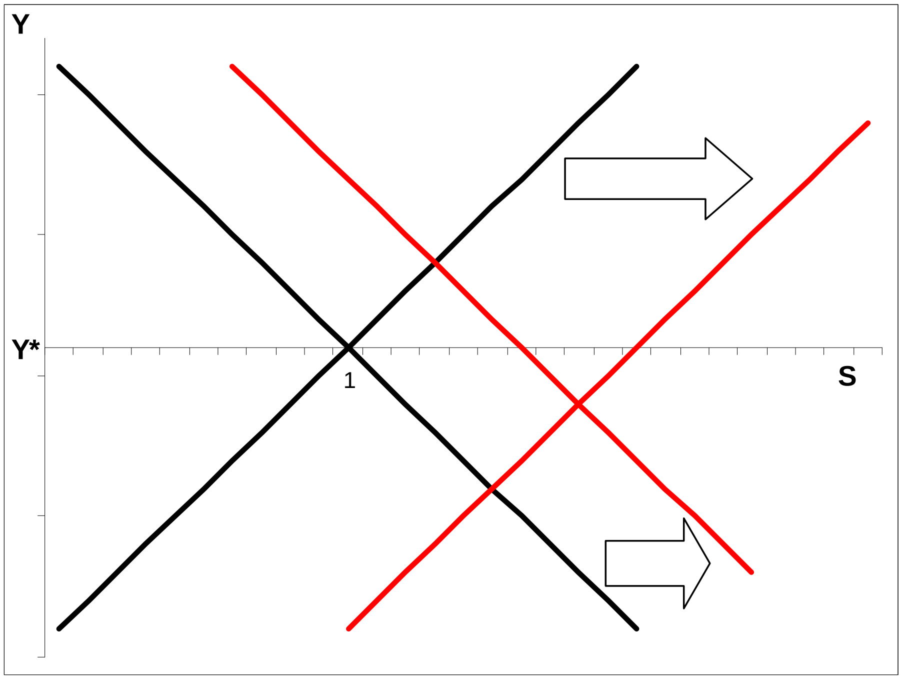
$$\begin{aligned}
C^\sigma N^\varphi &= \frac{W}{P} \\
&= A \frac{W}{P^H A} \frac{P^H}{P} \\
&= A \frac{1}{h(S)} MC
\end{aligned}$$

In the steady state we have $MC = 1 - \frac{1}{\varepsilon}$, hence, and because of the risk sharing condition, we get

$$\begin{aligned}
(\vartheta Y^*)^\sigma reer(S) \left(\frac{Y}{A} + \phi N^M \right)^\varphi &= A \frac{1}{h(S)} \left(1 - \frac{1}{\varepsilon} \right) \\
Y &= A \left[\left(\frac{A \frac{1}{h(S)} \left(1 - \frac{1}{\varepsilon}\right)}{(\vartheta Y^*)^\sigma reer(S)} \right)^{\frac{1}{\varphi}} - \phi N^M \right] \\
Y &= A \left[\left(\frac{A \left(1 - \frac{1}{\varepsilon}\right)}{(\vartheta Y^*)^\sigma S} \right)^{\frac{1}{\varphi}} - \phi N^M \right] \\
Y &= k(S) \qquad (42)
\end{aligned}$$

with $k'(S) < 0$. Jointly with (41), we have a system of two equations in the two unknowns Y and S given parameters, N^M and productivity A. Because in (41)

Figure 1: Change of Steady State when initial condition υ falls below one



Y is strictly increasing while in (42) Y is strictly decreasing in S , there is a unique solution for Y and S. In the fully symmetric, no-migration benchmark model, this unique solution is determined by $S = 1$ and $Y = Y^*$ (Gali and Monacelli, 2005). The original assymmetry ($\vartheta < 1$) clearly shifts both curves to the right raising the terms of trade as illustrated in figure 1. The effect of emigration is to lower $k(S)$ and shift the corresponding curve down (figure 2). Consequently, output can be calibrated to be below the world average while the terms of trade can be allowed to be above and below 1.

6.2 Figures and Tables

Figure 2: Change of Steady State when labour migration is allowed

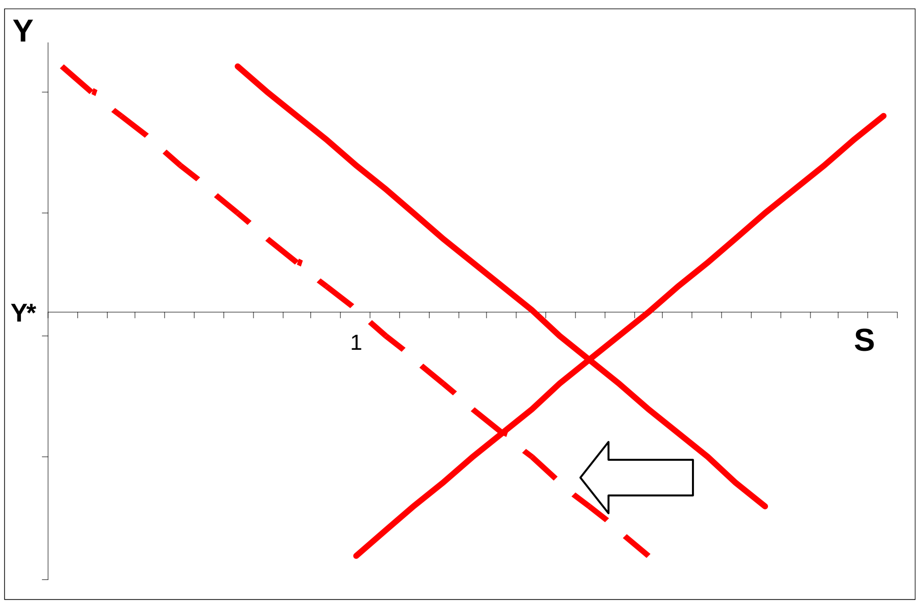


Figure 3: Change of $\sigma_{\alpha,S}$ when S falls from 2 to 1

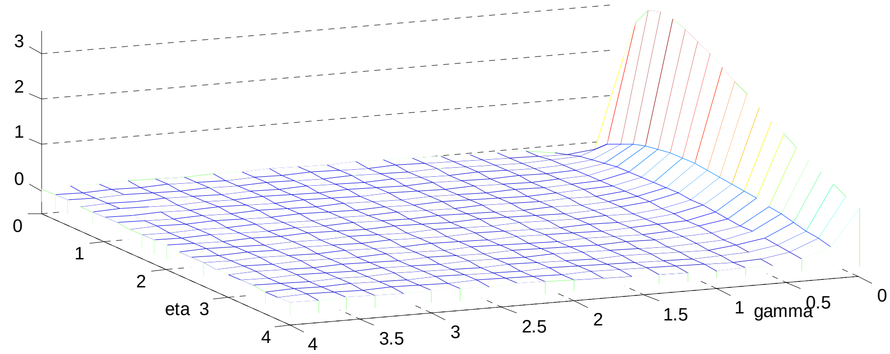


Figure 4: Assumptions for parameters: $\vartheta = 0.1$, $\sigma = 0.95$, $\alpha = 0.4$.

Figure 5: $\varphi\left(\nu-\zeta_Y\right)$ with $\varphi=3$ and $\frac{\partial N^M}{\partial Y}\frac{Y}{N^M}=0.1$

	Share of labour force abroad			
phi	**5%**	**10%**	**20%**	**30%**
2	2,69	2,40	1,90	1,48
5	2,31	1,82	1,17	0,75
10	1,86	1,26	0,64	0,32
20	1,31	0,72	0,25	0,04

Figure 6: $\varphi\left(\nu-\zeta_Y\right)$ with $\varphi=3$ and $\frac{\partial N^M}{\partial Y}\frac{Y}{N^M}=1$

	Share of labour force abroad			
phi	**5%**	**10%**	**20%**	**30%**
2	2,43	1,91	1,00	0,23
5	1,75	0,86	-0,33	-1,09
10	0,93	-0,16	-1,29	-1,86
20	-0,08	-1,14	-2,00	-2,37

Figure 7: $\varphi\left(\nu-\zeta_Y\right)$ with $\varphi=3$ and $\frac{\partial N^M}{\partial Y}\frac{Y}{N^M}=2$

	Share of labour force abroad			
phi	**5%**	**10%**	**20%**	**30%**
2	2,14	1,36	0,00	-1,15
5	1,13	-0,21	-2,00	-3,14
10	-0,10	-1,74	-3,43	-4,30
20	-1,62	-3,21	-4,50	-5,06

Figure 8: Unit Demand Shock

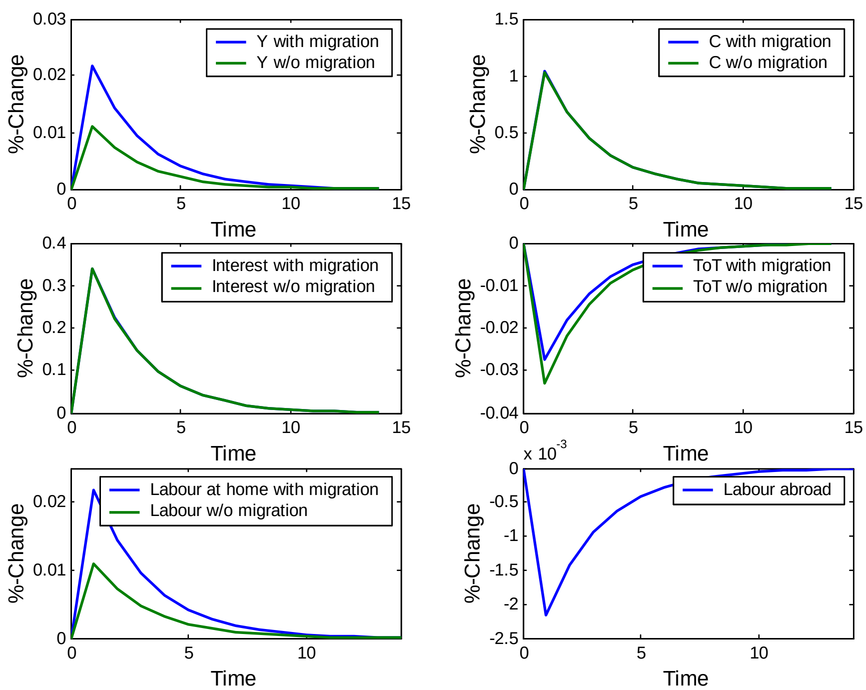


Figure 9: Unit productivity shock