ESTIMATING NAIRU IN SMALL OPEN ECONOMIES: MODELS WITH ADAPTIVE AND RATIONAL EXPECTATIONS

Daniel Nemec
Faculty of Economics and Administrations, Masaryk University, Czech Republic

and

Osvald Vasicek
Faculty of Economics and Administrations, Masaryk University, Czech Republic

ABSTRACT
This paper estimates the NAIRU and output gap in two small open economies (Czech Republic and New Zealand) in a Bayesian framework. The estimates of these unobserved states are made using two models: simple hysteretic model with adaptive expectations and dynamic (non-hysteretic) rational expectations model. Hysteretic model assumes a well known "law of motion" of the equilibrium unemployment. The results show that the hysteresis is a significant phenomenon in the Czech economy. Similar trajectories of the NAIRU are estimated in both models. On the other hand, effective and credible inflation targeting monetary policy may cause a false indication of the hysteresis as the estimates for the New Zealand economy show. Estimated path of the NAIRU is very sensitive to specification of the prior parameters in this case. We estimated three sets of the paths which are in accordance with the natural rate hypothesis (equilibrium unemployment is constant in time), hysteresis hypothesis (equilibrium unemployment follows the path of actual unemployment), and the theory of NAIRU (time-varying equilibrium unemployment). Fortunately, Bayesian inference allows us to decide which trajectory is the most probable. This may lower the uncertainty related to NAIRU estimation. The paper concludes that most of the decrease in New Zealand unemployment is driven by structural factors compared to Czech experience with hysteresis mechanisms staying behind the decreasing unemployment rate in the last years.

JEL Nos. C11, C51, E24, E31

Corresponding author: Daniel, Nemec
Email contact: nemecd@econ.muni.cz

1 Introduction

There are two important indicators of monetary stability and potential economic growth: the unemployment gap and the output gap. Both indicators are based on the differences between observable macroeconomic variables (unemployment rate and aggregate product) and their equilibrium unobservable counterparts (equilibrium unemployment rate, potential output). The equilibrium unemployment rate is usually connected to the nonaccelerating inflation rate of unemployment (NAIRU) because the issue of monetary stability is incorporated in this theoretical concept.

Recent studies have attempted to estimate the NAIRU or output gap using various models and techniques. A nice review of possible methods to estimate output gap are presented in Bjoernland et al. [4]. These methods may be applied to estimate equilibrium unemployment as well.

One of the first estimates of time-varying NAIRU was made by Gordon [10]. Berger and Evereart [3] estimate the NAIRU as an unobserved non-stationary process within the framework of a simple structural model including Phillips curve, Okun's law and a demand equation. Their model is estimated in a Bayesian framework (importance sampling). Similarly, Logeay and Tober [15] model the NAIRU as non-stationary trend and estimate it simultaneously with a Phillips curve using the Kalman-filter technique. They combine the hysteresis approach with a time-varying NAIRU. Apel and Jansson [1] estimate the NAIRU jointly with output gap in a system of structural equations (including Phillips curve and Okun relation) using an unobservable components approach. Hjelm [11] goes a step further and uses a structural VAR approach. Stephanides [21] uses univariate approach employing a common version of the Phillips curve.

It is evident that there are many models and estimation techniques implying different estimates of unobserved variables. The uncertainty arising from NAIRU and potential output estimates is closely connected with the uncertainty of model parameters, stochastic nature of the unobserved variables, and with various model specifications.

As Szeto and Guy [22] pointed out, most models require an estimate of inflationary expectations. Our approaches assume adaptive and rational expectations. Rational expectations are solved within the model identification in our approach.

Two alternative estimates of the NAIRU for small open economies (Czech Republic and New Zealand) are presented and analyzed in our paper. Estimates are made within the framework of the hysteretic model with adaptive expectations and also dynamic rational expectations model. A key part of both approaches is Phillips curve which is an important theoretical link between unemployment and inflation. The resulting trade-off played a fundamental role in practical monetary policy.

As a first model, we use the hysteresis model that is based on simple Phillips curve with adaptive expectations (see Gordon [9]). Hysteresis hypothesis posits a NAIRU that automatically follows the path of actual unemployment rate. Consequently, law of motion of equilibrium unemployment is known. That is very useful in estimating of the NAIRU which is generally treated as an unobserved variable. Beyond this, hysteresis phenomenon in the economy has major implications for the relationship between equilibrium unemployment and price stability. Any rate of unemployment is consistent with stable inflation. Equilibrium rate of unemployment can thus be reduced if actual unemployment declines as the result, for example, of expansionary demand policy.

Rational expectations are implemented into the framework of multi-equation dynamic macroeconomic (monetary) model which includes basic equations determining economic development. Equilibrium unemployment (NAIRU) follows a random walk in this case. Phillips curve equation is enriched by expected inflation and the model is solved as a model with rational expectations. This model is based on practical monetary model discussed in Laxton and Scott [14].

The aim of our research is to estimate and analyze the trajectory of unobservable level of equilibrium unemployment in two small open economies. The Czech Republic as a transitive economy was characterized by low unemployment at the beginning of the transformation process. During the late 1990s, a significant increase of unemployment rate occurred. This could be result of monetary policy shock in 1997 which led, in accordance with hysteresis hypothesis, to permanent shift in unemployment rate. Current successive decrease of unemployment may also be explained by hysteresis mechanism as a result of demand oriented monetary and fiscal policy. We verify the presence of the hysteresis in unemployment by estimating of the possible paths of the equilibrium unemployment within the framework of the hysteretic and non-hysteretic model.

The economy of New Zealand was influenced by number of shocks in the late 1980s which dramatically raised the unemployment rate. The question is if there may be similar hysteretic mechanisms as in the Czech economy. Our paper also compares both economies and evaluates the relevance of the hysteresis hypothesis (with all implications of this approach) in standard market economy of New Zealand as well. Especially, we answer the question whether the hysteresis in economy is permanent or temporal phenomenon.

We use techniques of Bayesian analysis (Gibbs sampler and Metropolis-Hastings algorithm) to identify the models. Unobserved states are estimated using the Kalman filter. Model is solved as Dynamic Stochastic General Equilibrium (DSGE) model with rational expectations using the Dynare toolbox.[1] The Bayesian approach has many advantages. First, we are able to incorporate our prior information about model properties (model parameter) in a simple and straightforward way. Second, we can also compare the competing models.

We present alternative estimates of trajectories of unobserved variables (NAIRU and potential output) and we identify dynamic properties of the economies in our paper. We derive probabilities of validity of the natural rate hypothesis, theory of NAIRU and the full hysteresis (based on Bayesian estimates of the hysteretic Phillips curve) by carrying out the Bayesian inference. This inference leads to conclusion about the efficiency of economic policy. We are also able to comment possibilities of sustainable growth in both economies (from the point of view of the monetary stability). Our comments are based on actual states of economies implied by unemployment gap and output gap.

A part of our paper deals with theoretical issues concerning relationship among the natural rate hypothesis, theory of NAIRU and hysteresis hypothesis. These issues are discussed within the framework of Phillips curve theory and corresponding implications for economic policy. Our conclusions involve evaluation of the effects of inflation targeting because both analyzed economies practice this kind of monetary policy.

The structure of our paper is as follows. The second section presents a short history and practical implications of the theory of NAIRU, natural rate hypothesis and hysteresis hypothesis. In the next two sections (section 3 and 4), two models and their estimates are presented. The last section discusses the estimates of unobserved variables (NAIRU and potential output), summarizes our main points concerning the state of the economy and draws some final conclusions about the implications for economic growth and the role of monetary policy.

2 Theory of the NAIRU and hysteresis hypothesis

The key questions for the economists since World War II have been concerned with the causes of changes in the rates of inflation and unemployment and with the relation between the price level and the level of unemployment and its determinants.

Theoretical concept of "Phillips curve" played very important role in post-war Keynesianism. Phillips [18] attempted to identify the relation between unemployment and the rate of change of money wage rates during business cycles. So-called "traditional Phillips curve" reflects this relation in terms of economic theory. The main idea was simple: *"In a year of rising business activity, with the demand for labour increasing and the percentage unemployment decreasing, employers will be bidding more vigorously for the services of labour than they would be in a year during which the average unemployment was the same but the demand for labour was not increasing. Conversely in a year of falling business activity employers will be less inclined to grant wage increase, and workers will be in a weaker position to press for them."* (Phillips [18], p. 283) Hence, changes in money wage rates can be explained by level of unemployment and its variations (except exogenous shocks in import prices).

Substitution of inflation for the wage changes was the next modification of Phillips curve. Keynesian economics had been able to take into account the level of unemployment and the level of inflation.

[1] Software available on www.cepremap.cnrs.fr/dynare/

Based on this theoretical framework (empirically justified in the economy of United States by Samuelson and Solow [19]), policymakers have the opportunity to choose from two evils: inflation or unemployment. Such policies, especially expansionary demand policies, were applied in the post-war period. These policies underestimate money inflation and expectations of the economic agents.

The collapse of the Phillips curve occurred in the 1970s because it was unable to cope with energy shocks that dominated in period 1972 to 1981. Keynesian theory was not able to explain concurrent rise in inflation and unemployment by its own (theoretical) tools. Better said, it was not able to explain these events quickly enough (see Blinder [5]). However, events in the 1970s and early 1980s were consistent with Milton Friedman's Natural Rate Hypothesis (NRH) and that gave rise to domination of new economic paradigms, namely monetarism and later new classical economics. Natural rate of unemployment is defined by Friedman [8] as *"The "natural rate of unemployment" is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labour and commodity markets, including market imperfections, stochastic variability in demand and supplies, the cost of gathering information about job vacancies and labour availabilities, the cost of mobility, and so on."* This level of unemployment is natural outlet of economic equilibrium.

There is temporary trade-off between inflation and unemployment due to an existence of "adaptive expectations". However, this trade-off does not work in the long run. Expansionary demand policies lowering unemployment below its natural rate are therefore accompanied by accelerating inflation.

Blinder [5] claims that the natural rate hypothesis played essentially no role in the intellectual combat of the period 1972-1985. He argues that Phillips curve that is vertical in the long run has been an integral part of Keynesian theory since about 1972. In the late 1970s, Keynesians added supply-side variables to empirical Phillips curves as well[2].

Changes in the main economics paradigm influenced economic policy. The first half of the 1980s was characterised by the disinflationary monetary policies. Blinder [5] called them "Reagan-Volcker fiscal and monetary policy shocks". In accordance with Friedman's natural rate hypothesis, these policies caused increase in the level of unemployment. Nevertheless, this hypothesis argues that Europe was operating closely to its natural rate of unemployment that is often called the "nonaccelerating inflation rate of unemployment" (NAIRU) because inflation in Europe has been stabilized and was no longer decelerating.

In reality, high and persistent unemployment prevailed in Europe during the late 1980s, 1990s and it is not even changing too much today. Two main interpretations of the rising European NAIRU have been proposed by Gordon [9]:

- The "structuralist view" advances specific supply-side impediments (including high real wages and government regulations) as the cause of the rising NAIRU. This view implies that the NAIRU cannot decline until these impediments are removed. The absence of convincing quantitative evidence that would explain why the European NAIRU has increased from 2 percent in the 1960s to 10 percent today is weakness of this interpretation. This approach implicitly claims that, for example, government regulations in the 1980s were higher than in the 1960s. That is not the most credible conclusion.
- The "hysteresis approach" posits NAIRU that automatically follows trend of the actual unemployment rate. The European NAIRU is high because actual unemployment is high. The NAIRU can be reduced if actual unemployment declines as the result of expansionary demand policies. The empirical counterpart of the hysteresis framework is a relation where inflation depends on the change in unemployment and not on the level of unemployment as in the NRH.

Existence of hysteresis in the unemployment has two important implications:

1. Any rate of unemployment is consistent with steady inflation at a rate that depends on the history of both inflation and unemployment.
2. Expansionary demand policies can be efficient in the battle against unemployment whereby economic policy in a Keynesian direction revives.

These implications reject Milton Friedman's NRH. Hysteresis hypothesis is thus one of the new theoretical foundations for Keynesian economics.

[2] Nevertheless it was too late. That is very interesting because Phillips [18] points out that the exogenous shocks in import prices are not included in his relations between wages and unemployment.

Let us return to the definition of the natural rate of unemployment and its relation to the NAIRU. There may be a question concerning their differences, in particular in the framework of hysteresis in unemployment. Natural rate of unemployment was defined above. It is an equilibrium level of unemployment determined by structural patterns of labour market. NAIRU is rather an empirical value. The theory of NAIRU is a response to monetarist critique regarding Philips curve and its implication for efficiency of economic (monetary) policy. This concept is based on reinterpretation of Phillips curve in a way that brings closer Keynesian and monetarist views of the unemployment-inflation relation but preserve effects of activist demand policy. NAIRU can be interpreted as a rate below which the actual unemployment causes inflation to accelerate. Mechanism behind this is an extension of traditional Phillips curve mechanism: low unemployment creates pressures on wages which leads to their increase. This increase raises costs of the firms and implicitly also consumer prices. When the unemployment is high the mechanism works in opposite directions. So, there is a rate of unemployment that does not change the level of inflation.

Theory of the NAIRU implies that low unemployment may cause accelerating inflation regardless of sources of this low unemployment. That is not general conclusion of the natural rate hypothesis. From Keynesian point of view, theory of the NAIRU successfully reformulates the natural rate hypothesis. The level of NAIRU may be a guide to monetary or fiscal policy: unemployment below NAIRU enables expansionary demand incentives of economic policy and actual unemployment above NAIRU requests restrictive policy interventions. NAIRU may be viewed as a practical guide for economic policy.

From the monetarist perspective, the NAIRU is a synonym for the natural rate. The NAIRU theory is misunderstanding of the ineffectiveness of government demand oriented policy. Espinoza-Vega and Russel [7] and Chang [6] discuss these issues in more detail.

On other hand, hysteresis hypothesis posits NAIRU that automatically follows the path of the actual unemployment. NAIRU and natural rate of unemployment are the same equilibrium values. Any rate of unemployment may be consistent with the stable inflation.

3 Hysteretic Phillips curve

The model of the Phillips curve, presented in this section, allows us to prove particular probabilities of the natural rate hypothesis, hysteresis hypothesis and the NAIRU theory. The specificity of hysteretic Phillips curve enables us to estimate the trajectory of the NAIRU (or equilibrium unemployment) based on macroeconomic data of the Czech Republic and of the New Zealand. Hysteretic Phillips curve offers macroeconomic view on hysteresis in unemployment. There are many hysteresis mechanisms in the background which serve as microfoundations of this theory. Moreover, hysteretic concept of unemployment may be set into the framework of growth theory (model with multiple equilibriums), which enhances theoretical and practical attractiveness of this hypothesis. Nemec and Moravansky [16] present a short review of hysteresis mechanism and their verification in the Czech economy. Calibrated growth model with hysteresis effects (calibrated on Czech data) is described in Nemec and Moravansky [17].

3.1 Basic Model

Hysteretic Phillips curve is similar to that one presented in Gordon [9]. It is the simplest version of the symmetric natural rate hypothesis relating unemployment and inflation:

(G1) $\pi_t = \alpha\pi_{t-1} + \beta(U_t - U_t^*)$

Inflation rate (year-on-year) is denoted as π_t, unemployment rate is U_t and U_t^* represents equilibrium rate of unemployment or NAIRU. Parameter α indicates the power of adaptive inflation expectations. Hysteresis hypothesis posits a NAIRU that automatically follows the path of actual unemployment rate. Consequently, law of motion of equilibrium unemployment is known. Hysteresis arises when equilibrium unemployment does not depend only on its microeconomic determinants represented by Z_t but also on the lagged unemployment rate U_{t-1}. These microeconomic determinants correspond to actual structural characteristics of the labour and commodity markets expressed by Friedman. Assuming that hysteresis exists, we obtain

(G2) $U_t^* = \eta U_{t-1} + Z_t$

Full hysteresis occurs when $\eta = 1$, which means that there is not unique U_t^*.[3] Substituting (G2) into (G1) results in

(G3) $\pi_t = \alpha\pi_{t-1} + \beta(U_t - \eta U_{t-1} - Z_t)$

By transforming (G3), we can see that full hysteresis implies a relation between inflation and the change of unemployment, not the level of unemployment. That is why hysteresis implies that any rate of unemployment may be consistent with steady inflation.

(G4) $\pi_t = \alpha\pi_{t-1} + \beta(1-\eta)U_t + \beta\eta(U_t - U_{t-1}) - \beta Z_t$

We consider Z being time invariant parameter. Estimated model is a normal linear regression model.

(G5) $\pi_t = \lambda_1 + \lambda_2\pi_{t-1} + \lambda_3 U_t + \lambda_4(U_t - U_{t-1}) + \epsilon_t$

Error term ϵ_t has common statistical properties.

3.2 Estimation techniques

We estimate the parameters of the normal linear regression model with independent normal-gamma prior using Gibbs sampler. The Gibbs sampler is a powerful tool for Bayesian posterior simulation.

The likelihood function

We assume $\epsilon_t \sim N(0, h^{-1})$. The likelihood function may be written as:

$$p(y|\lambda, h) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}}\left\{\exp\left[-\frac{h}{2}(y - X\lambda)'(y - X\lambda)\right]\right\}$$

The prior

We assume $p(\lambda, h) = p(\lambda)p(\beta)$ with $p(\lambda)$ being Normal and $p(h)$ being Gamma:

$$\lambda \sim N(\underline{\lambda}, \underline{V})$$
$$h \sim G(\underline{\nu}, \underline{s}^{-2})$$

The prior density may be enhanced by the information that $\eta \in (0,1)$. We use indicator function $1(\lambda \in A)$ and 0 otherwise.

$$p(\lambda) = \frac{1}{(2\pi)^{\frac{k}{2}}}|\underline{V}|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\lambda - \underline{\lambda})'\underline{V}^{-1}(\lambda - \underline{\lambda})\right] \cdot 1(\lambda \in A)$$

$$p(h) = c_G^{-1} h^{\frac{\underline{\nu}-2}{2}} \exp\left(-\frac{h\underline{\nu}}{2\underline{s}^{-2}}\right)$$

Where c_G is the integrating constant for the Gamma p.d.f. We use following notation: $\underline{\lambda} = E(\lambda|y)$ is the prior mean of λ and the prior mean and degrees of freedom of h are $\underline{s}^{-2}$ and $\underline{\nu}$ respectively. $\underline{V}$ is simply the prior covariance matrix of λ. X is the matrix of explanatory variables.

The posterior

The posterior is proportional to the prior times the likelihood:

$$p(\lambda, h|y) \propto \left\{\exp\left[-\frac{1}{2}\left\{h(y - X\lambda)'(y - X\lambda) + (\lambda - \underline{\lambda})'\underline{V}^{-1}(\lambda - \underline{\lambda})\right\}\right]\right\} \cdot 1(\lambda \in A) \times h^{\frac{N+\underline{\nu}-2}{2}} \exp\left[-\frac{h\underline{\nu}}{2\underline{s}^{-2}}\right]$$

This joint posterior density does not take the form of any known density. But, we are able to derive full conditional densities to carry out posterior inference (using Gibbs sampler). For further details see Koop [13] and note that our indicator function must be implemented in the appropriate equations. Bayesian approach allows us to derive Bayes factor comparing probabilities of particular model. We estimate model probabilities (Table 4) using Savage-Dickey density ratios. This method is a way of writing the Bayes factor for comparing nested models.

3.3 Estimation results

We use quarterly seasonally adjusted macroeconomic data of the Czech Republic (from the first quarter 1996 to the third quarter 2007) and of the New Zealand (from the last quarter 1990 to the third quarter 2007). In particular, year-to-year inflation is measured in the case of the Czech Republic as net inflation (reflecting transitory character of the economy, i.e. without regulated prices inflation) and for the New Zealand consumer prices inflation is used.

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[3] The unique equilibrium unemployment U^* exists if $\eta = 0$ (hysteresis is not present at labor market). In this case the NAIRU depends only on the structural determinants (Z_t). These determinants may be time-variant, but we expect they are constant in the short run.

All variables (inflation and unemployment) are seasonally adjusted using Kalman filter procedure. Source of data is the Czech Statistical Office, the Czech National Bank and the Reserve Bank of New Zealand.

Estimation results are presented in Table 1 and Table 2. Empirical first and second moments of marginal densities of the parameters are presented in second and third column. Geweke's convergence diagnostic CD is an indicator of convergence of the Gibbs sampler. It seems that the parameters have converged to the joint posterior density which we are interested in. Bayes factor (post odds ratio) is calculated using Savage-Dickey density ratio (last column of the tables). It shows ratio of probabilities of a restricted and unrestricted model. The restricted model is the model with $\lambda_i = 0$.

Table 1: Parameters estimation (hysteretic Phillips curve) – Czech Republic

	Prior mean (std. deviation)	Posterior mean (std. deviation)	Geweke's CD	Bayes factor $\lambda_i = 0$
λ_1	2.0000 (1.5000)	2.3339 (0.8058)	0.2637	0.0528
λ_2	0.5000 (0.2500)	0.7450 (0.0747)	-0.5451	0.0000
λ_3	0.5000 (1.0000)	-0.2443 (0.0921)	-0.0514	0.2726
λ_4	0.5000 (1.0000)	-0.7848 (0.4259)	0.3689	0.5026

Table 2: Parameters estimation (hysteretic Phillips curve) – New Zealand

	Prior mean (std. deviation)	Posterior mean (std. deviation)	Geweke's CD	Bayes factor $\lambda_i = 0$
λ_1	2.0000 (1.5000)	0.1398 (0.0464)	-0.2410	0.5529
λ_2	0.5000 (0.2500)	0.8429 (0.0526)	0.3404	0.0000
λ_3	0.5000 (1.0000)	-0.0111 (0.0041)	0.1832	4.5302
λ_4	0.5000 (1.0000)	-0.1202 (0.0433)	0.2162	0.3993

The Table 3 shows the structural parameters of the model. The Table 4 presents probabilities of particular models, i.e. the probabilities that natural rate hypothesis ($\eta = 0$), full hysteresis ($\eta = 1$) or theory of the NAIRU $\eta \in (0;1)$ can be computed using the data.

Table 3: Structural parameters

	α	β	η	Z
Czech Republic	0.7450	-1.0291	0.7627	2.268
New Zealand	0.8429	-0.1314	0.9151	1.064

Table 4: Posterior probabilities of the models

	$\eta = 0$	$\eta = 1$	$\eta \in (0;1)$
Czech Republic	0.2831	0.1537	0.5632
New Zealand	0.0673	0.7638	0.1689

We are able to simulate the trajectory of the NAIRU and unemployment gap using the values of estimated structural parameters (Table 3). These indicators are presented jointly with other estimated trajectories in Figure 3 - Figure 6. The results are discussed in following paragraphs but few comments on estimates of the hysteretic Phillips curve should be noticed here. The signs of the structural parameters for both economies are in accordance with economic theory. Increasing unemployment leads to decrease inflation rates (λ_4 is negative), high unemployment tends to cut inflation pressures (arising from labour market) as well (λ_3 is negative).

The value of parameter η in model for Czech economy suggests that there is hysteresis in unemployment (but the extreme case of full hysteresis has probability only 15 %). An appropriate demand-oriented economic policy may cut down unemployment rate in the long run leaving inflation rate unchanged. Relatively high value of estimated parameter α indicates strong adaptivity in inflation expectations. This may imply high confidence in inflation targeting and in the credibility of the Czech National Bank.

Estimates of hysteretic Phillips curve in the case of the New Zealand economy shows that the probability of full hysteresis is 76%. This suggests that decelerating unemployment in the 1990s was not accompanied by accelerating inflation. But, how plausible are the estimation results of the simple Phillips curve? Our first suggestions about hysteretic patterns of the New Zealand unemployment (as well of the Czech unemployment) will be verified in our second (non-hysteretic) model. In the case of the New Zealand economy, the consequences of the traditional hysteresis hypothesis do not seem to be valid. Lowering unemployment is more probably the result of structural changes on the New Zealand labor market than the success of the expansionary demand policy. Accompanied low and stable inflation was achieved by effective inflation targeting. This may be seen in the high value of the parameter α which measures the confidence in consistent monetary policy.

4 Dynamic macroeconomic model

This section presents a multi-equation dynamic (monetary) macroeconomic model of the open economy with rational expectations. There are not any explicit assumptions about possible hysteretic patterns of the unemployment in the economy. It is an equilibrium model which connects output dynamic, unemployment dynamics and inflation dynamics. The variables of NAIRU and potential output are modelled as unobserved states of the economic system in this model (or better said, in the model identification).

4.1 Basic model

The model concept is based on approach proposed by Laxton and Scott [14]. It consists of system of reduced causal equations which we will briefly describe.

(L1) Output gap definition

$$ygap_t = 100 * (gdp_t - \overline{gdp_t})$$

The first equation is the definition of output gap. It corresponds to difference between logs of gross domestic product (gdp_t) and potential output ($\overline{gdp_t}$). Output gap is measured as percentage deviation of actual output from the potential output.

(L2) Output gap dynamics

$$ygap_t = \alpha_1 ygap_{t-1} - \alpha_2 rgap_{t-1} - \alpha_3 zgap_{t-1} + \epsilon_t^{ygap}$$

This equation implies relation between actual output gap and its lagged value. Output gap is also influenced by the lagged real interest rate ($rgap_{t-1}$) and the real exchange rate ($zgap_{t-1}$). Both of them are measured as deviations from their equilibrium values. There are negative signs before parameters, so we expect positive values of the estimated parameters in all equations. Positive deviations of the real interest rate and of the real exchange rate decrease the output gap. Variable ϵ_t^{ygap} denotes the shock to output gap.

(L3) Unemployment gap definition

$$ugap_t = u_t - \bar{u}_t$$

Unemployment gap is defined as deviation of actual unemployment (u_t) from its equilibrium ($\bar{u}_t$) represented by NAIRU. The unemployment gap is therefore measured in percentage points.

(L4) Stochastic process of potential output

$$\overline{gdp}_t = \gamma_t + \overline{gdp}_{t-1} + \epsilon_t^{\overline{gdp}}$$
$$\gamma_t = \beta\gamma^{ss} + (1-\beta)\gamma_{t-1} + \epsilon_t^{\gamma}$$

Unobserved potential output is modelled as random walk with drift. This specification assumes that there is shock ($\epsilon_t^{\overline{gdp}}$), directly affecting the level of potential output each time period. In addition, there is persistent deviation in the growth rate of potential output (γ_t) from a constant steady-state growth rate (γ^{ss}).

(L5) Stochastic process of the NAIRU

$$\bar{u}_t = \bar{u}_{t-1} + \epsilon_t^{\bar{u}}$$

This equation is a stochastic process for the NAIRU. This equilibrium unemployment is modeled as random walk. This assumption is very useful and reasonable for identification of unobserved variables.

(L6) Unemployment gap dynamics

$$ugap_t = -\phi_1 ygap_t + \phi_2 ugap_{t-1} + \epsilon_t^{ugap}$$

This equation is a dynamic Okun's law which relates output gap and unemployment gap. Increase in the output gap should lead to decrease in unemployment gap. Possible persistence and exogenous shock are represented by lagged unemployment and by the shock ϵ_t^{ugap} respectively.

(L7) Inflation equation (Phillips curve)

$$\pi_t = \delta_1 \pi_t^m + \delta_2 E_t \pi_{t+4} + (1 - \delta_1 - \delta_2)\pi_{t-1} - \delta_4 ugap_t - \delta_5 \Delta ugap_t + \epsilon_t^{\pi}$$

The last equation is a standard open-economy inflation equation. In this equation year-on-year inflation depends on import price inflation (π_t^m), expectations of inflation over the next year and on the unemployment gap and its change (pressures from labour market). Thus, the model is backward-looking and forward-looking as well.

For estimation purposes, stochastic AR(1) processes for real interest rate gap, real exchange rate gap and import price inflation are added to system equations:

$$rgap_t = r \cdot rgap_{t-1} + \epsilon_t^{rgap}$$
$$zgap_t = z \cdot zgap_{t-1} + \epsilon_t^{zgap}$$
$$\pi_t^m = p \cdot \pi_{t-1}^m + \epsilon_t^{\pi^m}$$

4.2 Estimation techniques

Due to unobservable variables in the model, we need to use filtration techniques. Model parameters and the trajectories of unobservable states are estimated using Dynare toolbox for Matlab. Dynare is a powerful tool which use techniques of Bayesian inference, in particular Random Walk Metropolis-Hastings algorithm (see Koop [13] for details). Unobservable states (including shocks) are filtered and smoothed by implemented Kalman filter. The model is taken as dynamic stochastic general equilibrium model (DSGE). This allows us to solve rational expectations occurred in the model. In addition, model parameters and the NAIRU and potential output are estimated (filtered and smoothed) separately by extended Kalman filter for the case of the Czech Republic. Estimates parameters and trajectories are presented in the tables and figures below.

4.3 Estimation results

We use quarterly seasonally adjusted macroeconomic data of the Czech economy (from the first quarter 1996 to the third quarter 2007) and of the New Zealand economy (from the last quarter 1990 to the third quarter 2007). Observable variables are year-on-year inflation, unemployment rate, real output (in logarithms), import price inflation (excluding oil prices) and the gaps of the real interest rate and the real exchange rate. The gaps are obtained using Hodrick-Prescott filter (see Hodrick and Prescott [12]).

Table 5 - Table 8 present estimation results of model parameters and standard deviations of shocks. Prior and posterior means, standard deviations and prior densities as well as highest posterior density intervals (HPDIs, similar to confidence intervals in classical statistics) are showed in the tables. The signs of the parameters are in accordance with economic theory.

In the case of the Czech economy, the posterior results are not sensitive to prior hyperparameters. This is not the case of New Zealand. For this economy we present three estimations which differ in the values of prior shocks. Prior information about deep parameters is the same in all cases. The resulting parameter values and trajectories of unobserved variables correspond to three patterns of the economy (or equilibrium unemployment): economy with hysteresis, economy matching NRH and the economy with time-varying NAIRU. To decide which results are "the best" (the best fit of the data), we can use the estimates of the log-likelihood. This quantity is equivalent to the Bayesian marginal likelihood of the model and may be used to model comparison.

Table 5: Parameters estimation (dynamic model) – Czech Republic

Parameter	Prior mean	Posterior mean	HPDIs		Prior density	Prior std. deviations
α_1	0.400	0.6499	0.5277	0.8032	Beta	0.1000
α_2	0.200	0.1763	0.0578	0.3084	Beta	0.1000
α_3	0.150	0.1507	0.0437	0.2796	Normal	0.3000
β	0.800	0.8178	0.6792	0.9526	Beta	0.1000
ϕ_1	0.200	0.1615	0.0666	0.2448	Beta	0.1000
ϕ_2	0.850	0.9795	0.9613	0.9993	Beta	0.1000
δ_1	0.200	0.0847	0.0553	0.1223	Beta	0.1000
δ_2	0.300	0.1229	0.0632	0.1844	Beta	0.1000
δ_4	0.500	0.0351	0.0001	0.0694	Normal	0.2000
δ_5	0.500	0.4051	0.1221	0.7165	Normal	0.2000
Standard deviations of the shocks						
ϵ^{ygap}	1.000	0.7181	0.3143	1.1675	Inv. Gamma	30.0000
$\epsilon^{\overline{gdp}}$	0.500	0.0841	0.0664	0.1015	Inv. Gamma	30.0000
ϵ^{γ}	0.500	0.0842	0.0666	0.1027	Inv. Gamma	30.0000
$\epsilon^{\overline{u}}$	0.500	0.1234	0.0867	0.1553	Inv. Gamma	30.0000
ϵ^{ugap}	0.500	0.1213	0.0862	0.1557	Inv. Gamma	30.0000
ϵ^{π}	0.500	0.6705	0.4768	0.7955	Inv. Gamma	30.0000

The values of estimated parameters ϕ_1 and ϕ_2 suggests that there is high persistence of unemployment gap in the Czech economy. The influence of the output gap is relatively weak. From this point of view, empirical validity of Okun's law is not confirmed in the Czech Republic. It is not surprising that high growth rates of the output in the last years (before 2008) had minimal influence on the decrease of the unemployment. Decreasing rate of unemployment since 2007 has been caused mostly by retirement of long run unemployed. This factor is not incorporated in the Okuns's law.

Estimated parameters in the inflation equation indicate similar dependence of the lagged inflation as hysteretic Phillips curve (in previous model) implies. Influence of the imported inflation is very small. That is amazing because openness of the Czech economy is quite large. Explanation may be seen in the long term appreciation of the Czech currency. Standard deviations of the shocks characterize variability of the related quantities. Output gap and inflation are the most variable of them.

Table 6: Parameters estimation (dynamic model - hysteresis) – New Zealand

Parameter	Prior mean	Posterior mean	HPDIs		Prior density	Prior std. deviations
α_1	0.700	0.6493	0.5075	0.7984	Beta	0.1000
α_2	0.200	0.2074	0.0948	0.3020	Beta	0.1000
α_3	0.200	0.0194	0.0064	0.0321	Beta	0.1000
β	0.200	0.3184	0.1459	0.5027	Beta	0.1000
ϕ_1	0.300	0.2851	0.1568	0.4070	Beta	0.1000
ϕ_2	0.800	0.7607	0.6282	0.8961	Beta	0.1000
δ_1	0.200	0.0214	0.0111	0.0317	Beta	0.1000
δ_2	0.200	0.0874	0.0455	0.1288	Beta	0.1000
δ_4	0.500	0.5769	0.4128	0.7406	Normal	0.1000
δ_5	0.500	0.6106	0.4554	0.7719	Normal	0.1000
Standard deviations of the shocks						
ϵ^{ygap}	0.200	0.2780	0.1337	0.4130	Inv. Gamma	30.0000
$\epsilon^{\overline{gdp}}$	0.200	0.0265	0.0235	0.0293	Inv. Gamma	30.0000
ϵ^{γ}	0.100	0.0178	0.0138	0.0214	Inv. Gamma	30.0000
$\epsilon^{\overline{u}}$	0.200	0.2316	0.1926	0.2689	Inv. Gamma	30.0000
ϵ^{ugap}	0.200	0.0637	0.0417	0.0862	Inv. Gamma	30.0000
ϵ^{π}	0.300	0.1019	0.0652	0.1360	Inv. Gamma	30.0000

Log data density is -259.085031.

Table 6 - Table 8 present estimations of the model parameters for the New Zealand economy. The obtained results have been very sensitive to prior information. We set the same prior density of the

deep parameters. Priors of the standard deviations of the shocks were set differently for three specifications.

We have obtained three sets of model parameters which may be assigned to three patterns of the equilibrium unemployment in the economy. Table 6 corresponds to the hysteretic case, Table 7 describes economy which is in accordance with theory of NAIRU and Table 8 reflects natural rate hypothesis. Corresponding trajectories of the unobserved equilibrium unemployment (see Figure 4) reveals the adequacy of this notation. The trajectory denoted as "Laxton-Hysteresis" (dash line) is very similar to the path of equilibrium unemployment obtained from hysteretic model ("Gordon-Hysteresis"). The dot-and-dashed line "Laxton-NRH" is in accordance with the natural rate hypothesis, i.e. equilibrium unemployment is constant in time. Time-variant NAIRU is described by dashed line with circles ("Laxton-NAIRU").

Table 7: Parameters estimation (dynamic model - NAIRU) – New Zealand

Parameter	Prior mean	Posterior mean	HPDIs		Prior density	Prior std. deviations
α_1	0.700	0.7954	0.6676	0.8977	Beta	0.1000
α_2	0.200	0.1887	0.0913	0.3020	Beta	0.1000
α_3	0.200	0.0132	0.0028	0.0224	Beta	0.1000
β	0.200	0.2901	0.1261	0.4631	Beta	0.1000
ϕ_1	0.200	0.2800	0.1464	0.4104	Beta	0.1000
ϕ_2	0.800	0.9119	0.8860	0.9749	Beta	0.1000
δ_1	0.200	0.0123	0.0048	0.0202	Beta	0.1000
δ_2	0.200	0.0750	0.0277	0.1228	Beta	0.1000
δ_4	0.500	0.0624	0.0116	0.1137	Normal	0.3000
δ_5	0.500	0.9623	0.4050	1.4704	Normal	0.3000
Standard deviations of the shocks						
ϵ^{ygap}	0.100	0.2090	0.0358	0.3154	Inv. Gamma	30.0000
$\epsilon^{\overline{gdp}}$	0.100	0.0150	0.0122	0.0175	Inv. Gamma	30.0000
ϵ^{γ}	0.100	0.0153	0.0126	0.0179	Inv. Gamma	30.0000
$\epsilon^{\bar{u}}$	0.300	0.1278	0.0652	0.1792	Inv. Gamma	30.0000
ϵ^{ugap}	0.200	0.0598	0.0392	0.0812	Inv. Gamma	30.0000
ϵ^{π}	0.200	0.1887	0.0759	0.2669	Inv. Gamma	30.0000

Log data density is -225.351166.

Table 8: Parameters estimation (dynamic model - NRH) – New Zealand

Parameter	Prior mean	Posterior mean	HPDIs		Prior density	Prior std. deviations
α_1	0.700	0.7966	0.6942	0.9134	Beta	0.1000
α_2	0.200	0.1724	0.0752	0.2647	Beta	0.1000
α_3	0.200	0.0105	0.0016	0.0177	Beta	0.1000
β	0.200	0.3273	0.1408	0.4956	Beta	0.1000
ϕ_1	0.300	0.2659	0.1414	0.3896	Beta	0.1000
ϕ_2	0.800	0.9295	0.8724	0.9745	Beta	0.1000
δ_1	0.200	0.0107	0.0045	0.0168	Beta	0.1000
δ_2	0.200	0.0764	0.0274	0.1291	Beta	0.1000
δ_4	0.500	0.0322	0.0107	0.0536	Normal	0.3000
δ_5	0.500	0.6620	0.3243	0.9728	Normal	0.3000
Standard deviations of the shocks						
ϵ^{ygap}	0.100	0.2787	0.0830	0.4574	Inv. Gamma	30.0000
$\epsilon^{\overline{gdp}}$	0.200	0.0265	0.0235	0.0295	Inv. Gamma	30.0000
ϵ^{γ}	0.100	0.0177	0.0141	0.0210	Inv. Gamma	30.0000
$\epsilon^{\bar{u}}$	0.200	0.0799	0.0450	0.1227	Inv. Gamma	30.0000
ϵ^{ugap}	0.200	0.0608	0.0423	0.0791	Inv. Gamma	30.0000
ϵ^{π}	0.400	0.2441	0.1963	0.2905	Inv. Gamma	30.0000

Log data density is -247.011838.

We use the values of the "Log data densities" (marginal likelihood of the models) to determine which model fit the data best. It is possible to compare this density values because the same prior for the structural parameters were chosen. It is clear, that the "NAIRU model" is the best one. There is relative low persistence of the unemployment gap (parameter ϕ_2) in the "Hysteresis case" The value of the parameters of unemployment gap (δ_4) and of the difference in unemployment gap (δ_5) is the same in the Phillips curve equation (but estimated unemployment gap has minimal volatility–see Figure 6). The persistence of the unemployment gaps is significantly lower in all cases in comparison to the estimation results of the model of the Czech economy.

Variability of the shocks affecting the output gap and inflation is high in the Czech economy in comparison with the New Zealand economy. That corresponds to the transition process of the Czech Republic in the 1990s and to the accompanied structural changes of the economy. Higher persistence in the trend growth rate (of the potential output) of the New Zealand denotes stability of this economy in the last 18 years.

When we compare the values of parameters δ_2, we can see that the role of rational expectations (expected inflation) is more remarkable in the Czech economy. This may suggest that inflation targeting is more successful in New Zealand where the influence of past inflation is significantly larger.

5 Two small open economies – comparison

In this section, estimates of trajectories of the NAIRU (unemployment gap) and potential output (output gap) are discussed. Estimates made by Hodrick-Prescott (HP) filter are added to the set of our alternative estimates. This estimate may serve as a benchmark.

We can see in all figures that estimated paths of NAIRU (unemployment gap) and output gap differ in levels (the difference amounts to 2 percentage points). However, dynamics of these unobserved states are very similar. All estimates imply similar changes and turning points. This conclusion is similar to the one made by Scott [20]. His estimations of output gap have been generally in accordance with the historical profile of the output gap.

All estimates of the output gap perfectly illustrate the periods of economic recessions in the Czech Republic with beginning in the year 1997 and 2002. The Czech economy has been characterized by significantly higher growth rates of the output in the last two years. Figure 1 suggests that these growth rates are unsustainable when we use estimates of Laxton model by Kalman filter. Economy should be 2.5 % above the potential output in 2007. The results must be taken with reservations because Kalman filter is very sensitive to the choice of initial conditions. We obtain similar results using HP filter. HP filter has been used with usual parameter $\lambda = 1600$ (for quarterly data) which can also influence the results. We prefer the estimates of Laxton model using Dynare toolbox. These estimates reflect that the real output in the Czech Republic is little above the potential output during last two years. Actual growth rates of 5% (approximately) are consistent with the economic potential of the Czech economy. The economic growth is sustainable.

Estimations of the output gap for the New Zealand show that the cyclical behaviour of the economy is very mild. There were no strong cyclical fluctuations. This may be the result of effective monetary policy in the last decade. Actual position of the economy seems to be at the potential. We use the outcomes of Laxton model in this evaluation.

As mentioned in the introduction, the level of the NAIRU determines the equilibrium unemployment (and resulting sustainable employment, i.e. sustainable utilization of the labour force). Alternative estimates of the NAIRU of the Czech economy are presented in Figure 3. Equilibrium unemployment has been increasing significantly since 1996. This may be connected with the transformation process and with structural changes in the economy. It must be pointed out that the trajectory of NAIRU in the framework of hysteretic Phillips curve (denoted as Gordon-hysteresis) is very similar to Laxton approach (Laxton-Dynare). It seems that hysteresis hypothesis play essential role in the description of unemployment pattern in the Czech economy. Monetary shock (or restriction) in 1997 negatively influenced low unemployment rates that prevailed before 1996. This shock has amplified structural changes in the economy. Inflation had been decelerating and unemployment rate grown. Since 2000, the inflation has been stabilised at relatively low rates but high and persistent unemployment as well as equilibrium unemployment prevailed (in accordance with hysteresis hypothesis).

There is a break in the trend of unemployment in 2005. One factor causing decreasing unemployment may be changes in methods of calculations of unemployment. More plausible explanation is connected with the demand-oriented government policy (including investment incentives and debt financing of the government spending). Unemployment has decreased and inflation has not accelerated. That is next reasoning of hysteresis effects in the Czech economy.

Output gap (see Figure 1) and unemployment gap (see Figure 5) in the Czech Republic in the last two years suggest that actual low unemployment is sustainable. Moderate (wage) inflation pressures resulting from negative unemployment gap are induced especially by lack of employees. Aged employees retire and the size of labour force is shrinking. This is a natural process not included in the framework of presented models.[4] Higher rate of decrease of the unemployment rate (compared with development of equilibrium unemployment) may be understand as a temporary shock which disappears in the near future.

The outcomes of identification of the model for the New Zealand economy reflect significantly decelerating unemployment gap. To justify this statement, we use the most probable "NAIRU model" denoted as "Laxton-NAIRU". We reject the validity of hysteretic models (in the simple model of the Phillips curve) because of low probability of this model in the Laxton framework. Simple hysteretic Phillips curve fails to reveal appropriate patterns of the New Zealand unemployment. This model is too simple to take into account inflation targeting which causes that the inflation is stable regardless of the movements in unemployment gap. It is true that the unemployment was lowered without negative inflation pressures but there were probably no traditional hysteresis mechanisms behind this decrease.

We can accept the crucial factors for lowering New Zealand unemployment described in Szeto and Guy [22]. These factors are consistent with the "structuralist view" and they include policy change in wage bargaining and benefit reforms.

Apel and Jansson [1] pointed out that models (or theories) explaining only potential output or only the NAIRU cannot provide adequate insights if there are important interactions between these two. The case of the New Zealand is a very good example.

The trajectories of output gap and unemployment gap show that the New Zealand economy is on its potential level. There are only little inflation pressures stemming from labor market because there is a negative unemployment gap. Thus, they are not a threat for a sustainable economic growth.

[4] Of course, the retirement age (or early retirement age) is given by parameters of pension system. These parameters are outcomes of political decision which may not be "natural". But this decision may be treated as social (optimal) consensus which justifies the term "natural process".

Figure 1: Output gap - Czech Republic

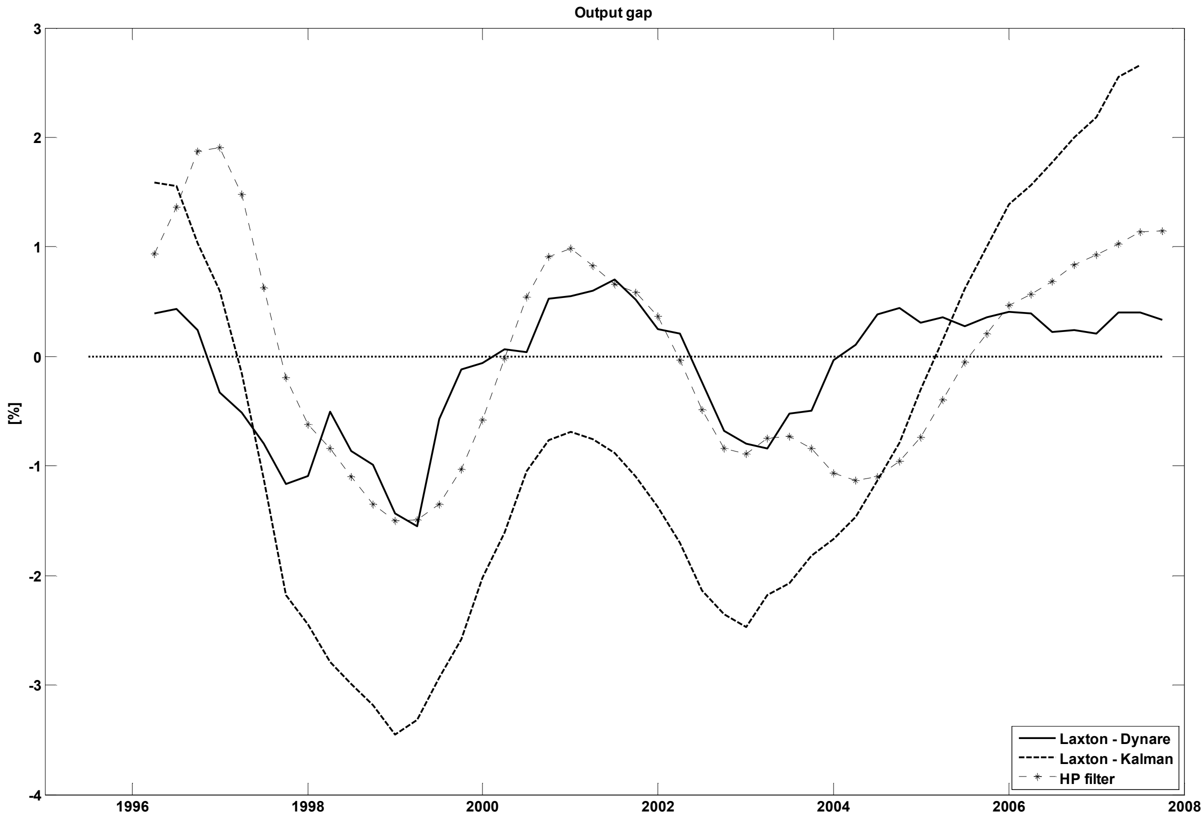


Figure 2: Output gap - New Zealand

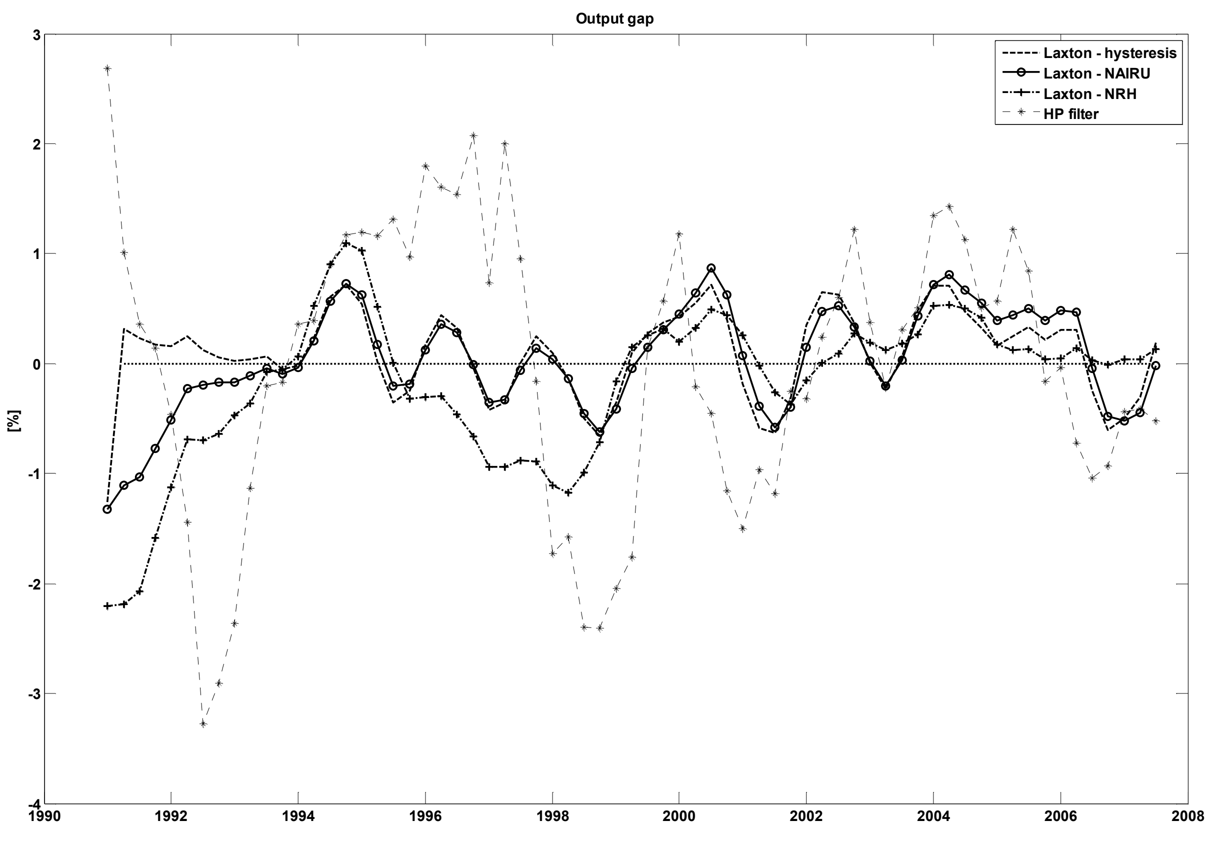


Figure 3: NAIRU and unemployment - Czech Republic

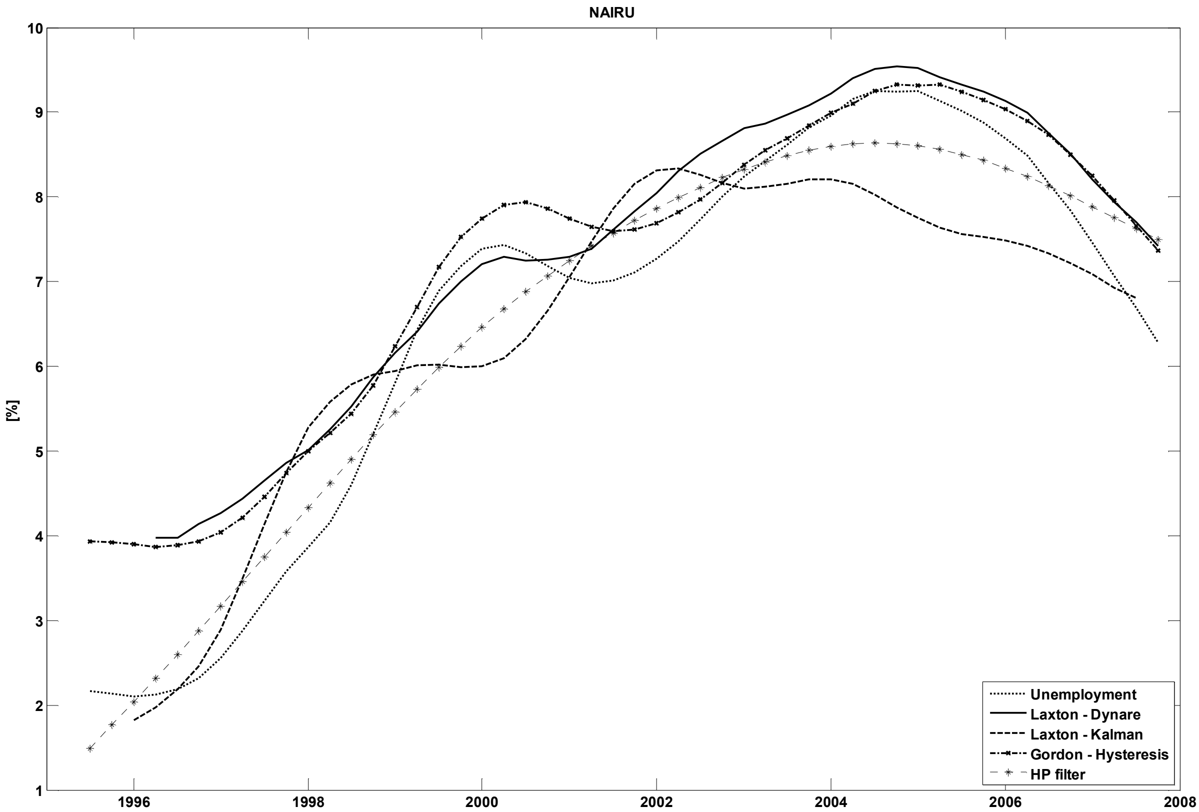


Figure 4: NAIRU and unemployment – New Zealand

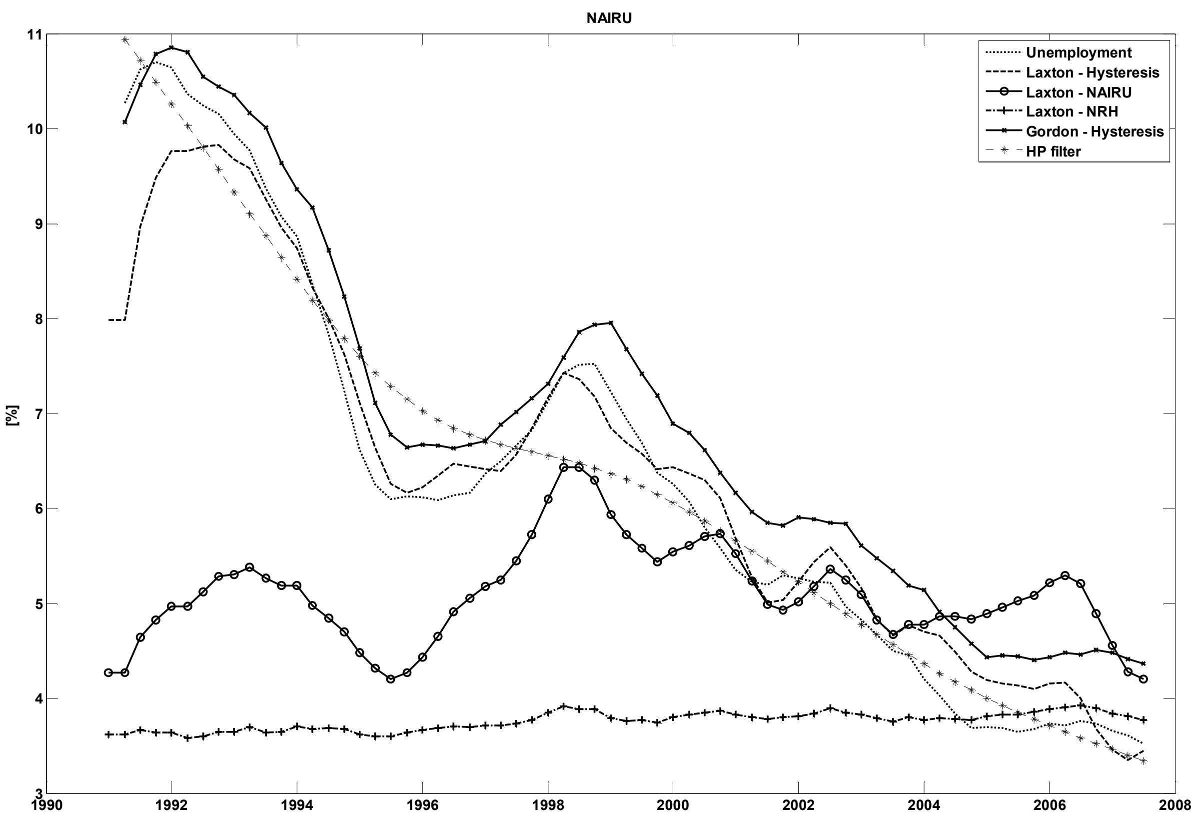


Figure 5: Unemployment gap - Czech Republic

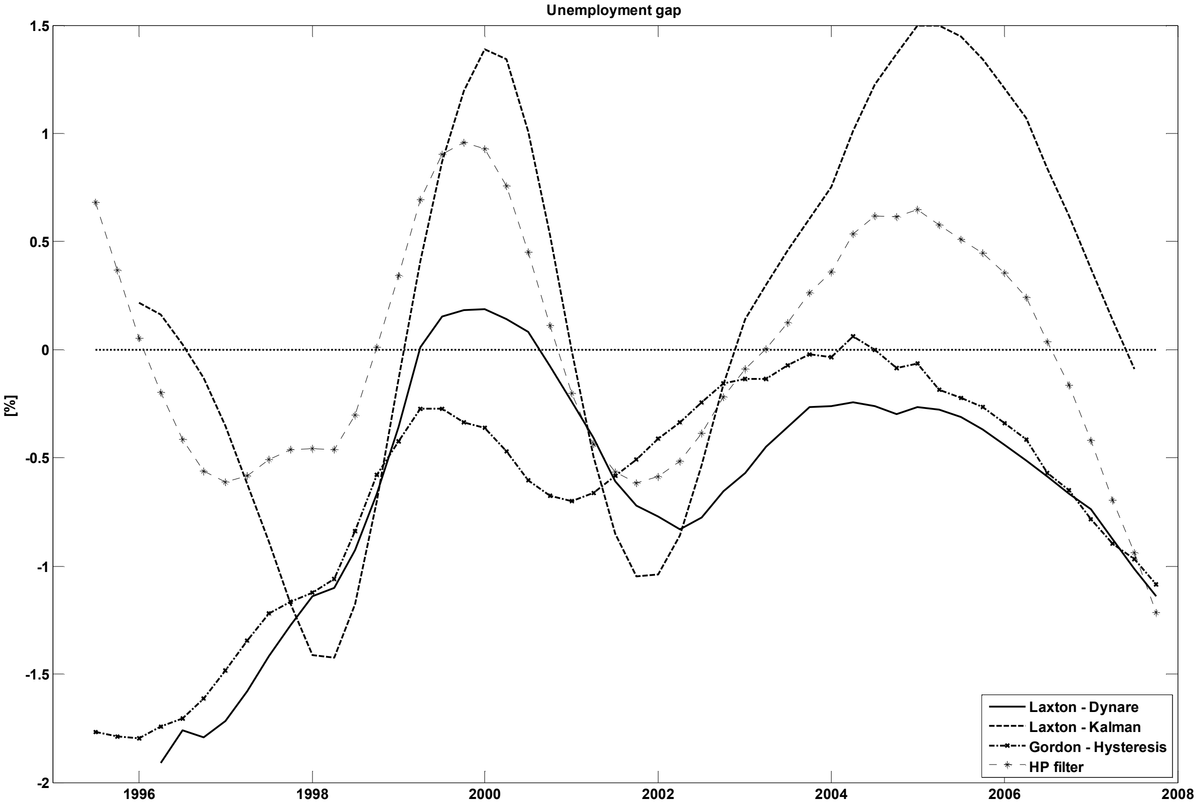
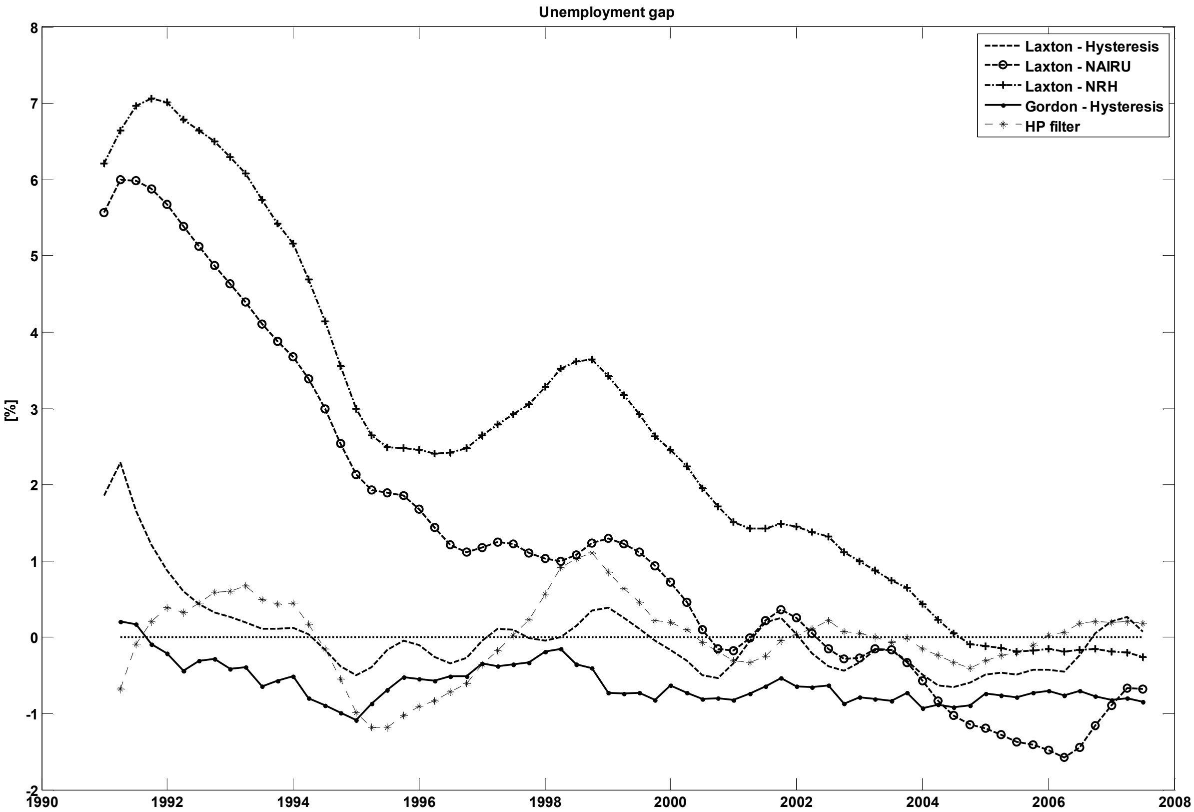


Figure 6: Unemployment gap – New Zealand



6 Conclusion

Bayesian estimates of equilibrium unemployment (NAIRU), unemployment gap and output gap imply that the present state of the Czech and New Zealand economy corresponds to the potential state. Output gap is relative small and unemployment gap does not mean that the labor market is the source of monetary instability. Actual high growth rates of GDP and decreasing unemployment rates are thus sustainable in the long run.

Moreover, the model estimates indicate hysteretic patterns of the Czech unemployment. The existence of hysteresis has therefore important implications for economic policy and its instruments because they can be efficient in the battle against unemployment in the long run. Because the hysteresis is identified in the Czech economy, we can accept that unemployment could be reduced (and probably has been reduced) by using expansionary policies without negative inflation consequences. The NAIRU is thus compatible with any level of unemployment as the hysteresis hypothesis suggests. The hysteresis may be permanent phenomenon in the economy.

On the other hand, the estimate of NAIRU based on simple Phillips curve framework is a false indicator of hysteresis hypothesis for the New Zealand unemployment. To prove hysteresis hypothesis (as in the case of the Czech Republic), a complex model is necessary. This model enables jointly estimation of the NAIRU and potential output. Estimated path of the NAIRU was very sensitive to specification of the prior parameters in this case. We estimated three sets of the paths which have been in accordance with the natural rate hypothesis (equilibrium unemployment is constant in time), hysteresis hypothesis (equilibrium unemployment follows the path of actual unemployment), and the theory of NAIRU (time-varying equilibrium unemployment). Bayesian inference allows us to decide that time-varying NAIRU is the most probable. This lowered the uncertainty related to NAIRU estimation. Most of the decrease in New Zealand unemployment was driven by structural factors compared to Czech experience with hysteresis mechanisms staying behind the decreasing unemployment rate in the last years.

Further research will be focused on identification of an extended dynamic model which includes equations for interest rate gap, real exchange rate gap and a monetary rule (as proposed by Berg et al. [2]). This kind of model may be more appropriate helpful for inflation targeting economies.

Acknowledgement

This work was supported by MSMT project Research centers 1M0524, and funding of specific research at ESF MU. We would like to thank Miroslav Hlousek for his valuable assistance and suggestions.

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