Does Competition Reduce the Risk of Bank Failure?

David Martinez-Miera
CEMFI and UIMP

Rafael Repullo
CEMFI and CEPR

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Abstract

A large theoretical literature shows that competition reduces banks' franchise values and induces them to take more risk. Recent research contradicts this result: When banks charge lower rates, their borrowers have an incentive to choose safer investments, so they will in turn be safer. However, this argument does not take into account the fact that lower rates also reduce the banks' revenues from non-defaulting loans. This paper shows that when this effect is taken into account, a U-shaped relationship between competition and the risk of bank failure generally obtains.

JEL Classification: G21; D43; E43.

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1 Introduction

There is a large theoretical literature that shows that competition reduces banks' franchise values and induces them to take more risk.[1] A key assumption in this literature is that banks invest in assets with exogenous distributions of returns. Recent work by Boyd and De Nicolo (2005) and Boyd et al. (2006) replaces this by the assumption that banks invest in loans. Following the seminal paper on credit rationing by Stiglitz and Weiss (1981), they also assume that the risk of these loans is increasing in the loan interest rate. Hence a reduction in loan rates due to greater bank competition reduces the loans' probability of default. They also assume that loan defaults are perfectly correlated, in which case the loans' probability of default coincides with the bank's probability of failure. Hence they conclude that competition reduces the risk of bank failure.

However, this result does not necessarily hold in the (arguably more realistic) case of imperfect correlation of loan defaults, because then greater bank competition also reduces the interest payments from non-defaulting loans, that provide a buffer to cover loan losses. Thus in addition to the *risk-shifting effect* there is a *margin effect* that goes in the opposite direction, so the final effect on the risk of bank failure is in principle ambiguous.

Our basic setup is identical to that of Boyd and De Nicolò (2005), except for the introduction of imperfect correlation in loan defaults. Specifically, we use a static model of Cournot competition in a market for entrepreneurial loans in which the probability of default of these loans is privately chosen by the entrepreneurs. The banks are funded with fully insured deposits and have no capital. To model imperfect default correlation, we use the single risk factor model of Vasicek (2002),[2] according to which the default of an individual loan is driven by the realization of two risk factors: A systematic risk factor that is common to all loans, and an idiosyncratic risk factor. When the weight of the systematic risk factor is zero we have statistically independent defaults, and when the weight of the idiosyncratic

[1] See Keeley (1990), Besanko and Thakor (1993), Suarez (1994), Matutes and Vives (2000), Hellmann et al. (2000), and Repullo (2004), among others.

[2] This model is used for the computation of the capital requirements in the new framework for bank capital regulation proposed by the Basel Committee on Banking Supervision (2004), known as Basel II.

risk factor is zero we have perfectly correlated defaults.

We show that the result in Boyd and De Nicolò (2005) is not robust to the introduction of even a small deviation from perfect correlation in loan defaults. Specifically, when the number of banks is sufficiently large, the risk-shifting effect is always dominated by the margin effect, so any additional entry would increase the risk of bank failure.

In less competitive loan markets the effect is ambiguous, so we resort to numerical solutions for a large range of parameterizations of the model. We show that in general there is a U-shaped relationship between competition (measured by the number of banks) and the risk of bank failure. In other words, in very concentrated markets the risk-shifting effect dominates, so entry reduces the probability of bank failure, whereas in very competitive markets the margin effect dominates, so further entry increases the probability of failure.

To check the robustness of these results we first consider a dynamic version of the model of Cournot competition in the loan market, in which banks that do not fail in one period have the opportunity to lend to a new set of entrepreneurs in the next period. This generates an endogenous franchise value that is lost upon failure, so banks have an incentive to be prudent. We show that the same U-shaped relationship between competition and the risk of bank failure obtains. We also show that the introduction of franchise values enhances bank stability (relative to the static setup), except in the case in which the number of banks is the one that minimizes the probability of failure, where it has no effect. Finally, the same results (both static and dynamic) obtain when the Cournot model is replaced by a circular road model of competition in the loan market. The fact that the results are robust to the change of strategic variable from quantities (loan supplies) to prices (loan rates) suggests that they are likely to hold for a wide set of model of imperfect competition. Hence the conclusion is that when loan defaults are imperfectly correlated the probability of bank failure is lowest in loan markets with moderate levels of competition, with higher probabilities of failure in either very competitive or very monopolistic markets.

To simplify the presentation, and in contrast with Boyd and De Nicolò (2005), we abstract from competition in the deposit market by assuming that banks face a perfectly elastic supply of (fully insured) deposits at an interest rate that is normalized to zero. In models where banks invest in assets with exogenous distributions of returns imperfect competition can only be introduced in the deposit market, but in models where banks face a downward-sloping demand for loans this is no longer necessary. It should be noted that our results for the Cournot model are robust to the introduction of an upward-sloping supply of deposits.[3] Indeed the margin effect that we have identified is stronger in such a model, because greater bank competition not only reduces the interest payments from non-defaulting loans, but also increases the cost of deposit financing.

The assumption that deposits are fully insured is also made to simplify the presentation. In the absence of deposit insurance, and assuming that depositors are risk-neutral, the deposit rate would be determined in equilibrium by the condition that the expected return of bank deposits equals the risk-free rate.[4] Hence uninsured deposit rates would be increasing in the risk of bank failure, but our results would be essentially unchanged.

The rest of the paper is structured as follows. Section 2 presents the model of Cournot competition with imperfectly correlated defaults. Section 3 characterizes the equilibrium of this model, and analyzes the effect of an increase in the number of banks on loan rates and probabilities of bank failure. Section 4 presents the numerical results on the U-shaped relationship between competition and the risk of bank failure. Section 5 shows that these results also obtain in a dynamic version of the Cournot model with endogenous franchise values, and in a circular road model of loan rate competition. Section 6 discusses the implications of our results for the relationship between monetary and financial stability, and the welfare maximizing competition policy in banking. Section 7 contains our concluding remarks.

2 The Model

Consider an economy with two dates ($t = 0, 1$) and three classes of risk-neutral agents: entrepreneurs, banks, and depositors.

[3] Models where banks compete by simultaneously setting deposit and loan rates are more complicated to analyze, because the balance sheet identity (loans = deposits) is in general not satisfied; see, for example, the discussion in Yanelle (1997).

[4] See Repullo (2005) for a model of information-based bank runs with endogenous uninsured deposit rates.

2.1 Entrepreneurs

There is a continuum of penniless entrepreneurs characterized by a continuous distribution of reservation utilities with support in $\mathbb{R}_+$. Let $G(u)$ denote the measure of entrepreneurs that have reservation utility less than or equal to u.

Each entrepreneur i has a project that requires a unit investment at date 0, and yields a stochastic payoff at date 1

$$R(p_i) = \begin{cases} 1+\alpha(p_i), & \text{with probability } 1-p_i \\ 1-\lambda, & \text{with probability } p_i \end{cases}$$

where the probability of failure $p_i \in [0,1]$ is privately chosen by the entrepreneur at date 0. Following Allen and Gale (2000, chapter 8) we assume that the success return of the project $\alpha(p_i)$ is positive and increasing in p_i. Thus riskier projects have a higher success return. In order to get interior solutions to the entrepreneur's choice of risk we also assume that $\alpha(p_i)$ is concave with $\alpha(0) < \alpha'(0)$. The project's loss given failure λ is positive and smaller than one, and to simplify the presentation we assume that it does not depend on p_i.

To fund their projects entrepreneurs borrow from banks. For any given loan rate r, entrepreneur i will choose p_i in order to maximize its expected payoff from undertaking the project, which is the success return net of the interest payment, $\alpha(p_i) - r$, times the probability of success, $1-p_i$.[5] Let

$$u(r) = \max_{p_i}(1-p_i)(\alpha(p_i)-r) \tag{1}$$

denote the maximum expected payoff that an entrepreneur can obtain when the loan rate is r. Since entrepreneurs only differ in their reservation utilities, the solution $p(r)$ to this problem and hence $u(r)$ do not depend on i. Moreover for $\alpha(0)-\alpha'(0) < r < \alpha(1)$ the solution $p(r)$ will be interior[6] and characterized by the first-order condition

$$(1-p)\alpha'(p) - \alpha(p) + r = 0 \tag{2}$$

[5] With probability p_i the project fails, in which case by limited liability the entrepreneur gets a zero payoff, and the bank recovers $1-\lambda$.

[6] The corner $p=0$ cannot be a solution if $\alpha'(0)-\alpha(0)+r>0$, which gives $\alpha(0)-\alpha'(0)<r$, while the corner $p=1$ cannot be a solution if $-\alpha(1)+r<0$, which gives $r<\alpha(1)$.

Hence by the envelope theorem we have $u'(r) = -(1-p) < 0$. Also, since $\alpha''(\cdot) \leq 0$ and $\alpha'(\cdot) > 0$, differentiating the first-order condition we get

$$p'(r) = -\frac{1}{(1-p)\alpha''(p) - 2\alpha'(p)} > 0$$

Thus the higher the loan rate the higher the probability of failure chosen by the entrepreneurs. The positive effect of loan rates on entrepreneurs' optimal choice of risk will be called the *risk-shifting effect*.

Entrepreneur i will want to undertake her project when the loan rate is r if her reservation utility u_i is smaller than or equal to $u(r)$. Hence the measure of entrepreneurs that want to borrow from the banks at the rate r is given by $G(u(r))$. Since each one requires a unit loan, the *loan demand function* is

$$L(r) = G(u(r)). \tag{3}$$

Clearly for $0 \leq r < \alpha(1)$ we have $L(r) > 0$ and $L'(r) = G'(u(r))u'(r) < 0$. Let $r(L)$ denote the corresponding inverse loan demand function.

In contrast with Boyd and De Nicolò (2005), we assume that project failures and consequently loan defaults are imperfectly correlated. We use the *single risk factor model* of Vasicek (2002), according to which the outcome of the project of entrepreneur i is driven by the realization of a random variable

$$y_i = -\Phi^{-1}(p_i) + \sqrt{\rho}\, z + \sqrt{1-\rho}\, \varepsilon_i \tag{4}$$

where $\Phi(\cdot)$ denotes the cdf of a standard normal random variable and $\Phi^{-1}(\cdot)$ its inverse, z is a systematic risk factor that affects all projects, ε_i is an idiosyncratic risk factor that only affects the project of entrepreneur i, and $\rho \in [0,1]$ is a parameter that determines the extent of correlation in project failures. It is assumed that z and ε_i are standard normal random variables, independently distributed from each other as well as, in the case of ε_i, across projects.

The project of entrepreneur i fails when $y_i < 0$. The deterministic term $-\Phi^{-1}(p_i)$ in (4) ensures that the probability of failure satisfies

$$\Pr(y_i < 0) = \Pr[\sqrt{\rho}\, z + \sqrt{1-\rho}\, \varepsilon_i < \Phi^{-1}(p_i)] = \Phi[\Phi^{-1}(p_i)] = p_i$$

Notice that for $\rho = 0$ the systematic risk factor does not play any role and we have statistically independent failures, while for $\rho = 1$ the idiosyncratic risk factor does not play any role and we have perfectly correlated failures. In what follows we focus on the imperfect correlation case $\rho \in (0, 1)$.

Consider now the continuum of entrepreneurs that want to undertake their projects when the loan rate is r. By our previous argument they all choose the same probability of failure $p = p(r)$. But then the aggregate *failure rate* x (the fraction of projects that fail) is only a function of the realization of the systematic risk factor z. Specifically, by the law of large numbers the failure rate x coincides with the probability of failure of a (representative) project i conditional on the realization of z:

$$\gamma(z) = \Pr\left[-\Phi^{-1}(p) + \sqrt{\rho}\, z + \sqrt{1-\rho}\, \varepsilon_i < 0 \mid z\right] = \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}\, z}{\sqrt{1-\rho}}\right)$$

Hence using the fact that $z \sim N(0, 1)$, the cdf of the failure rate is

$$F(x) = \Pr\left[\gamma(z) \leq x\right] = \Phi\left(\frac{\sqrt{1-\rho}\, \Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right) \tag{5}$$

For $\rho \in (0, 1)$ the cdf $F(x)$ is continuous and increasing, with $\lim_{x\to 0} F(x) = 0$ and $\lim_{x\to 1} F(x) = 1$. It can also be shown that $E(x) = \int_0^1 x\, dF(x) = p$. Note that $\partial F/\partial p < 0$, so changes in the probability of failure p lead to a first-order stochastic dominance shift in the distribution of the failure rate x, and $\partial F/\partial \rho \geq 0$ if and only if $x \leq \Phi(\sqrt{1-\rho}\, \Phi^{-1}(p))$, so changes in the correlation parameter ρ lead to a mean-preserving spread in the distribution of the failure rate x. Note also that when $\rho \to 0$ (independent failures) the distribution of the failure rate approaches the limit $F(x) = 0$, for $x < p$, and $F(x) = 1$, for $x \geq p$. The single mass point at $x = p$ implies that a fraction p of the projects fail with probability 1. And when $\rho \to 1$ (perfectly correlated failures) the distribution of the failure rate approaches the limit $F(x) = \Phi(-\Phi^{-1}(p)) = 1 - \Phi(\Phi^{-1}(p)) = 1 - p$, for $0 < x < 1$. The mass point at $x = 0$ implies that with probability $1 - p$ no project fails, and the mass point at $x = 1$ implies that with probability p all projects fail.

2.2 Banks

There are n identical banks that at date 0 are funded with fully insured deposits, have no capital, and invest in a portfolio of entrepreneurial loans. The supply of deposits is perfectly elastic at an interest rate that is normalized to zero, and there are no intermediation costs. Following Boyd and De Nicolò (2005) we assume that banks compete for loans à la Cournot, so the strategic variable of bank $j = 1, ..., n$ is its supply of loans l_j. The aggregate supply of loans $L = \sum_{j=1}^{n} l_j$ determines the loan rate $r(L)$, which in turn determines the probability of failure chosen by the entrepreneurs $p(r(L))$.

The return of bank j's portfolio is stochastic: a random fraction x of its loans default, in which case the bank loses the interest r as well as a fraction λ of the principal. Thus the bank gets $l_j(1+r)$ from the fraction $1-x$ of the loans that do not default, recovers $l_j(1-\lambda)$ from the fraction x of defaulted loans, and has to pay back l_j to the depositors, so its payoff at date 1 is

$$l_j(1+r)(1-x) + l_j(1-\lambda)x - l_j = l_j[r - (r+\lambda)x]$$

The bank fails when $r - (r+\lambda)x < 0$, that is when the default rate x is greater than the bankruptcy default rate

$$\widehat{x}(L) = \frac{r(L)}{r(L)+\lambda} \tag{6}$$

Hence, by limited liability, bank j's objective function is

$$\pi(l_j, l_{-j}) = l_j \int_0^{\widehat{x}(L)} [r(L) - (r(L)+\lambda)x] \; dF(x; p(r(L))) \tag{7}$$

where l_{-j} denotes the vector of loan supplies of the other $n-1$ banks, and the distribution function of the default rate is written so as to keep track of the effect of the (endogenous) probability of default of the loans. Thus when choosing its supply of loans l_j, bank j takes into account the direct effect on the loan rate $r(L)$, as well as the indirect effect on the probability of default of the loans $p(r(L))$ and hence on the probability distribution of the default rate x.

3 Equilibrium

This section characterizes the Cournot-Nash (symmetric) equilibrium of our model of competition in the loan market, and analyzes the effect of an increase in the number of banks on equilibrium loan rates and equilibrium probabilities of bank failure.

Integrating by parts, the banks' objective function (7) can be written more compactly as

$$\pi(l_j, l_{-j}) = l_j h(L)$$

where

$$h(L) = \int_0^{\widehat{x}(L)} (r(L) + \lambda) F(x; p(r(L)))\, dx \tag{8}$$

is the banks' expected payoff per unit of loans.

In what follows we are going to assume that functional forms and parameter values are such that $h'(L) < 0$ and $h''(L) < 0$, so there is a unique symmetric equilibrium characterized by the first-order condition[7]

$$Lh'(L) + nh(L) = 0 \tag{9}$$

It should be noted that these assumptions are stronger than the assumption that the inverse loan demand function $r(L)$ is decreasing and concave. To see this observe that

$$h'(L) = (r(L)+\lambda)F(\widehat{x}(L); p(r(L)))\widehat{x}'(L) + \int_0^{\widehat{x}(L)} \left[F(x; p(r(L))) + (r(L) + \lambda)\frac{\partial F}{\partial p} p'(r(L)) \right] r'(L)\, dx$$

Now $r'(L) < 0$ implies

$$\widehat{x}'(L) = \frac{\lambda r'(L)}{(r(L) + \lambda)^2} < 0$$

so the first term is negative. The sign of the integral is however ambiguous, because $F(x; p(r(L))) > 0$, while $\partial F / \partial p < 0$ (the first-order stochastic dominance effect on the probability distribution of the default rate) and $p'(r) > 0$ (the risk-shifting effect). To understand the source of the ambiguity consider the simple case with $\rho = 1$ (perfectly correlated defaults) for which $F(x; p) = 1 - p$ for $0 < x < 1$, so by (6) and (8) we have $h(L) = (1 - p(r(L)))r(L)$ and

$$h'(L) = (1 - p(r(L)))r'(L) - r(L)p'(r(L))r'(L)$$

[7] Note that in this model $h(L)$ plays the role of the inverse demand function in a standard Cournot model. The simple proof of existence and uniqueness of equilibrium may be found in Tirole (1988, p. 225).

The first term in this expression is the standard negative one: Higher aggregate loans reduce the banks' expected payoff per unit of loans. The second term is however positive, reflecting the working of the risk-shifting effect: Lower loan rates increase the probability of loan repayment and hence increase the banks' expected payoff per unit of loans. So to get $h'(L) < 0$ the risk-shifting effect must not be very large.[8] A similar argument applies for the assumption $h''(L) < 0$.

The effects of competition on equilibrium aggregate lending and loan rates are stated in the following result.

Proposition 1 *An increase in the number of banks n increases equilibrium aggregate lending L and consequently reduces the equilibrium loan rate r.*

Proof Differentiating the first-order condition (9) and using the assumptions $h'(L) < 0$ and $h''(L) < 0$ gives

$$\frac{dL}{dn} = -\frac{h(L)}{Lh''(L) + (n+1)h'(L)} > 0$$

But then $r'(L) < 0$ implies

$$\frac{dr}{dn} = r'(L)\frac{dL}{dn} < 0 \quad \square$$

Proposition 1 implies that the higher the competition among banks the lower the probability of default of the loans in their portfolios. However, this does not necessarily imply a reduction in the banks' probability of failure. To see this observe that banks fail whenever the default rate x is greater than the bankruptcy default rate $\widehat{x}(L)$ defined in (6). Using the probability distribution of the default rate (5), the *probability of bank failure* is given by

$$q(L) = \Pr\left(x > \widehat{x}(L)\right) = \Phi\left(\frac{\Phi^{-1}(p(r(L))) - \sqrt{1-\rho}\ \Phi^{-1}(\widehat{x}(L))}{\sqrt{\rho}}\right) \tag{10}$$

Hence we have

$$\frac{dq}{dn} = q'(L)\frac{dL}{dn}$$

[8]For example, for small probabilities of default and $r = 4\%$ we would need $p'(r)$ to be smaller than, approximately, $1/0.04 = 25$. This condition is very likely to be satisfied: an increase of 100 basis points in the loan rate r should not lead to an increase of 2,500 basis points in the probability of default p.

Since $dL/dn > 0$ by Proposition 1, it follows that higher competition leads to lower risk of bank failure if and only if the slope of the function $q(L)$ is negative.

Now differentiating (10) we get

$$q'(L) = \frac{\Phi'(\cdot)}{\sqrt{\rho}} \left[\frac{d\Phi^{-1}(p(r(L)))}{dp} p'(r(L))r'(L) - \sqrt{1-\rho}\, \frac{d\Phi^{-1}(\widehat{x}(L))}{dx} \widehat{x}'(L) \right] \tag{11}$$

Since $\Phi'(\cdot) > 0$ (it is a normal density), the sign of $q'(L)$ is the same as the sign of the term in square brackets, which has two components. The first one is negative, since $d\Phi^{-1}(\cdot)/dp > 0$, $p'(r) > 0$, and $r'(L) < 0$, while the second one is positive (whenever $\rho < 1$), since $d\Phi^{-1}(\cdot)/dx > 0$ and $\widehat{x}'(L) < 0$. The negative effect is the *risk-shifting effect* identified by Boyd and De Nicolò (2005): More competition leads lower loan rates, which in turn lead to lower probabilities of default, and hence safer banks. The positive effect is what may be called the *margin effect*: More competition leads to lower loan rates, and consequently lower revenues from non-defaulting loans, which provide a buffer against loan losses, so we have riskier banks. Depending on which of the two effects dominates, the impact of competition on the risk of bank failure may be positive or negative.

A few special cases are worth mentioning. When $\rho = 1$ (perfectly correlated defaults) the margin effect in (11) disappears,[9] so we get the result in Boyd and De Nicolò (2005): Competition always reduces the risk of bank failure. When $\rho = 0$ (independent defaults) the default rate is deterministic (a fraction p of the loans default with probability 1), in which case it is easy to show that for any number of banks the probability of failure is zero. And when $p'(r) = 0$ the risk-shifting effect in (11) disappears, so competition always increases the risk of bank failure. For $0 < \rho < 1$ and $p'(r) > 0$ the result is in general ambiguous. However the following result shows that the margin effect dominates in very competitive markets.

Proposition 2 *For any correlation parameter $\rho \in (0, 1)$, when the number of banks n is sufficiently large additional increases in n increase the probability of bank failure q.*

Proof When n tends to infinity the first-order condition (9) that characterizes the Cournot equilibrium becomes $h(L) = 0$, which by (8) implies $\widehat{x}(L) = 0$ and hence $r(L) = 0$. Thus

[9] Alternatively, substituting $\rho = 1$ in (10) gives $q(L) = p(r(L))$, which implies $q'(L) = p'(r(L))r'(L) < 0$.

in very competitive markets the loan rate approaches the deposit rate, which has been normalized to zero. But then we have

$$\lim_{n\to\infty} \frac{d\Phi^{-1}(\widehat{x}(L))}{dx} = \frac{1}{\Phi'[\Phi^{-1}(0)]} = \infty$$

and $\lim_{n\to\infty} p(r(L)) = p(0) > 0$, by assumption $\alpha(0) < \alpha'(0)$, which implies

$$\lim_{n\to\infty} \frac{d\Phi^{-1}(p(r(L)))}{dp} = \frac{1}{\Phi'[\Phi^{-1}(p(0))]} < \infty$$

Hence by (11) we have $q'(L) > 0$ for sufficiently large n, which implies $dq/dn > 0$. □

Proposition 2 shows that the result in Boyd and De Nicolò (2005) is not robust to the introduction of even a small deviation from perfect correlation in loan defaults. Specifically, in very competitive loan markets the risk-shifting effect is always dominated by the margin effect, so any additional entry would increase the risk of bank failure. The intuition for this result is that as we get close to perfect competition the margin between loan and deposit rates converges to zero. But since the probability of default of the loans is bounded away from zero (and banks have no capital buffer), in the limit the loan losses will always be greater than the intermediation margin, so banks will fail with probability 1. Hence in the limit the relationship between the number of banks and the probability of bank failure will be increasing. The open question is what happens in less competitive loan markets. To answer this question we will resort to numerical solutions for simple parameterizations of the model.

4 Numerical Results

This section computes the equilibrium of the model of competition in the loan market for a simple parameterization in which the inverse demand for loans $r(L)$ and the entrepreneurial risk-shifting function $p(r)$ are linear, and examines how the probability of bank failure q changes with the number of banks n.

The critical parameters that determine the shape of the relationship between q and n are the correlation parameter ρ and the risk-shifting parameter $b = p'(r)$. By the results in

Section 3 we know that q is decreasing in n when $\rho \to 1$ (the case of perfectly correlated defaults), and it is increasing in n when $b \to 0$ (the case of no entrepreneurial risk-shifting). By Proposition 2 we also know that the relationship is increasing for sufficiently large n. Our numerical results shed light on what happens for $0 < \rho < 1$ and $b > 0$, and for smaller values of n.[10]

Specifically, we postulate an entrepreneurial risk-shifting function of the form

$$p(r) = a + br. \tag{12}$$

where $a > 0$ and $b > 0$, and an inverse demand for loans of the form

$$r(L) = c - dL \tag{13}$$

where $c > 0$ and $d > 0$. The linear function $p(r)$ can be derived from a success return function of the form

$$\alpha(p) = \frac{1 - 2a + p}{2b} \tag{14}$$

which implies the expected payoff function[11]

$$u(r) = \frac{(1 - a - br)^2}{2b} \tag{15}$$

In this setup, parameter a is the minimum probability of default of a project (that is, the probability of default that would be chosen by the entrepreneurs for a zero loan rate), and the ratio c/d is the maximum demand for loans (that corresponds to a zero loan rate).[12]

In our benchmark parameterization we take $a = 0.01$, $b = 0.5$, $c = 1$, and $d = 0.01$. This means that the demand for loans goes from 100 to 0 as loan rates go from 0% to 100%, and that the probability of default that corresponds to a loan rate of 2% is 2%. The loss given default parameter λ is set at 0.45, and the correlation parameter ρ is set at 0.2.[13] It should

[10] The computations are carried out in Matlab. The program is available upon request.

[11] Since $L(r) = G(u(r))$, one can show that (13) and (12) also imply $G(u) = (a + bc - 1 + \sqrt{2bu})/bd$.

[12] Since we have normalized to zero the interest rate on (fully insured) deposits, loan rates should be interpreted as spreads over a risk-free rate.

[13] The value of λ is the one specified in the Internal Ratings-Based formula (foundation approach) of Basel II for senior claims on corporate, sovereign, and bank exposures not secured by recognized collateral. The value of ρ for these exposures ranges from 0.12 to 0.24.

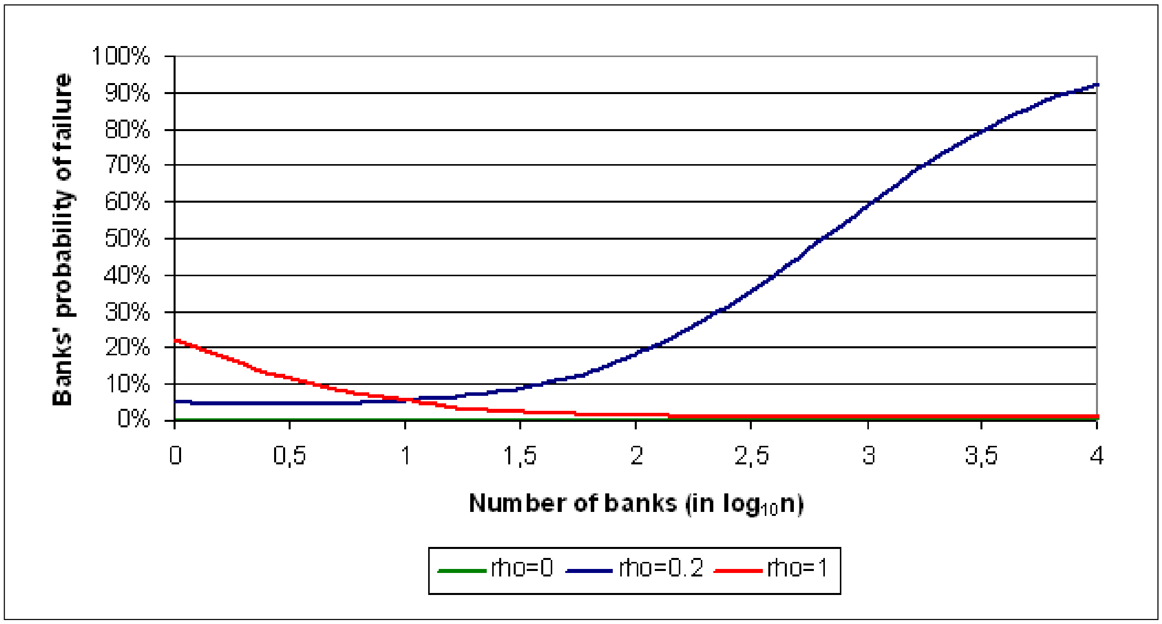

Figure 1: Relationship between the number of banks and the probability of bank failure in the Cournot model for different values of the risk-shifting parameter.

be noted that these parameters are chosen for the purpose of illustrating the possible shapes of the relationship between the number of banks and the risk of bank failure. They are not intended to produce realistic values of variables such as the loan rate r, the probability of loan default p, or the probability of bank failure q.

Figure 1 shows the relationship between the number of banks n (expressed in $\log_{10} n$, so n ranges from 1 to 10,000 banks) and the probability of bank failure q for three different values of the correlation parameter, $\rho = 0$, 0.2, and 1, with the other parameters at their benchmark levels. As noted in the previous section, with independent defaults ($\rho = 0$) banks never fail ($q = 0$), because the (deterministic) interest income per unit of loans, $(1 - p(r(L)))r(L)$, is greater than the (deterministic) loan losses per unit of loans, $p(r(L))\lambda$. With perfectly correlated defaults ($\rho = 1$) the probability of bank failure is decreasing in the number of banks, which is the result in Boyd and De Nicolò (2005). Interestingly, when $\rho = 0.2$ we have a U-shaped relationship between competition and the risk of bank failure, with a minimum q for $n = 3$.

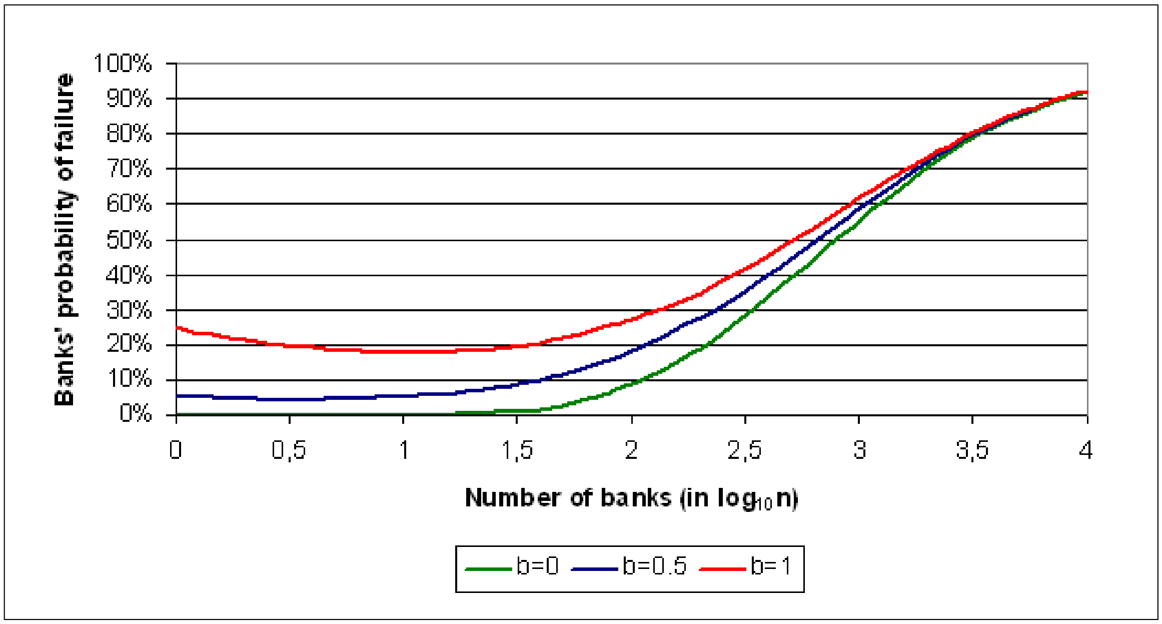

Figure 2: Relationship between the number of banks and the probability of bank failure in the Cournot model for different values of the risk-shifting parameter.

Figure 2 shows the relationship between the number of banks n (expressed in $\log_{10} n$) and the probability of bank failure q for three different values of the entrepreneurial risk-shifting parameter, $b = 0$, 0.5, and 1, with the other parameters at their benchmark levels. As noted in the previous section, when $b = 0$ the risk-shifting effect disappears, so the margin effect makes the probability of bank failure increasing in the number of banks. For $b = 0.5$ we have the same U-shaped relationship already depicted in Figure 1. For higher values of the risk-shifting parameter, such as $b = 1$, the risk-shifting effect becomes stronger, but the slope of the relationship eventually becomes positive (in this case for $n = 10$). In all these cases, as we get close to perfect competition the probability of bank failure converges to one. By Proposition 2, this is because when n tends to infinity the margin between loan and deposit rates converges to zero, but with $\rho \in (0, 1)$ and $p(0) = a > 0$ the loan losses are positive with probability 1, which gives the result.

It turns out that the U-shaped relationship between the number of banks n and the probability of bank failure q obtains for a very large set of parameter values. Let $n_{\min}$

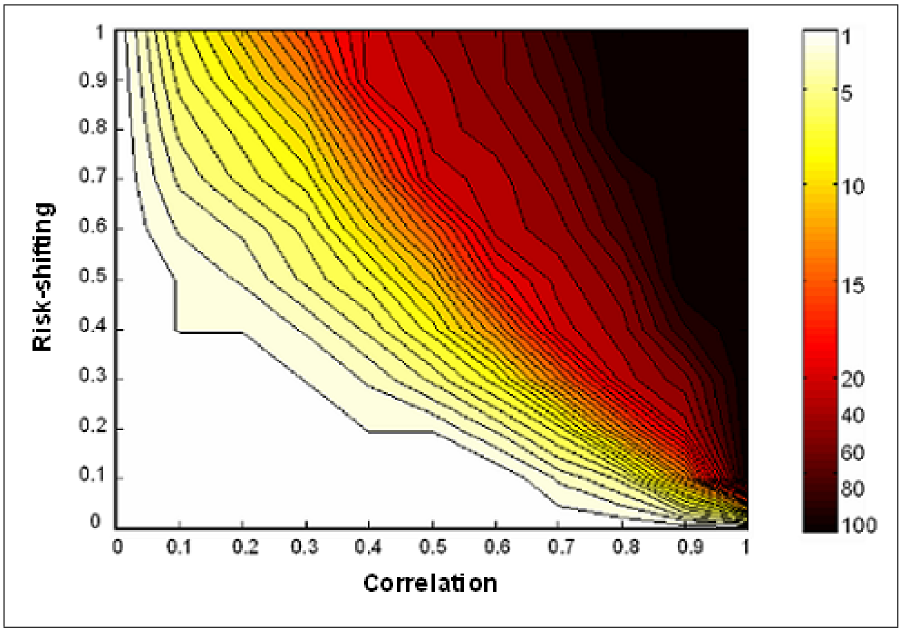

Figure 3: Number of banks that minimize the probability of bank failure in the Cournot model for different values of the correlation and risk shifting parameters.

denote the number of banks that minimize q. Figure 3 illustrates the way in the correlation parameter ρ and the entrepreneurial risk-shifting parameter b determine $n_{\min}$. Specifically, it shows the combinations of ρ and b for which $n_{\min} = 1, 2, 3, ...$. For low values of ρ or low values of b we have $n_{\min} = 1$, so a monopolistic bank would minimize the probability of failure. Otherwise we have a U-shaped relationship: When the actual number of banks n is below (above) the corresponding $n_{\min}$, more (less) competition would reduce q. Higher correlation ρ and higher risk-shifting b increase $n_{\min}$, which reaches values greater than 100 when $\rho \to 1$, i.e., with perfectly correlated defaults.

It should be noted that similar results obtain for other functional forms for the inverse demand for loans $r(L)$, such as $r(L) = c - dL^{\delta}$ with $\delta > 1$ (to ensure concavity), and for the entrepreneurial risk-shifting function $p(r)$, such as $p(r) = a + br^{\eta}$ with $\eta > 0$. Thus we conclude that in the static model of Cournot competition in the loan market considered by Boyd and De Nicolò (2005), too little and too much competition are generally associated with higher risks of bank failure.

Although our analysis has focussed on the impact of changes in competition within the banking sector, the results could be easily extended to a situation in which the banking sector faces increased "outside" competition from financial markets. In particular, suppose that the entrepreneurs have the option of funding their projects in a public debt market at an interest rate $\overline{r} > 0$. This outside option truncates the loan demand function at the rate $\overline{r}$. If the truncation is binding, the equilibrium loan rate would be $r = \overline{r}$, so an increase in competition coming from the financial markets would lead to a reduction in equilibrium loan rates. As before, the effect on the risk of bank failure would result from the combination of a negative risk-shifting effect and a positive margin effect, with the margin effect dominating for sufficiently small values of the market rate $\overline{r}$.

5 Extensions

This section analyzes two extensions of our model. First we consider a dynamic version of the model of Cournot competition in the loan market in which banks that do not fail at any date t have the opportunity to lend to a new set of entrepreneurs at date $t+1$. This generates an endogenous franchise value that is lost upon failure, so banks have an incentive to be prudent. The second extension is to replace the Cournot model by a circular road model of competition in the loan market, in which loan rates are the banks' strategic variables. In both extensions our previous results on the relationship between competition and the risk of bank failure remain unchanged.

5.1 A dynamic Cournot model

Consider a discrete time, infinite horizon model with n identical banks that at each date $t = 0, 1, 2, ...$ in which they are open raise fully insured deposits at an interest rate that is normalized to zero in order to compete à la Cournot for loans to the continuum of entrepreneurs described in Section 2.1.

Assuming that banks are closed whenever the default rate x is greater than the bankruptcy default rate $\widehat{x}(L)$ defined in (6),[14] the Bellman equation that characterizes the symmetric equilibrium of the dynamic model is

$$V_j = \max_{l_j} \beta \left[l_j h(L) + (1 - q(L)) V_j \right] \tag{16}$$

where $\beta < 1$ is the bank shareholders' discount factor, $h(L)$ is the banks' expected payoff per unit of loans given by (8), and $q(L)$ is the probability of bank failure given by (10). According to this expression, the franchise value of a bank that is open results from maximizing with respect to its supply of loans l_j (taking as given the supplies of the other $n-1$ banks that jointly determine the aggregate supply of loans L) an objective function that has two terms: The first one is the discounted expected payoff from current lending, $l_j h(L)$, and the second one is the discounted expected payoff of remaining open at the following date, which is the product of the probability of survival (one minus the probability of failure $q(L)$) and the franchise value V_j.[15] Solving the Bellman equation (16) and setting $l_j = L/n$ gives the equilibrium aggregate lending L as well as the banks' equilibrium franchise value V (the same for all j).

In what follows we are going to assume that functional forms and parameter values are such that the dynamic model has a unique equilibrium for all n. Then the relationship between the equilibrium of the static and the dynamic model is stated in the following result.

Proposition 3 *Let L_s and L_d denote equilibrium aggregate lending in the static and the dynamic model, respectively, for a given number of banks n. Then $q'(L_d) < 0$ implies $L_d > L_s$, $q'(L_d) > 0$ implies $L_d < L_s$, and $q'(L_d) = 0$ implies $L_d = L_s$.*

Proof The first-order condition (9) that characterizes the symmetric equilibrium of the static model is

$$L_s h'(L_s) + n h(L_s) = 0$$

[14] See Suarez (1994) and Elizalde and Repullo (2007) for a discussion of alternative closure rules.

[15] See Fudenberg and Tirole (1991, Chapter 4) for a proof that one-stage-deviations are sufficient to characterize subgame perfect equilibria. It should be noted that with a single systematic risk factor when one bank fails all of them fail, so there is no need to consider situations, like those in Perotti and Suarez (2002), where one bank may survive while others fail, so the surviving bank may increase its market power in the following period.

Differentiating the bank's objective function in (16) we get the first-order condition that characterizes the symmetric equilibrium of the dynamic model

$$L_d h'(L_d) + nh(L_d) = nq'(L_d)V$$

Hence when $q'(L_d) < 0$ we have $L_d h'(L_d) + nh(L_d) < L_s h'(L_s) + nh(L_s)$. But since $h'(L) < 0$ and $h''(L) < 0$ imply that the function $Lh'(L) + nh(L)$ is decreasing, it follows that $L_d > L_s$. The second and third results are proved in the same manner. □

The result in Proposition 3 shows that there is one case, namely when $q'(L_d) = 0$, in which aggregate lending, and consequently the probability of bank failure, are the same in the static and in the dynamic model. In all other cases we know the effect on aggregate lending, but to establish the effect on the probability of bank failure we need to know the form of the function $q(L)$. Assuming that $q(L)$ is U-shaped, Proposition 3 implies that when the number of banks n is such that $q'(L_d) < 0$ we have $q(L_d) < q(L_s)$, and when the number of banks n is such that $q'(L_d) > 0$ we also have $q(L_d) < q(L_s)$. Hence in both cases the probability of bank failure in the dynamic model is smaller than the probability of bank failure in the static model, so banks are generally safer in the model with endogenous franchise values. It is only in the case with $q'(L_d) = 0$ when both probabilities coincide.

We illustrate this result for our benchmark parameterization (for which the function $q(L)$ is indeed U-shaped) and $\beta = 0.96$. Figure 4 shows the relationship between the number of banks n and the probability of bank failure q in the static and in the dynamic model. In both cases the relationship is U-shaped, with the curve for the static model being everywhere above the curve for the dynamic model, except at the minimum in which the two curves are tangent. The two curves are also tangent when n tends to infinity, because the equilibrium franchise value V tends to zero as we approach the perfect competition limit, in which case the banks' objective function in the dynamic model coincides with the objective function in the static model.

The tangency result implies that Figure 3 also shows for the dynamic model the way in which the correlation parameter ρ and the entrepreneurial risk-shifting parameter b determine the effect of the number of banks on the probability of bank failure. Hence we conclude

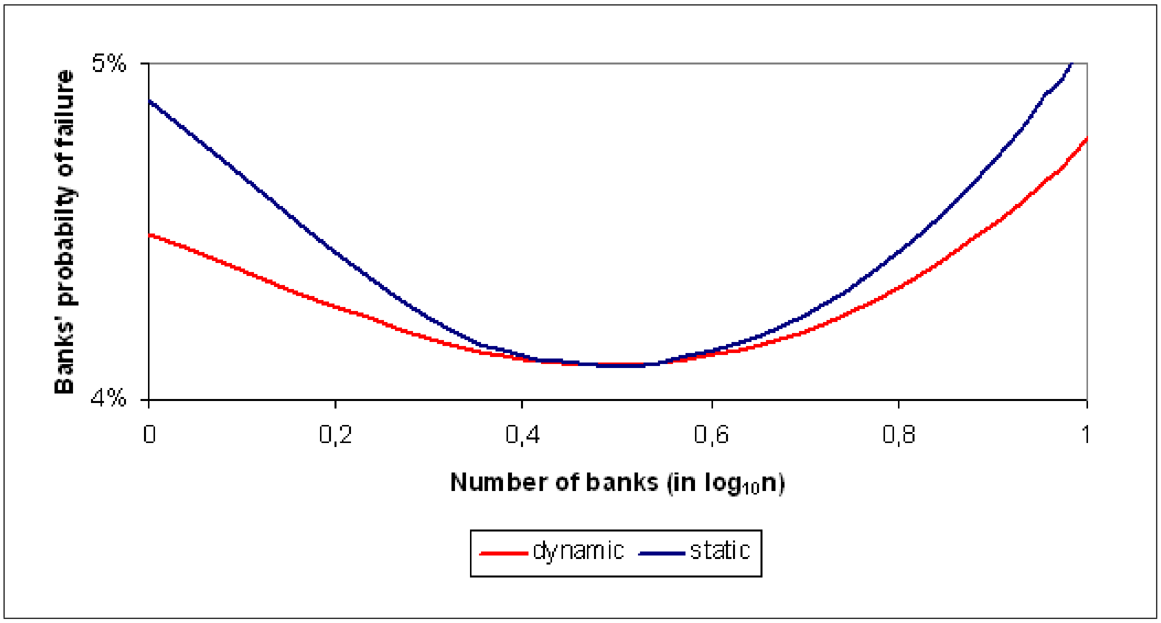

Figure 4: Relationship between the number of banks and the probability of bank failure in the static and the dynamic Cournot model.

that our results on the relationship between competition and the probability of bank failure are robust to the introduction of endogenous franchise values in our model of Cournot competition in the loan market with imperfectly correlated defaults.[16]

5.2 A circular road model

We now examine the robustness of our results to changes in the nature of competition among banks. Specifically, we consider Salop's (1979) circular road model of price competition.[17] There are $n \geq 2$ banks located symmetrically on a circumference of unit length, and a

[16]It should be noted that we are ignoring the possibility of collusive equilibria in which the banks restrict their lending under the threat of reverting to the noncooperative equilibrium if a deviation occurs. Let V_n denote the (noncooperative) franchise value of a bank when there are n banks in the market, and let L_1 denote the lending of a monopoly bank in the dynamic model. Then the n banks could sustain the monopoly outcome if $\max_{l_j} \beta\,[l_j h(l_j + (n-1)L_1/n) + (1 - q(l_j + (n-1)L_1/n))V_n] \leq V_1/n$. For our numerical parameterization, this condition is satisfied for $n \leq 74$. The qualitative result is that as n increases it is more difficult to sustain collusive equilibria.

[17]In the context of banking this model has been used, among others, by Chiappori et al. (1995) and Repullo (2004). In this model the number of banks can be endogenized by introducing a fixed cost of entry. Hence an increase in competition (in the sense of an increase in the number of banks) would be equivalent to a reduction in the cost of entry.

continuum of measure 1 of entrepreneurs distributed uniformly on this circumference. We focus on the static version of the model, since the results for the dynamic version are similar to those obtained for the model of Cournot competition.

Entrepreneurs have the investment projects described in Section 2.1. They are ex-ante identical except for their location on the circumference, and have a zero reservation utility. To fund their projects they have to travel to a bank, which involves a transport cost μ per unit of distance.

To obtain the symmetric Nash equilibrium of the model of spatial competition we first compute the demand for loans of bank j when it offers a loan rate r_j while the remaining $n-1$ banks offer the rate r.[18] Assuming that the transport cost μ is not too high, the market will be "totally covered," and bank j will have two effective competitors, namely banks $j-1$ and $j+1$. An entrepreneur located at distance θ from bank j and distance $1/n-\theta$ from bank $j+1$ will be indifferent between borrowing from j and borrowing from $j+1$ if her utility net of transport costs is the same, that is, if

$$u(r_j) - \mu\theta = u(r) - \mu\left(\frac{1}{n} - \theta\right)$$

Solving for θ in this equation yields

$$\theta(r_j, r) = \frac{1}{2n} + \frac{u(r_j) - u(r)}{2\mu}$$

Taking into account the symmetric market area between bank j and bank $j-1$, and the fact that each entrepreneur requires a unit loan, we get the following demand for loans of bank j

$$l(r_j, r) = \frac{1}{n} + \frac{u(r_j) - u(r)}{\mu} \tag{17}$$

Notice that for $r_j = r$ we have $u(r_j) = u(r)$, so $\theta(r,r) = 1/2n$, i.e., the mid point between two adjacent banks. In this case $l(r,r) = 1/n$, so banks would be equally sharing the unit mass of borrowers. Since we have shown that $u'(\cdot) < 0$, it follows that the demand function $l(r_j, r)$ is decreasing in r_j and increasing in r.

[18] As in the original Salop (1979) model, we assume that banks do not price discriminate borrowers by their location.

Assuming that the supply of deposits is perfectly elastic at an interest rate that is normalized to zero, and following the same steps as in Section 2.2, bank j's objective function may be written as

$$\pi(r_j, r) = l(r_j, r)h(r_j)$$

where

$$h(r_j) = \int_0^{\widehat{x}(r_j)} [r_j - (r_j + \lambda)x] \ dF(x; p(r_j)) \tag{18}$$

is the bank j's expected payoff per unit of loans, and $\widehat{x}(r_j)$ is bank j's bankruptcy default rate defined by

$$\widehat{x}(r_j) = \frac{r_j}{r_j + \lambda} \tag{19}$$

Hence the first-order condition that characterizes the symmetric equilibrium is

$$h'(r) + nl_1(r, r)h(r) = 0 \tag{20}$$

where $l_1(r_j, r) = \partial l/\partial r_j$.

In correspondence with our assumptions in Section 3, here we assume that functional forms and parameter values are such that $h'(r) > 0$, $h''(r) < 0$,[19] and $l_{11}(r_j, r) = \partial^2 l/\partial r_j^2 < -l_1(r, r)h'(r)/h(r)$.[20] We can then prove the following result.

Proposition 4 *An increase in the number of banks n reduces the equilibrium loan rate r.*

Proof Differentiating the first-order condition (20) and using $l_1(r, r) < 0$ together with the assumptions $h'(r) > 0$, $h''(r) < 0$, and $l_{11}(r_j, r) < -l_1(r, r)h'(r)/h(r)$ gives

$$\frac{dr}{dn} = -\frac{l_1(r, r)h(r)}{h''(r) + nl_{11}(r, r)h(r) + nl_1(r, r)h'(r)} < 0 \quad \square$$

Proposition 4 implies that the higher the competition among banks the lower the probability of default of the loans in their portfolios. However, as in the case of the model of Cournot competition, this does not imply a reduction in the banks' probability of failure. To

[19] As before, assuming that $h'(r) > 0$ and $h''(r) < 0$ requires that the risk-shifting effect $p'(r)$ must not be very large.

[20] In the linear risk-shifting case where $p(r) = a + br$ and $u(r)$ is given by (15), using (17) this condition reduces to $b < (1 - a - br)h'(r)/h(r)$, so it will hold if the risk-shifting effect $b = p'(r)$ is not be very large.

see this observe that banks fail whenever the default rate x is greater than the bankruptcy default rate $\widehat{x}(r)$ defined in (19). Using the probability distribution of the default rate (5), the *probability of bank failure* is given by

$$q(r) = \Pr\left(x > \widehat{x}(r)\right) = \Phi\left(\frac{\Phi^{-1}(p(r)) - \sqrt{1-\rho}\ \Phi^{-1}(\widehat{x}(r))}{\sqrt{\rho}}\right) \tag{21}$$

Hence we have

$$\frac{dq}{dn} = q'(r)\frac{dr}{dn}$$

Since $dr/dn < 0$ by Proposition 4, it follows that higher competition leads to lower risk of bank failure if and only if the slope of the function $q(r)$ is positive.

Now differentiating (21) we get

$$q'(r) = \frac{\Phi'(\cdot)}{\sqrt{\rho}}\left[\frac{d\Phi^{-1}(p(r))}{dp}p'(r) - \sqrt{1-\rho}\ \frac{d\Phi^{-1}(\widehat{x}(r))}{dx}\widehat{x}'(r)\right] \tag{22}$$

Since $\Phi'(\cdot) > 0$ (it is a normal density), the sign of $q'(r)$ is the same as the sign of the term in square brackets, which has two components. The first one is positive, since $d\Phi^{-1}(\cdot)/dp > 0$ and $p'(r) > 0$, while the second one is negative (whenever $\rho < 1$), since $d\Phi^{-1}(\cdot)/dx > 0$ and $\widehat{x}'(r) > 0$. As in the model of Cournot competition, the first component captures the *risk-shifting effect:* More competition leads lower loan rates, which in turn lead to lower probabilities of default, and hence safer banks. The second component captures the *margin effect:* More competition leads to lower loan rates, and consequently lower revenues from non-defaulting loans, which provide a buffer against loan losses, so we have riskier banks. Depending on which of the two effects dominates, the impact of competition on the risk of bank failure may be positive or negative. But as in the model of Cournot competition, one can show that when loan defaults are imperfectly correlated the margin effect dominates in very competitive markets.

Proposition 5 *For any correlation parameter* $\rho \in (0, 1)$, *when the number of banks* n *is sufficiently large additional increases in* n *increase the probability of bank failure* q.

Proof When n tends to infinity the first-order condition (20) that characterizes the equilibrium becomes $h(r) = 0$, which by (18) implies $\widehat{x}(r) = 0$ and hence $r = 0$. Thus in very competitive markets the loan rate approaches the deposit rate, which has been normalized to zero. But then we have

$$\lim_{n\to\infty} \frac{d\Phi^{-1}(\widehat{x}(r))}{dx} = \frac{1}{\Phi'[\Phi^{-1}(0)]} = \infty$$

and $\lim_{n\to\infty} p(r) = p(0) > 0$, by assumption $\alpha(0) < \alpha'(0)$, which implies

$$\lim_{n\to\infty} \frac{d\Phi^{-1}(p(r))}{dp} = \frac{1}{\Phi'[\Phi^{-1}(p(0))]} < \infty$$

Hence by (22) we have $q'(r) < 0$ for sufficiently large n, which implies $dq/dn > 0$. □

Proposition 5 shows that in very competitive loan markets the risk-shifting effect is always dominated by the margin effect, so any additional entry would increases the risk of bank failure.[21] To illustrate what happens in less competitive markets we resort to numerical solutions for simple parameterizations of the model.

As in Section 4 we postulate the linear risk-shifting function $p(r) = a + br$, for which the the expected payoff function $u(r)$ is given by (15). Hence demand for loans of bank j is

$$l(r_j, r) = \frac{1}{n} + \frac{1}{2b\mu}\left[(1-a-br_j)^2 - (1-a-br)^2\right]$$

For our benchmark parameterization we take the minimum probability of default $a = 0.01$, the loss given default parameter $\lambda = 0.45$, the correlation parameter $\rho = 0.2$, and the transport cost parameter $\mu = 1$.

As in the case of the model of Cournot competition, we get a U-shaped relationship between the number of banks n and the probability of bank failure q. If we let $n_{\min}$ denote the number of banks that minimize the probability of failure, Figure 5 illustrates the way in the correlation parameter ρ and the entrepreneurial risk-shifting parameter b determine $n_{\min}$. For low values of ρ or low values of b we have $n_{\min} = 2$, so a duopoly would minimize the probability of failure. Higher correlation ρ and higher risk-shifting b increase $n_{\min}$, which reaches values greater than 100 when $\rho \to 1$, i.e., with perfectly correlated defaults.

Hence we conclude that our results remain unchanged when we replace the Cournot model by a circular road model of competition in the loan market. The fact that the results

[21] Using (17) it is immediate to check that Proposition 5 also holds when the transport cost parameter μ is sufficiently small.

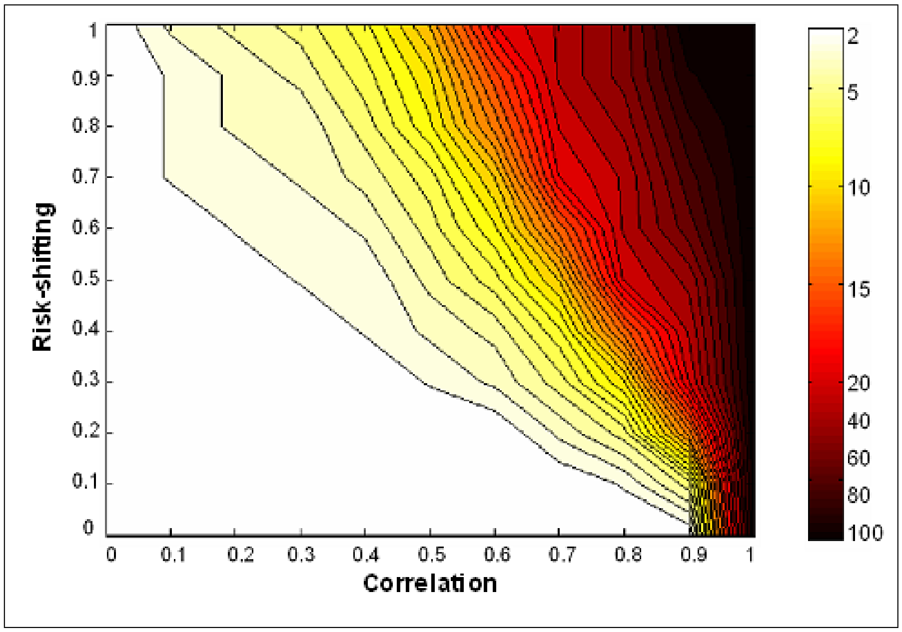

Figure 5: Number of banks that minimize the probability of bank failure in the circular road model for different values of the correlation and risk shifting parameters.

are robust to the change of strategic variable from quantities (loan supplies) to prices (loan rates) suggests that they are likely to hold for a wide set of models of imperfect competition. The general conclusion is that when loan defaults are imperfectly correlated the probability of bank failure is lowest in loan markets with moderate levels of competition, with higher probabilities of failure in either very competitive or very monopolistic markets.

6 Discussion

This section considers some implications of our results. To simplify the presentation, the discussion will be conducted in the context of the static model of Cournot competition analyzed in Section 3. We first consider the effect of changes in the deposit rate in order to assess the relationship between monetary and financial stability, and then we look at the implications for the welfare maximizing competition policy in banking.

6.1 Monetary and financial stability

Our model could also be used to analyze the relationship between monetary and financial stability, in particular the effect of a tightening of monetary policy on the risk of bank failure. For this we replace the deposit rate by a risk-free rate i, which proxies the policy rate set by the central bank. The banks' objective function now becomes

$$\pi(l_j, l_{-j}) = l_j h(L, i)$$

where

$$h(L, i) = \int_0^{\widehat{x}(L,i)} [r(L) - i - (r(L) + \lambda)x] \; dF(x; p(r(L)))$$

is the banks' expected payoff per unit of loans, and $\widehat{x}(L, i)$ is the bankruptcy default rate defined by

$$\widehat{x}(L, i) = \frac{r(L) - i}{r(L) + \lambda}$$

Hence the first-order condition that characterizes a symmetric equilibrium is

$$Lh_1(L, i) + nh(L, i) = 0$$

where $h_1(L, i) = \partial h / \partial L$.

The comparative static analysis of the effects of an increase in the risk-free rate i is not straightforward, so we illustrate them numerically. Figure 6 shows the relationship between the number of banks n (expressed in $\log_{10} n$) and the probability of bank failure q for three different values of the risk-free rate, $i = 0$, 0.01, and 0.02, with the other parameters at their benchmark levels. In all cases the relationship is U-shaped, with higher values of the risk-free rate associated with higher values of the probability of bank failure. The fact that a tightening of monetary policy leads to an increase in the probability of bank failure is explained by a combination of a positive risk-shifting effect (that follows from the increase in the loan rate r) and a positive margin effect (that follows from the reduction in the intermediation margin $r - i$). Hence in this case the margin effect reinforces the risk-shifting effect.

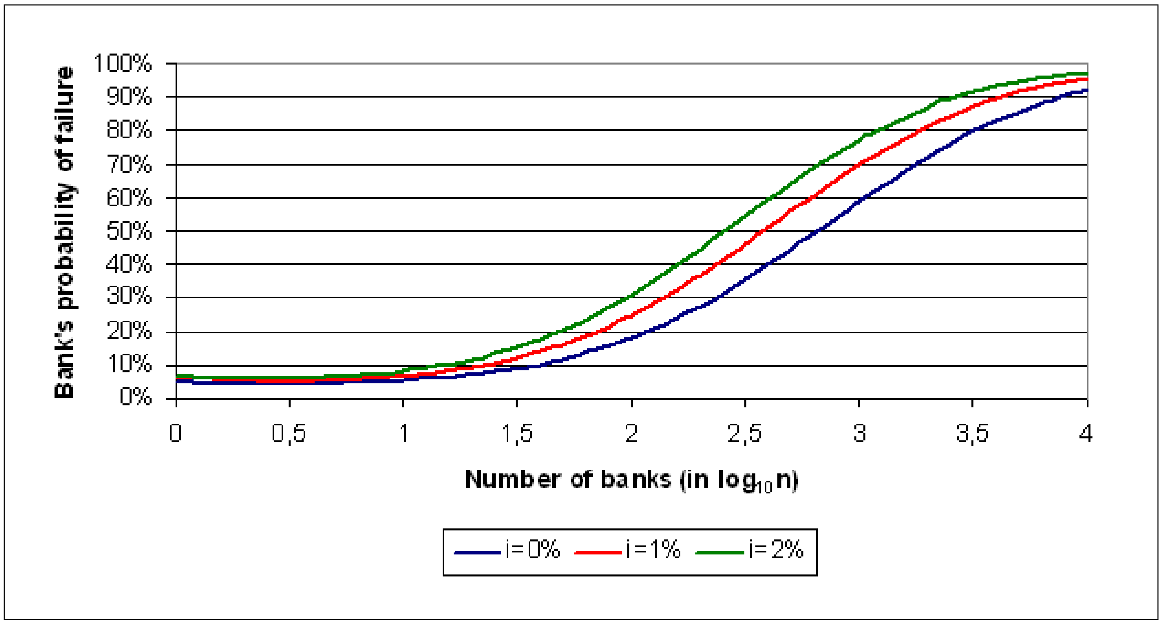

Figure 6: Relationship between the number of banks and the probability of bank failure in the Cournot model for different values of the cost of financing.

Thus our model provides a framework for understanding the historical evidence of cases where the concern of a central bank for the solvency of its banks was a major factor in an excessively expansionary monetary policy; see Goodhart and Schoenmaker (1995) and the references therein.

6.2 Welfare analysis

In our risk-neutral economy, social welfare may be evaluated by simply adding the expected payoffs of entrepreneurs, bank shareholders, depositors, and the government (as deposit insurer). In addition, we are going to assume that the failure of the banking system entails a social cost $C > 0$,[22] which captures the administrative costs of liquidating the banks and paying back depositors, as well as the negative externalities associated with such failure (breakup of lending relationships, distortion of the payment system, etc.).

Since depositors are fully insured, they get a return that just covers the opportunity cost of their funds, so their net payoff is zero. The net expected payoff of an entrepreneur i that

[22] Recall that since there is a single factor of systematic risk, if one bank fails all of them fail.

undertakes her project at the rate r is $(1-p(r))(\alpha(p(r)-r)-u_i$, where u_i denotes her reservation utility. Hence the net expected payoff of the entrepreneurs that undertake their projects at the rate r is

$$L(r)(1-p(r))(\alpha(p(r))-r)-\int_0^{u(r)} u\ dG(u)$$

The expected payoff of bank shareholders is

$$L(r)\int_0^{\widehat{x}(r)} [r-(r+\lambda)x]\ \ dF(x;p(r))$$

Finally, the expected payoff of the government is

$$L(r)\int_{\widehat{x}(r)}^{1} [r-(r+\lambda)x]\ \ dF(x;p(r))-q(r)C$$

where the first term is the expected liability of the deposit insurer (the expected value of the bank losses that obtain when the default rate x is greater than the bankruptcy default rate $\widehat{x}(r)$), and the second term is the product of the probability of bank failure $q(r)$ by the social cost of such failure C. Adding up the previous expressions, taking into account that the expected value of the default rate is the probability of default $p(r)$, and making use of the fact that $r=r(L)$, we obtain the following social welfare function

$$W(L)=L\left[(1-p(r(L)))\alpha(p(r(L)))-\lambda p(r(L))\right]-\int_0^{u(r(L))} u\ dG(u)-q(L)C \tag{23}$$

The first term in this expression is the expected return of the projects that are undertaken, the second term is the opportunity cost of the entrepreneurs that undertake them, and the third term is the expected social cost of bank failure.

Differentiating (23) with respect to aggregate lending L, and making use of the definition of $u(r)$ in (1), the definition of $L(r)$ in (3), and the fact that $G(u(r(L)))=L$ implies $G'(u(r(L)))u'(r(L))r'(L)=1$, we get

$$\begin{aligned} W'(L) &= L\left[(1-p(r(L)))\alpha'(p(r(L)))-\alpha(p(r(L)))-\lambda\right]p'(r(L)r'(L) \\ &\quad +\left[(1-p(r(L)))r(L)-\lambda p(r(L))\right]-q'(L)C \end{aligned}$$

The first term in this expression is positive, since $p'(r(L)) > 0$, $r'(L) < 0$, and the function $(1-p)\alpha(p) - \lambda p$ is concave, with a slope for $p = p(0)$ equal to $-\lambda < 0$.[23] The second term is the expected payoff of a bank loan, which should be positive except for large values of n for which it approaches the limit $-\lambda p(0) < 0$. Finally, we have seen that $q(L)$ is generally U-shaped, so the third term is positive (negative) for low (high) values of L.

Since by Proposition 1 aggregate lending L is an increasing function of the number of banks n, we conclude that the number of banks that maximizes social welfare, denoted n^*, is in general greater than $n_{\min}$ and satisfies $dn^*/dC < 0$. Moreover as the social cost of bank failure C goes up, the optimal number of banks n^* approaches the number $n_{\min}$ that minimizes the probability of bank failure. Hence we conclude that if bank failures generate some negative externalities, the welfare maximizing competition policy in banking will be characterized by entry restrictions that leave banks some monopoly rents in order to reduce their risk of failure.

7 Concluding Remarks

This paper has investigated the effects of increased competition on the risk of bank failure in the context of a model in which (1) banks invest in entrepreneurial loans, (2) the probability of default of these loans is endogenously chosen by the entrepreneurs, and (3) loan defaults are imperfectly correlated. We show that there are two opposite effects. The *risk-shifting effect* identified by Boyd and De Nicolò (2005) follows from (1) and (2) and works as follows: More competition leads lower loan rates, which in turn lead to lower probabilities of default, and hence safer banks. The *margin effect* follows from (3) and works as follows: More competition leads to lower loan rates, and consequently lower revenues from non-defaulting loans, which provide a buffer against loan losses, so we have riskier banks. The results show that the risk-shifting effect tends to dominate in monopolistic markets, whereas the margin effect dominates in competitive markets, so a U-shaped relationship between competition and the risk of bank failure generally obtains.

[23] To see this use the fact that for $r = 0$ the first-order condition (2) that characterizes the entrepreneurs' choice of p becomes $(1-p)\alpha'(p) - \alpha(p) = 0$.

These results could be tested empirically by regressing some measure of bank solvency (such as the Z-score[24]) on the level and the square of some measure of concentration (such as the Herfindahl-Hirschmann index[25]) or of market power (such as the Lerner index[26]). According to the predictions of our model, the coefficient of the linear term should be positive, and that of the quadratic term should be negative. Also, the presence of a risk-shifting effect could be tested by regressing some measure of the average probability of default of the banks' loan portfolios (such as a non-performing loans ratio) on any of the above measures of concentration or market power. According to our theoretical model, the coefficient of these variables should be positive.

Finally, it should be stressed that our results are derived from a model in which the banks are assumed to have no capital. Allowing bank shareholders to contribute costly capital would not change the results (at least in any static model without franchise values), because one can show that we would always get a corner solution with zero capital. The open question is then how competition affects the risk of bank failure when there is a capital requirement. We plan to explore this issue in future research.

[24] The Z-score is defined as $Z = (ROA + EA)/\sigma(ROA)$, where ROA is the rate of return on assets, EA is the ratio of equity to assets, and $\sigma(ROA)$ is the standard deviation of the rate of return on assets. This measure has been used by Boyd et al. (2006).

[25] The Hirschmann-Herfindahl index is defined as the sum of the squares of the banks' market shares, that is $HHI = \sum_{j=1}^{n}(l_j/L)^2$. This measure has been used by Boyd et al. (2006).

[26] The Lerner index is defined as $L = (r - MC)/r$, where r is the average loan rate and MC is an estimate of the loans' marginal cost. This measure has been used by Jiménez et al. (2006).

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