Asymmetric Trade Integration, Expectations, and Growth

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Abstract

We consider a many country endogenous growth model with adaptive heterogenous expectations and international trade in complementary capital goods. We analyze the impact of asymmetric trade integration on the world long run steady state. Levels of endogenous variables can differ across countries even if the long run growth rate is common and such differences are solved from the model. We apply heterogenous learning to generate transition dynamics that are also affected by trade policy. The model generates endogenous long run income clubs (country groups in which technology and output levels persistently differ) and endogenous short run growth clubs (country groups that share short run growth experiences).

Key words: Endogenous growth, complementary capital goods, preferential trade agreements, adaptive expectations.

JEL codes: F43, F15.

1 Introduction:

The impact of international trade on economic growth is a topic of empirical and policy importance that has attracted much attention in economic literature. The proliferation of preferential trade agreements (PTAs) over the last fifteen years has motivated new interest in asymmetric trade integration but less attention has been directed toward its dynamic impact.[1] In this paper, we present an endogenous growth model that we then apply to analyze these dynamic effects.[2]

We expand the closed economy growth model of Evans, Honkapohja and Romer (EHR) (1998) and the two country symmetric trade model of Honkapohja and Turunen-Red (HTR) (2002). We include several countries that may differ from each other in terms of trade policy, size, cost of innovation, and overall productivity of resources. The inclusion of three or more countries allows us to study effects of trade integration on several levels but our main attention is directed toward understanding the growth impact of PTAs in which two or more countries form a trade bloc against the remaining outsiders. Direct effects of PTAs can be usefully isolated if countries are restricted to be structurally symmetric. We can then ask whether, e.g., a customs union that raises a common trade barrier against the rest of the world inevitably slows down long run growth in the world. Whether such an PTA may benefit the member countries by providing an asymmetric, even if transitory, boost in growth is also of interest.

Structural asymmetries between countries in size, cost of innovation, and factor productivity allow for the possibility that PTAs may either exacerbate or mitigate the growth impact of other fundamental differences. Notably, empirical evidence suggests that growth effects of PTAs are asymmetric: agreements that take place between advanced industrialized countries ("North-North" PTAs) appear to be growth enhancing for members whereas the growth impact of mixed "North-South" PTAs is inconclusive and, for "South-South" agreements, even negative (Berthelon (2004)).

In its basic structure, our model is analogous to other "idea" growth models. Three production sectors are included (production of aggregate consumption, invention of new capital goods, production of capital goods) and the source of growth is the endogenous invention of new intermediate capital goods. Following EHR, we assume that intermediate capital goods are complementary to each other in final production and, as in HTR, countries are connected through trade in capital good varieties. Each new innovation contributes to the output of the final consumption commodity and, because of the complementarity of capital goods, improves the marginal productivity of other capital goods. This raises the productivity and value of new capital goods over time and provides increasing incentives for innovation. Balanced growth is nevertheless obtained because the cost for innovation also increases as technology advances. The cumulative investment in innovation and production of capital goods defines the stock of aggregate capital in each time period. A production possibility frontier between this aggregate capital and consumption yields an endogenous opportunity cost for aggregate capital that increases as production of capital goods and innovation expand.

There are several aspects to the growth process that we consider. First, we characterize the steady states of the model. These define the pace of technological advance in the world and thus

[1] The term asymmetric trade integration here encompasses all PTAs involving trade policy (e.g., free trade areas and customs unions). We do not consider economic unions that include joint decision making regarding monetary policy, factor movements, or other institutional arrangements.

[2] Comprehensive discussions of static effects of PTAs can be found in Baldwin and Venables (1995), Bhagwati, Greenaway and Panagariya (1998), and World Bank (2005). PTAs as "stumbling blocks" for multilateral trade liberalization have been recently discussed in Limao (2006).

yield a common rate of long run growth for all countries. This common rate of long run growth is affected by all symmetries and asymmetries between countries, including asymmetries in trade policy (PTAs). Secondly, levels of endogenous variables can differ across nations even if the long run growth rate is common, and any such differences are solved from the model. Country-specific values of a growth multiplier convert asymmetries in trade policy (PTAs) and other exogenous differences between countries into endogenous long run "income clubs"; countries in such clubs exhibit common long run growth but persistently differ in levels of technology and output. A trade bloc that raises its trade barrier against the rest of the world has a negative effect on the technology levels of the outsiders, while retaliation by the nonmembers of an PTA alleviates losses.

Thirdly, we augment the model with transition dynamics that generate short run differences in growth without violating the common long run growth prediction. We assume that individuals observe current values of economic variables and form expectations that are adjusted as the economy evolves; in this process, errors in expectations are used so as to learn about the future course of the economy. Steady states that are approached through such learning dynamics are called stable with respect to learning.[3]

Adjustment of expectations may not be symmetric across countries. Learning heterogeneity is particularly natural when structural heterogeity exists as is the case in our model.[4] Honkapohja and Mitra (2006) have studied stability conditions for learning dynamics in the presence of structural heterogeneity. In the present context, their results suggest that any asymmetries between countries (including PTAs) will affect national transition dynamics. This suggests that endogenous (steady state) income clubs that involve persistent differences in the level variables across countries and arise from exogenous and policy asymmetries can be associated with short run differences in growth rates (endogenous "growth clubs"). The observed relatively slow growth of some low-income countries (that violates the Solow type convergence hypothesis) may thus at least partly reflect heterogenous learning and associated transition dynamics.

Related Literature: The theoretical and empirial literature that directly addresses dynamic effects of PTAs is rather sparse. The single theoretical treatment of asymmetric trade integration is by Walz (1997, 1999). The Walz model is built on the Grossman and Helpman (1991) approach and therefore differs from ours in several respects. The significant recent empirical study of the growth effects of PTAs is Berthelon (2004). Berthelon constructs an explanatory variable that takes into account the extent of the new market that is made accessible through the PTA. Berthelon obtains that the market size of the partner countries matters and that PTAs contribute positively to growth. North-North type agreements are found to be most potent in improving growth, while the evidence for South agreements suggests that the growth impact may even be negative.[5]

Other literature that does not consider PTAs but discusses related themes includes recent work on international income and growth differences. Waugh (2007) uses a static general equilibrium model to quantify the contribution of trade to relative incomes of countries. His results indicate that asymmetries in trade costs with poorer countries facing higher costs for exports have a significant impact on relative incomes. A large literature addresses the cross-sectional distribution of income across countries and the observed lack of convergence in incomes and growth between the poorest and richest nations. Pritchett (2000, 2006) has argued that the large differences in incomes

[3] See Evans and Honjapohja (2001) for a thorough discussion of learning dynamics.

[4] Pfajfar (2007) presents evidence of expectational heterogeneity in the United States.

[5] Kali, Mendez and Reyes (2007) find that the structure of a country's trade pattern affects growth.

and growth can be understood in terms of growth regimes (convergence clubs) of countries that experience different steady states, each with its own transition dynamics; the longer run growth experience of a particular country then reflects not only the transition dynamics of the country's initial steady state but also the transitions from one growth regime to another. Since the transition dynamics from one state to another can differ, growth processes of countries can widely vary.

2 Model:

In this section, we describe our model and characterize the long run equilibrium solutions. We allow for several asymmetries across countries that roughly reflect factors that have been identified in the literature as contributing toward income and growth divergence.

2.1 Basic Assumptions:

We assume that there are N (≥ 3) countries, indexed by i. The aggregate consumer in each country maximizes the discounted utility expression

$$U_{it} = \sum_{j=0}^{\infty} \frac{\beta^{t+j} C_{i,t+j}^{1-\sigma}}{1-\sigma}, \qquad 0 < \sigma < 1, \tag{1}$$

where $C_{i,t+j}$ denotes final consumption in country i in period $(t+j)$. Given a constant interest rate, r, each aggregate consumer's preferred rate of consumption growth, g_c, is obtained from the Euler equation

$$\frac{C_{i,t+1}}{C_{it}} \equiv g_c = [\beta(1+r)]^{1/\sigma}. \tag{2}$$

Financial capital is taken to be freely mobile, so that the interest rate equalizes worldwide.

Final consumption is produced by a competitive production sector according to the production function

$$Y_{it} = \widehat{L}_i^{1-\alpha} \left(\sum_{k=1}^{N} \int_0^{A_{kt}} x_{ikt}(j_k)^{\gamma} \, dj_k \right)^{\phi}, \quad \widehat{L}_i \equiv \psi_i^{\frac{1}{1-\alpha}} L_i . \tag{3}$$

In (3), L_i denotes the (fixed) endowment of immobile labor (country size). The quantity of intermediate capital goods imported from country k is indicated by $x_{ikt}(j_k)$, where j_k indexes varieties of capital goods supplied by producers in country k. The number of different capital goods produced in a country at time t, A_{kt}, defines the technology level in each location at a point in time. Parameter ψ_i represents total factor productivity; differences in ψ_i reflect institutions and policy environments that have an impact on resource allocation in a country and imperfections in international technology transfer (including differing costs for the adoption of new capital goods). Parameter ϕ determines the degree of technological substitutability among capital inputs; if $\phi > 1$, as we assume, all capital varieties are complements in production. The restriction $\alpha = \gamma\phi$ is imposed in order to preserve linear homogeneity of the production process with respect to labor and intermediate inputs. The source of growth is the endogenous invention of new capital goods; in (3), output of final consumption grows as each A_{kt} increases over time and, owing to the complementarity of capital goods, the marginal product of each intermediate capital variety improves with growth in each A_{kt}.

We assume that each country (i) may impose trade barriers, denoted by τ_{ik} (≥ 1), against imported capital goods (from country k); for domestic production trade barriers do not exist ($\tau_{ii} = 1$).[6] All tariff revenues are distributed to the consumption sectors as lump sum income.

Final production sectors take domestic prices as given (all prices are measured with respect to the world market price of final consumption). Maximizing profit given technology (3), final goods producers equate the marginal product of each capital good with its domestic rental price.[7] These equations yield the demand for all varieties of capital goods in all countries ($i, k = 1, ..., N$):

$$\tau_{ik} R_{ikt}(j_k) = \widehat{L}_i^{1-\alpha} \left(\sum_{l=1}^{N} \int_0^{A_{lt}} x_{ilt}(j_l)^{\gamma} \, dj_l \right)^{\phi-1} \alpha x_{ikt}(j_k)^{\gamma-1}. \tag{4}$$

In (4), $R_{ikt}(j_k)$ denotes, in country i, the rental prices of capital varieties imported from country k, excluding the trade barriers τ_{ik}.

Intermediate capital goods are supplied by separate monopolistic competitors; patent protection is complete so that there is no replicative innovation. A unit of each capital good is produced by converting one unit of aggregate (general purpose) capital (Z) into a specific capital variety. Production is realized at the end of a time period so that, at the end of a period, a capital goods producer (in country i) receives revenue $R_{kit}(j_i)x_{kit}(j_i)$ from sales in country k. In the beginning of a time period, $x_{kit}(j_i)$ units of Z are needed to produce the capital units that are rented out. The rental cost for the units of general purpose capital over the time period is $r_t p_t^z x_{kit}(j_i)$, where p_t^z is the opportunity cost of Z in final consumption. Each capital goods producer in country i observes the inverse demand obtained from equations (4) and chooses output $x_{kit}(j_i)$ so as to maximize the (end of the time period) profit

$$\pi_{it}(j_i) = \sum_{k=1}^{N} R_{kit}(j_i) x_{kit}(j_i) - r_t p_t^z \left[\sum_{k=1}^{N} x_{kit}(j_i) \right], \quad j_i \in [0, A_{it}]. \tag{5}$$

After substituting (4) into (5), this maximization yields capital producers' mark-up rules for the three markets:

$$R_{kit} = \frac{r_t p_t^z}{\gamma} \equiv R_t(r_t, p_t^z), \quad i, k = 1, ..., N. \tag{6}$$

Since all varieties of capital goods are priced equally, the index j_i for capital goods (produced in country i) is subsequently dropped.

While we assume that the rate of technological progress is the same in all countries in the long run, exogenous asymmetries are reflected in differences of technology levels. We set

$$A_{it} = \theta_i A_{1t}, \quad i = 2, ..., N, \ (\theta_1 \equiv 1), \tag{7}$$

and the proportionality factors θ_i are to be solved from the model.

[6] We interpret τ_{ik} as an ad valorem tariff (usually expressed as $(1 + \tau_{ik})$) but τ_{ik} may include other nonpolicy trade costs as well.

[7] We treat intermediate capital goods as a service flow from durable capital goods. For simplicity, we exclude depreciation from the model.

Equations (4) together with (6) and (7) give the provision of each capital variety in all markets $(i = 1, ..., N,\ k \neq i)$:

$$x_{iit} = \widehat{L}_i (A_{1t} S_i)^{\xi} \left(\frac{R_t}{\alpha}\right)^{\frac{1}{\alpha-1}},\ x_{ikt} = x_{iit} \tau_{ik}^{\frac{1}{\gamma-1}},\ \ \xi \equiv \frac{\phi - 1}{1 - \alpha}, \tag{8}$$

where

$$S_i \equiv \sum_k \theta_k \tau_{ik}^{\frac{\gamma}{\gamma-1}} = \theta_i + \sum_{k \neq i} \theta_k \tau_{ik}^{\frac{\gamma}{\gamma-1}}. \tag{9}$$

Aggregate output in country i at time t equals

$$Y_{it} = \widehat{L}_i^{1-\alpha} x_{iit}^{\alpha} (A_{1t} S_i)^{\phi} = \widehat{L}_i (A_{1t} S_i)^{1+\xi} \left(\frac{R_t}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}. \tag{10}$$

According to (8), capital goods imports to a country (x_{ikt}) decrease with trade barriers (τ_{ik}) and increase with country size and total factor productivity ($\widehat{L}_i$) and the level of technology (A_1), *ceteris paribus.*

Terms S_i defined in (9) give an import tariff -deflated sum of the relative technology levels of a country's trade partners, thus reflecting the accessibility of world technology to the aggregate production sector in country i. In (8), the S_i state the impact of a country's trade policy and trade pattern on domestic capital goods production (x_{iit}) and imports of intermediate capital. Each S_i decreases (thus reducing x_{iit} and x_{ikt}) as tariffs τ_{ik} increase and this effect is the larger the higher the technology level of the trade partners (θ_k) and the larger the contribution of capital goods in aggregate production (γ). Keeping trade policies τ_{ik} fixed, S_i increases if the country itself or its trade partners become more developed (θ increase).

The solution for aggregate output in (10) shows that, in addition to institutional and other exogenous factors (ψ_i), a country's total factor productivity depends the country's openness for technology imports (S_i) that magnifies the contribution of the technology level (A_1); when the degree of complementarity among capital goods (ϕ) increases, the contributions of technology and openness to total factor productivity become more important.

The firms that produce intermediate capital goods are treated as the innovators of each original design. For a new capital good variety to be introduced, a fixed innovation cost must be paid. Following EHR (1998) and HTR (2002), we assume that the cost of a new invention, when developed in country i, equals $\upsilon_i^{\varsigma} p^Z j^{\xi}$ ($\varsigma, \xi > 0$) units of final consumption. In this specification, j^{ξ} equals the cost of an invention in aggregate capital (Z) and ($\upsilon_i^{\varsigma} p^Z$) converts the cost of each capital unit into consumption. This specification implies that later innovations are more costly but, owing to capital complementarity, they are also more valuable. The parameters υ_i reflect any factors that impact research productivity in a country across sectors, e.g., differences in initial human capital and variation in institutions and policies that support innovation; ς that is common for all countries determines the dispersion of the innovation costs.[8]

[8] Alesina and Giavazzi (2006: Ch. 5) discuss differences in research productivity in the U.S. and among European countries.

At each time period, the quantity of new innovations is determined by the zero profit condition of the monopolistically competitive capital producers, i.e., in equilibrium,

$$\upsilon_i^{\varsigma} p_t^z \theta_i^{\xi} A_{1t}^{\xi} = \sum_{s=0}^{\infty} (1 + r_{t+s})^{-(s+1)} \pi_{i,t+s}, \tag{11}$$

where

$$\pi_{i,t} = (1-\gamma) R_t \left[\sum_{k=1}^{N} x_{ki} \right] = k_i \Omega A_{1t}^{\xi} (r_t p_t^Z)^{\frac{\alpha}{\alpha-1}}, \tag{12}$$

$$k_i(\theta; \tau, \widehat{L}) \equiv \sum_{k=1}^{N} \widehat{L}_k S_k^{\xi} \tau_{ki}^{\frac{1}{\gamma-1}}, \tag{13}$$

and $\Omega \equiv (1-\gamma)\gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}$. The profit (at the end of period t) for each intermediate good produced in country i is given by (12). In addition to the interest rate (r), opportunity cost of general purpose capital (p^Z) and technology level (A_1) that impact producers in all locations identically, asymmetries yield the country-specific multipliers k_i defined in (13). According to (13), profits are positively affected by the productive size of each market ($\widehat{L}_k$) and the openness and trade pattern of each economy in technology imports (S_k); an increase in the tariff barriers against imports elsewhere (τ_{ki}) reduces k_i.

The opportunity cost of general purpose capital is determined by the competitive production sectors' trade-off between final consumption and general purpose capital. We assume that both final consumption and aggregate capital are tradable so that a world production possibility frontier can be expressed as

$$Y_t^w = C_t^w + Z_t^w \Gamma \left(\frac{Z_{t+1}^w - Z_t^w}{Z_t^w} \right). \tag{14}$$

In (14), Y_t^w and C_t^w denote the world total output of final consumption and the amount of this output that is directly consumed; Z_t^w equals the world stock of aggregate capital in time period t. The function Γ is a convex cost function that expresses the cost of aggregate capital in consumption units.[9] Equation (14) yields

$$p^Z = -\frac{dC_t^w}{dZ_{t+1}^w} = \Gamma' \left(\frac{Z_{t+1}^w - Z_t^w}{Z_t^w} \right). \tag{15}$$

The stock of capital at time period t is obtained by adding up the cumulative investment in capital goods (K_t^w) and innovation ($Z_{in,t}^w$) so that

$$\begin{aligned} Z_t^w = K_t^w + Z_{in,t}^w &= \sum_{i=1}^{N} \left[\sum_{k=1}^{N} \int_0^{A_{kt}} x_{kit}(j_k)\, dj_k \right] + \int_0^{A_{1t}} j^{\xi} dj + \sum_{i \neq 1} \int_0^{\theta_i A_{1t}} j^{\xi} dj \\ &= A_{1t}^{1+\xi} \left[\left(\frac{R_t}{\alpha} \right)^{\frac{1}{\alpha-1}} \left(\sum_{i=1}^{N} \theta_i k_i \right) + \frac{1}{1+\xi} \left(\sum_{i=1}^{N} \theta_i^{1+\xi} \right) \right]. \end{aligned} \tag{16}$$

[9] If a unit of foregone consumption converts to a unit of aggregate capital, (14) corresponds to the accumulation equation $Y_t^w = C_t^w + (Z_{t+1}^w - Z_t^w)$. When $\Gamma(.)$ is not an identity function, the opportunity cost of Z^w is not constant, i.e., the production possibility frontier between consumption and general purpose capital is nonlinear.

2.2 Balanced Growth:

Steady state solutions are characterized by a constant interest rate (r), a constant opportunity cost of aggregate capital (p^Z) and a common rate of technology growth, defined by

$$g_A \equiv \frac{A_{1,t+1}}{A_{1t}}. \tag{17}$$

Equation (16) implies that the aggregate capital stock grows at the rate

$$g_Z = (g_A)^{1+\xi} \tag{18}$$

at a long run steady state. By (10), aggregate output must grow at the rate g_Z as well and, due to equations (2) and (14), consumption also grows at this same rate (g_Z).

Using definition (17) in (11) and (12) we obtain the equations $(i = 1, ..., N)$

$$g_A = \left[1 + r - \widehat{k}_i \Omega r^{\frac{\alpha}{\alpha-1}} (p^Z)^{\frac{1}{\alpha-1}}\right]^{\frac{1}{\xi}}, \quad \widehat{k}_i \equiv \frac{k_i}{\upsilon_i^{\varsigma} \theta_i{}^{\xi}}. \tag{19}$$

Since the growth rate g_A is common for all countries at a steady state, the equilibrium values of multipliers $\widehat{k}_i$ in (19) must also be equal for all nations that innovate. If all countries do then, at a steady state,

$$\widehat{k}_1(\theta; \tau, \widehat{L}, \upsilon) = \widehat{k}_i(\theta; \tau, \widehat{L}, \upsilon), \quad i = 2, ..., N. \tag{20}$$

According to (20), relative innovation levels θ_i adjust, depending on all policy and other asymmetries, so as to maintain equal profitability of inventions, per unit of final consumption spent in product development, in all countries (recall that $\theta_1 \equiv 1$).[10]

Substituting (18) into (19) and (2) and taking into account equations (15) and (20) we obtain the following equilibrium conditions:

$$g_Z = [\beta(1 + r)]^{1/\sigma}, \tag{21}$$

$$p^Z = \Gamma'(g_Z - 1), \tag{22}$$

$$g_Z = \left[1 + r - \Omega \widehat{k}_1 r^{\frac{\alpha}{\alpha-1}} (p^Z)^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}}, \tag{23}$$

$$\widehat{k}_1(\theta; \tau, \widehat{L}, \upsilon) = \widehat{k}_i(\theta; \tau, \widehat{L}, \upsilon), \quad i = 2, ..., N. \tag{24}$$

These equations determine the steady state solutions for $(g_Z, r, p_Z;\ \theta_2, ..., \theta_N)$, given the exogenous parameters $(\tau, \widehat{L}, \upsilon)$. The equilibrium rate of technology advance (g_A) is obtained from (18).

While the rate of technology growth is the same in all countries at a long run steady state, levels of technological attainment can vary as determined by the θ_i solutions. Since equations (24) are separable from the rest of the model, equilibrium differences in technology levels can be solved using only information regarding policy and other asymmetries between countries.[11] Given the steady state values of all θ_i, we can then apply equations (24) to obtain the *growth multiplie*r $\widehat{k}_1$ that appears in (23); this multiplier transmits the impact on growth of any asymmetries in trade policy

[10] If complete symmetry is imposed, then equations (20) yield $\theta_2 = \theta_3 = 1 (= \theta_1)$.

[11] This separability occurs because capital goods are assumed not to depreciate. If depreciation were included, variables θ would appear in (22).

(τ_i), country size (L_i), total factor productivity (ψ_i), or cost of innovation (υ_i). Via equations (22) and (21) such asymmetries also affect the equilibrium values of p^Z and r. By (11)-(12) and (19), the growth multiplier represents the steady state impact of country-specific heterogeneities on the profits of the monopolistically competitive innovators. It is this effect on the profitability of innovation that is of importance in the equilibrium condition (23).

By (24), relative technology levels satisfy the equations

$$\theta_i = \left(\frac{k_i(\theta; \tau, \widehat{L})/\upsilon_i^{\varsigma}}{k_1(\theta; \tau, \widehat{L})/\upsilon_1^{\varsigma}} \right)^{\frac{1}{\xi}}, \quad i = 2, ..., N, \tag{25}$$

i.e., technology levels will reflect the relative profitability of innovation in each location, taking into account trade policies and equilibrium trade patterns (k_i) and differences in research productivity (υ_i). Differences in technology levels partly determine equilibrium levels of aggregate output since, by (10) and (9),

$$y_i \equiv \frac{Y_i}{\widehat{L}_i}, \quad y_j \equiv \frac{Y_j}{\widehat{L}_j} \quad \Rightarrow \quad \frac{y_i}{y_j} = \left(\frac{S_i(\theta, \tau)}{S_j(\theta, \tau)} \right)^{1+\xi}. \tag{26}$$

Thus, aggregate output in a country per effective unit of labor is the larger the more open the country's trade policy and the more advanced its trade partners (as indicated by $S_i(.)$).

Given (14), total expenditures and revenues (measured in aggregate consumption) equalize worldwide in every time period. Country-specific balances of payments are maintained by allowing consumption levels to adjust to maintain the equality of export revenues and import costs in final consumption and capital goods; due to the balanced growth of output, consumption, and capital in all countries, trade remains balanced at a steady state.

Figure 1 illustrates a possible configuration of world steady states obtained using equations (21)-(24). The upward sloping curve CC graphs the r and g_Z combinations that satisfy (21). Curve TT represents the points (r, g_Z) where the zero profitability condition (23) is maintained, given the opportunity cost of capital (obtained from (22)) and the technology levels (determined by (24)).[12] The slope of the TT curve depends on the degree of technological complementarity among capital goods (ϕ) and the curvature of the production possibility frontier between consumption and aggregate capital (Γ''). In particular, if $\Gamma'' = 0$, curve TT is upward sloping (complementarity dominates) but when Γ'' is positive the slope of the TT curve can be negative.

As shown in Figure 1, depending on the shape and position of the TT curve, multiple equilibria (at intersections of CC and TT curves) can arise; these solutions yield distinct long run growth paths for the world and the associated rates of growth can significantly vary. Furthermore, at each steady state, depending on structural heterogenieties between countries, there can be several long run income clubs (country groups that persistently differ in their relative technology and output levels). According to equations (25) and (26), differences in the level of innovation and output between such income clubs are largely influenced by the relative profitability of innovation, the degrees of openness, and the patterns of trade among various countries. The stability properties of all equilibria under adaptive learning are discussed in Section 5.

[12] This TT curve applies in the long run analysis (after all countries have adjusted to a long run steady state). Short run dynamics are considered in Section 5.

3 World Steady States and Trade:

We next analyze equations (21)-(24) under alternative assumptions about trade policy and other structural heterogeneities.

3.1 Autarky Equilibria:

We begin at autarky, i.e., we assume for the moment that all countries exist in isolation, without financial or trade interconnections. Under complete autarky, equilibrium conditions (21)-(22) apply to each country separately and, assuming that innovation occurs in all countries, equations (23) and (24) are replaced by the requirement[13]

$$g_Z = \left[1 + r - \Omega \widehat{k} r^{\frac{\alpha}{\alpha-1}} (p^Z)^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}}, \quad \widehat{k} \equiv \frac{\widehat{L}}{\upsilon^{\varsigma}}. \tag{27}$$

The equilibrium conditions (21)-(22) and (27) can be used to draw illustrations analogous to Figure 1 for each country separately. In such figures, the TT curves will differ if the multipliers $\widehat{k}$ in (27) do. Consequently, autarky growth solutions can vary.

Proposition 1 *Let the structural parameters of two countries, i and j, be such that*

$$\widehat{k}_i > \widehat{k}_j. \tag{28}$$

Then, if the (stable) autarky equilibrium in each country is unique, country i will grow faster than country j. If multiple autarky equilibria exist in either country then, at every corresponding growth regime (autarky solution) in the two countries, growth in country i is faster than in country j; however, the realized rate of growth in autarky may still be higher in country j than in country i.

Proposition 1 yields the expected general conclusion about single country growth. Alternatively, if we regard equations (21)-(22) and (27) as a description of a symmetric world economy under free trade, a similar conclusion follows: a larger and more productive world will grow faster. When two separate economies are compared, an analogous result still holds as long as the autarky steady states are unique. However, if multiple equilibria exist, we can only be sure that the growth potential of country i, as represented by the set of (stable) autarky steady states for country i (such as $E1_i$ and $E3_i$ along TT_i in Figure 2b), is higher than the growth potential of country j (equilibria $E1_j$ and $E3_j$ along TT_j in Figure 2b).[14] But, the realized rate of growth may still be higher in country j because the actual autarky equilibrium in country j may correspond to a high growth equilibrium ($E3_j$), whereas autarky in country i may occur at a low growth state ($E1_i$).

Our last conclusion indicates that while a country may possess a higher potential for growth (higher $\widehat{k}$) this may not in all cases translate to a superior outcome. Whether this in fact occurs depends in a complicated manner on the fundamentals of the economy that govern the behavior of the innovating monopolistically competitive sector and additional forces that may guide expectations.

[13] We have dropped the country subscript i from equation (27).

[14] Equilibria such as $E1_i$ and $E3_i$ at which the TT_i curve cuts the CC curve from above are stable under adaptive learning. Equilibria such as $E2_i$ where this condition is violated are unstable.

3.2 Structural Symmetry and Tariffs:

We next consider an open and structurally symmetric world in which countries may nevertheless choose divergent trade policies. Given symmetry, we set $\widehat{L}_i \equiv \widehat{L}$ and $\upsilon_i \equiv \upsilon$ for all $i = 1, ..., N$, in equations (21)-(24); thus, every country is equally productive and of equal effective size. Then, using (19), equations (24) can be replaced by the equilibrium conditions

$$k_i(\theta; \tau, \widehat{L}) = \theta_i^{\xi} k_1(\theta; \tau, \widehat{L}), \quad i = 2, ..., N, \tag{29}$$

and, by (13), the terms k_i in (29) equal

$$k_i(\theta; \tau; \widehat{L}) = \widehat{L} \sum_{j=1}^{N} S_j^{\xi} \tau_{ji}^{\frac{1}{\gamma-1}}. \tag{30}$$

According to (29)-(30), the steady state values of innovation activity, θ_i, are influenced by the patterns of tariffs against capital goods (τ_{ji}) and the overall openness of each market (S_j). The importance of market openness increases with the complementarity of capital goods (ξ). If all trade is free ($\tau_{ji} = 1$ for all i, j), equations (29) yield $\theta_i = 1$ for all i.

3.2.1 Unilateral Trade Restrictions:

We first consider the impact of unilateral policy, i.e., we assume that one of the countries (country 1, say) imposes a tariff $\tau_{1i} \equiv \tau$ (> 1) against all imported capital goods but that others do not retaliate ($\tau_{ij} = 1$ for all other i, j).

As all other countries remain symmetric, we set $\theta_i \equiv \theta$, $i = 2, ..., N$, and obtain from (29) that

$$\theta^{\xi}(S_1^{\xi} + (N-1)S_j^{\xi}) = S_1^{\xi}\tau^{\frac{1}{\gamma-1}} + (N-1)S_j^{\xi}, \tag{31}$$

$$S_1 = 1 + (N-1)\theta\tau^{\frac{\gamma}{\gamma-1}}, \quad S_j = 1 + (N-1)\theta, \quad j = 2, ..., N. \tag{32}$$

Substitution of (32) into (31) yields the following equilibrium condition for θ:

$$\left(\frac{1 + (N-1)\theta\tau^{\frac{\gamma}{\gamma-1}}}{1 + (N-1)\theta}\right)^{\xi} = (N-1)\left(\frac{1-\theta^{\xi}}{\theta^{\xi} - \tau^{\frac{1}{\gamma-1}}}\right), \quad \theta^{\xi} \neq \tau^{\frac{1}{\gamma-1}}. \tag{33}$$

Proposition 2 *When a country imposes a uniform tariff* τ (> 1), *the steady state solution for the relative technology level in the rest of the world,* θ, *satisfies the condition*

$$\theta \in \left(\left(\frac{(N-1) + \tau^{\frac{1}{\gamma-1}}}{N}\right)^{\frac{1}{\xi}}, 1\right). \tag{34}$$

Furthermore, compared to symmetric free trade, the reduction in the θ *solution is the larger the higher the tariff.*

Proposition 2 shows that by unilaterally restricting imports of capital goods from the rest of the world (without retaliation) a country can raise its share of innovation and lower the level of innovation elsewhere. This effect comes about because of the changes in the relative prices of imported and domestic capital goods which, in turn, alter the profitability of innovation in all locations.

Nevertheless, if all countries initially innovate, a unilateral tariff cannot eradicate innovation in the rest of the world: by (34), there is a positive lower limit for the level of innovation in other locations, and this lower limit reflects the relative size of these countries' markets in the world. The significance of the market size is emphasized if we allow for a simple asymmetry in country size and/or total factor productivity, i.e., we set $\widehat{L}_i = \widehat{L}_{Row}$ for all $i = 2, ..., N$, and $\widehat{L}_1 > \widehat{L}_{Row}$.

Corollary 3 *Given $\widehat{L}_1 > \widehat{L}_{Row}$ and τ (> 1), the steady state solution for the relative technology level in the rest of the world satisfies the condition*

$$\theta \in \left(\left(\frac{\widehat{L}_{Row}}{\widehat{L}_1} \right)^{\frac{1}{\xi}} \left(\frac{(N-1) + \tau^{\frac{1}{\gamma-1}}}{N} \right)^{\frac{1}{\xi}}, 1 \right). \tag{35}$$

Thus, *the larger or more productive a country is in relation to the rest of the world, the more we can expect it to gain in innovation share from a unilateral tariff.* The degree of complementarity between capital goods also matters. The relative gain in innovation experienced by a country tends to be the larger the lower the degree of technical complementarity (ϕ or ξ).

The aggregate output ratio for country 1 and the rest of the world is obtained using (26). Given structural symmetry and when $\tau > 1$,

$$\frac{Y_1}{Y_i} = \left(\frac{1 + (N-1)\theta\tau^{\frac{\gamma}{\gamma-1}}}{1 + (N-1)\theta} \right)^{1+\xi} < 1, \quad i = 2, ..., N, \tag{36}$$

i.e., the unilateral tariff lowers the output ratio from its free trade equilibrium level (equal to unity). This reduction reflects the distortion in the usage of capital goods that follows the introduction of the import tariff. It is also clear, using (36), that the Y_1/Y_i ratio further decreases when the unilateral tariff becomes larger. The following numerical example illustrates the impact of the unilateral tariff on the technology and output levels when $N = 3$.

Example 1: Let the three countries be structurally symmetric and assume that $\alpha = 0.39$ and the value of the parameters ξ and γ are as stated in the two tables.[15]

TABLE 1A: $\xi = 0.49, \gamma = 0.3$

τ	θ	Y_1/Y_i
1.1	0.92	0.96
1.2	0.85	0.93
1.5	0.73	0.86
1.7	0.68	0.83

TABLE 1B: $\xi = 0.40, \gamma = 0.27$

τ	θ	Y_1/Y_i
1.1	0.90	0.97
1.2	0.83	0.94
1.5	0.69	0.90
1.7	0.63	0.88

In Example 1 and more generally under structural symmetry, the reduction in the innovation level of the rest of the world is larger than the reduction in the protectionist country's relative

[15] The parameter values in Table 1A were used in HTR (2002).

aggregate output (compare (36) and (25)). This suggests that *by imposing a tariff against imported capital goods a country may attain a sizable relative lead in technological innovation at a modest loss of relative production efficiency*; as illustrated by the calculations in Tables 1, if capital goods are not very complementary to each other this effect could be particularly significant. If, furthermore, there should be external gains from R&D activities that have not been considered here (e.g., learning from doing, improvement in total factor productivity), the temptation to restrict imports of foreign capital goods could become considerable.

The above observations give some basis for speculation about the past quantitative effects of developed country protectionism against the less developed countries (LDC's). As observed by Waugh (2007), trade costs are significantly skewed against the LDC's, and the pattern of protection is known to cascade. Our results suggest that unfavorable trade costs may help explain the present relative disadvantage of LDC's in technological innovation. Moreover, if in contrast to what has been assumed here some factors of production are internationally mobile, asymmetries in trade costs may also play a part in sorting the mobile factors across locations, thus further exacerbating differences in technological attainment (e.g., developed countries' lead in technological innovation attracts skilled labor from LDC's, further lowering the relative technology level in LDC's).

In addition to long term changes in the level variables, a unilateral tariff alters the world long run growth rate (g_Z).

Proposition 4 *Let τ_1 (≥ 1) be the initial value of the unilateral tariff and let g_Z^1 denote the corresponding growth solution at a stable steady state. Then, if the unilateral tariff is raised to τ_2 ($> \tau_1$), the long run growth rate g_Z^2 attained at the corresponding stable steady state is lower than g_Z^1. The reduction in the long run rate of growth is the larger the higher the unilateral tariff.*

According to Proposition 4, compared to symmetric free trade, the world economy grows more slowly when one of the countries imposes a unilateral tariff; the negative growth effect is also monotonically increasing in the size of the tariff.

Figure 3 illustrates. Suppose curve TT_1 (TT_2) in Figure 3 applies when $\tau = \tau_1$ ($\tau = \tau_2$). Then, assuming that the initial (stable) equilibrium occurs at $E3_1$ or at $E1_1$, the new (stable) steady state corresponding to the higher tariff occurs at $E3_2$ or at $E1_2$, respectively. In both cases, the higher tariff is a cause for a *local decline in growth.* If the increase in the unilateral tariff and its impact on the world economy are sufficiently large, however, there may be a downward jump in growth (bifurcation). This occurs in Figure 3 if the TT curve shifts from TT_1 to a position such as shown by curve TT_2'. In this case, the high growth equilibrium regime is eliminated, and the movement from the initial equilibrium at $E3_1$ to $E1_2$ involves a much larger reduction in growth than the movement from equilibrium $E3_1$ to $E3_2$.

Example 1 continued: Let $\alpha = 0.39$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L} = 0.014$ and $N = 3$. The values of ξ and γ are as stated in the two tables. Then, the $k_1(\tau)$ and $g_Z(\tau)$ solutions are: [16]

[16] The parameter values in Example 1 need to be modified to attain more reasonable values for g_Z. Without depreciation, the growth rate tends to be high. The values of k_1 reported in Tables 2 exclude the $\widehat{L}$-term in (30).

TABLE 2A: $\xi = 0.49, \gamma = 0.3$ **TABLE 2B:** $\xi = 0.40, \gamma = 0.27$

τ	k_1	g_Z
1.0	5.14	1.60
1.1	4.98	1.58
1.2	4.84	1.57
1.5	4.59	1.54
1.7	4.48	1.52

τ	k_1	g_Z
1.0	4.66	1.43
1.1	4.52	1.42
1.2	4.41	1.42
1.5	4.20	1.40
1.7	4.10	1.38

Tables 2 illustrate the increasingly negative effect of a unilateral tariff on long run growth as the tariff escalates. These calculations also suggest that the reduction in growth may be smaller (albeit from a lower initial level of growth) when capital goods are less complementary to each other. This appears interesting in the light of our earlier observations and Tables 1 which indicate that the reduction caused by the unilateral tariff on the technology level of the rest of the world is likely become more severe when the degree of complementarity is smaller.

Together, Propositions 2 and 3 suggest that imports of intermediate capital goods are more likely to be restricted in countries that are large and highly productive; such restrictions are also likely to be directed against smaller and less productive nations. While these types of trade distortions reduce growth in the long run, they can cause significant shifts of innovation from one location to another. When capital goods are not particularly complementary, the shifting of innovation can be important while the ensuing reduction in long run growth is likely to be smaller.

3.2.2 Preferential Trade Agreements:

Expanding the scope of Propositions 2 and 4, we now assume that n_1 of the N nations (say countries $1, 2, ..., n_1$) form a customs union, while n_2 $(= N - n_1)$ countries remain nonmembers. The members of the customs union impose a common trade barrier against imports of capital goods from other locations ($\tau_{ij} = \tau$ (> 1), $i = 1, 2, ..., n_1$, $j = n_1 + 1, ..., N$) but maintain free trade within the union ($\tau_{ij} = 1$, $i, j = 1, 2, ..., n_1$). We first assume that the nonmembers do not retaliate ($\tau_{ji} = 1$, $i = 1, 2, ..., n_1$, $j = n_1 + 1, ..., N$) and then add some comments about the impact of such retaliation.

Without retaliation and because of symmetry, we can set $\theta_i = 1$ for all countries in the customs union and $\theta_j \equiv \theta$ in the rest of the world.

Proposition 5 *i) When n_1 (≥ 1) countries form a customs union, the steady state solution for the relative technology level in the rest of the world satisfies the condition*

$$\theta \in \left(\left(\frac{n_1 \tau^{\frac{1}{\gamma - 1}} + n_2}{N} \right)^{\frac{1}{\xi}}, 1 \right). \tag{37}$$

Compared to symmetric free trade, the increase in the relative technology level of the customs union is the larger the higher the union tariff. ii) Let τ_1 (≥ 1) be the initial value of the union tariff and let g_Z^1 denote the corresponding growth solution at a stable steady state. Then, if the union tariff is raised to τ_2 $(> \tau_1)$, the long run growth rate g_Z^2 attained at the corresponding stable steady state is lower than g_Z^1. The reduction in the long run rate of growth is the larger the higher the union tariff.

In agreement with Propositions 2 and 4, Proposition 5 demonstrates that a subgroup of countries that protects its monopolistically competitive sector can gain an asymmetric share of future innovation. Compared to a symmetric free trade outcome, the world economy is bound to grow more slowly when a customs union exists. The magnitude of the change in the relative technological attainment of the customs union and the negative impact on long term growth depend on the height of the tariff wall in the union and, as indicated by our previous observations, are also likely to be influenced by the degree of complementarity among capital goods. According to condition (37), the relative size of the markets inside the customs union and in the rest of world also matter: in (37), the larger the customs union (n_1) and, therefore, the smaller the rest of the world (n_2), the smaller the low limit for θ, i.e., the larger the potential shift in future innovation from the rest of the world to the customs union.

Proposition 6 *Keeping the external tariff of a customs union fixed, the relative technology level of the members in the customs union increases as the union expands (the ratio n_1/n_2 increases).*

Thus, each additional member to the customs union benefits itself and every other member of the union by offering an additional market which, by expanding the reach of free trade for intermediate capital goods, further stimulates innovation within the union. Conversely, as n_1/n_2 increases, in the ever smaller rest of the world incentives for further innovation decline thus reducing the value of θ in the long run.

Analogously to inequality (36), the customs union experiences some efficiency loss in aggregate production and so the output ratio Y_1/Y_j must be smaller than unity. However, this loss in relative output decreases with the size of the union.

Proposition 7 *Keeping the external tariff of a customs union fixed, the output ratio Y_1/Y_j increases as the union expands (the ratio n_1/n_2 increases).*

According to Proposition 7, the expansion of innovation within a larger customs union can compensate for some of the productivity loss that is caused by the union's external tariff barrier (new members not only benefit all union members by inducing new innovation but also lower the cost of the price distortions that support the union). Example 2 illustrates with some numerical calculations.

Example 2: Let $\alpha = 0.39$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L} = 0.014$ and $N = 3$. The customs union is formed by countries 1 and 2 ($n_1 = 2,\ n_2 = 1$). The values of ξ and γ are as stated in the two tables. The column θ_{Low} gives the lower limit of θ obtained from (37) and the column θ states the steady state value.

TABLE 3A: $\xi = 0.49, \gamma = 0.3$

τ	θ_{Low}	θ	Y_1/Y_3	k_1	g_Z
1.1	0.83	0.84	0.98	4.98	1.58
1.2	0.71	0.72	0.97	4.87	1.57
1.5	0.49	0.50	0.95	4.65	1.54
1.7	0.41	0.41	0.95	4.56	1.53

TABLE 3B: $\xi = 0.40, \gamma = 0.27$

τ	θ_{Low}	θ	Y_1/Y_3	k_1	g_Z
1.1	0.81	0.81	0.99	4.52	1.43
1.2	0.67	0.67	0.98	4.42	1.42
1.5	0.43	0.44	0.97	4.26	1.40
1.7	0.35	0.35	0.96	4.19	1.39

In Tables 3, when compared to the free trade equilibrium, the formation of the customs union causes a reduction in the steady state values of θ, Y_1/Y_j, and g_Z. These changes increase with the external tariff of the customs union. Further, as suggested by Proposition 6 and 7, the customs union gains a larger advantage in the relative technology level and loses less in relative output (Y_1/Y_j), when the union is larger ($n_1 = 1$ in Tables 1 whereas $n_1 = 2$ in Tables 3). Analogously to Tables 1, the effect on the technology level θ is larger when the degree of technical compelementarity is smaller (Table 3b) and the reduction in Y_1/Y_j is somewhat smaller. Overall, the conclusions of Section 3.2.1 are being reinforced by Tables 3: the larger the customs union, either in terms of its effective size ($\widehat{L}$) or in terms of the numbers of members (n_1), the more the union gains innovation share and the less it loses in relative aggregate output.

One may wonder whether the rate of long run growth is affected by the size of a customs union: does the expansion of a customs union necessarily imply a slowdown in growth? Tables 2 and 3 do not evince such an effect and, in general, the conclusion cannot be clear-cut. This is because the inclusion of additional members in a customs union is followed by two types of growth effects. First, by joining a customs union, a new member attains free access to markets where its exports previously were subject to a tariff. This new market access expands trade (trade creation) and, as a result, innovation becomes more profitable. Thus, the expansion of a customs union partly speeds up growth (*growth creation*). On the other hand, there is also a trade diversion effect because the new member country must raise tariffs against the rest of the world. This limits access to innovations developed in the rest of the world and causes a reduction in the profitability of innovation within the customs union; growth must therefore slow down (*growth diversion*). The relative size of the two opposite effects determines whether the expansion of a customs union raises or lowers growth in the long run.

Since the conditions obtained are complex, we do not state them here but Proposition 5 and the obtained conditions are consistent with the mixed empirical evidence of Berthelon (2004) and others regarding the growth implications of PTAs. While the formation of a customs union reduces long term growth when compared to the free trade outcome (Proposition 5), there is little reason to believe that the growth consequences of an additional country's joining an existing PTA should be uniformly positive or negative for any country in the short run or the world in the long run (Proposition 8). The overall impact depends on the relative magnitude of growth creating and growth diverting effects and these, in turn, depend on the size of the markets involved, the tariff barrier imposed by the PTA, and the reallocation of innovation that follows the adjustment in the PTA membership.[17]

Countries outside a customs union may retaliate by raising a tariff wall of their own. Suppose, as above, that countries $1, 2, ..., n_1$ form a customs union (denoted by U) with an external tariff equal to $\tau_U \ \ (> 1)$. Let countries $(n_1 + 1), ..., N$ retaliate by forming a competing customs union with an external tariff τ_R (> 1). This retaliation always improves the relative technology position of countries R but the effect on their relative aggregate output and long run growth are ambiguous.

Proposition 8 *Let customs union U impose an external tariff $\tau_U \ \ (> 1)$, and let τ_R (≥ 1) be the external tariff of countries in customs union R. i) Then, at any τ_U, the long run technology level of countries in customs union R is an increasing function of the retaliation, τ_R. ii) The effect of a retaliatory tariff on the output ratio Y_1/Y_j, $j \in R$, and the rate of long run growth are ambiguous.*

That the competing customs union R gains in terms of its technology level by retaliating is not surprising in the light of Propositions 2 and 5. The retaliatory tariff raises profitability of innovation

[17] Berthelon (2004) found strong evidence supporting the importance of new market access as fostering country growth (the growth creation effect).

in R and therefore attracts a larger proportion of future innovation there. The effect of the relative correction in the location of innovation on aggregate output remains uncertain, however. The effect of the retaliatory tariff on the long run steady state depends on the change in the growth multiplier $\widehat{k}_1$ and this effect is also ambiguous. Example 3 offers some numerical experiments.

Example 3a: Let $\alpha = 0.39$, $\xi = 0.49$, $\gamma = 0.3$, $\beta = 0.90$, $\sigma = 0.22$, $\widehat{L} = 0.014$ and $N = 3$. Customs union U is formed by countries 1 and 2 and union R consist of country 3 ($n_1 = 2$, $n_2 = 1$).

TABLE 4A: $\tau_U \equiv 1.2$

τ_R	θ	Y_1/Y_3	k_1	g_Z
1.0	0.72	0.97	4.87	1.57
1.1	0.78	1.01	4.69	1.55
1.2	0.84	1.05	4.55	1.53
1.5	0.97	1.14	4.28	1.50

TABLE 4B: $\tau_U \equiv 1.5$

τ_R	θ	Y_1/Y_3	k_1	g_Z
1.0	0.49	0.95	4.65	1.54
1.1	0.54	1.00	4.46	1.52
1.2	0.58	1.03	4.32	1.50
1.5	0.67	1.13	4.03	1.48

Example 3a expands the experiment of Table 3A: countries 1 and 2 comprise the customs union U but now country 3 that does not belong to U retaliates. In Table 4A, union U imposes the tariff $\tau_U \equiv 1.2$ and in Table 4B the tariff is higher ($\tau_U \equiv 1.5$). The first row of each table states the long run steady state values of θ, Y_1/Y_3, and g_Z, without retaliation (this row is obtained from Tables 3). When the retaliatory tariff increases, the equilibrium level of innovation outside union U increases in both Tables 4 (Proposition 9 i)). Furthermore, the output ratio Y_1/Y_3 monotonically increases with τ_R and the long run growth rate decreases. These observations demonstrate that, at least in this numercial example, the distortionary impact of tariff τ_R that works toward raising Y_1/Y_3 and lowering g_Z is stronger than the opposite effect that comes about when the allocation of innovation among countries adjusts to a higher retaliatory tariff (θ increases). Table 4B indicates that the higher union tariff has a considerable impact on the allocation of innovation (θ is lower in Table 4B than in Table 4A (Proposition 5)) but the effects on the output ratio and long term growth appear much smaller. Example 3b offers a comparison in which the customs union U is smaller.

Example 3b: Let the model parameters be the same as in Tables 4 and let customs union U consist of country 1 only. Customs union R includes countries 2 and 3.

TABLE 5A: $\tau_U \equiv 1.2$

τ_R	θ	Y_1/Y_3	k_1	g_Z
1.0	0.85	0.93	4.84	1.57
1.1	1.02	0.95	4.67	1.54
1.2	1.20	0.95	4.56	1.52
1.5	1.73	0.97	4.32	1.50

TABLE 5B: $\tau_U \equiv 1.5$

τ_R	θ	Y_1/Y_3	k_1	g_Z
1.0	0.73	0.90	4.59	1.54
1.1	0.88	0.87	4.41	1.52
1.2	1.03	0.88	4.28	1.50
1.5	1.50	0.88	4.03	1.48

Tables 5 confirm the important role that the size of a customs union and its markets play in determining the impact of an PTA. Since union U is smaller in Tables 5 than in Tables 4, its tariff wall has much less effect on innovation in the rest of the world (the initial decline in θ in Tables 5 is smaller than in Tables 4), and even a modest retaliation results in a significant correction in the equilibrum value of θ. A small customs union is therefore much less likely to significantly gain in new innovation and, according to Tables 5, the cost in reduced relative output for the union remains even as the rest of the world retaliates ($Y_1/Y_3 < 1$ in Tables 5 where as Y_1/Y_3 significantly increases with retaliation in Tables 4). The growth solutions in Tables 5 are about the same as in Tables 4, showing a small improvement in growth when the customs union U is smaller.

Finally, we may interpret Examples 1-3 so as to consider the impact of unilateral and multilateral *reductions of tariffs*. In this model, unilateral tariffs and tariffs imposed by customs unions slow down growth, given that there is no retaliation (Proposition 4 and 5). Accordingly, *reductions of unilateral or common tariffs in a customs union will speed up growth*, assuming that no other distortions are present.[18] In Tables 2 and 3, there is little difference in the improvement of the long term growth rate as tariffs are reduced so that the number of countries participating in the tariff reduction does not appear to be significant. This changes in Tables 4 and 5 in which a customs union faces retaliation by the rest of the world. In both Tables 4 and 5, the increase in the steady state rate of growth is larger when both the union and retaliatory tariffs decline, i.e., a *multilateral reduction of trade restrictions yields a larger boost in growth than a partial reduction.* This conclusion is in the spirit of Rivera-Batiz and Xie (1992) who demonstrated, in a knowledge-driven model of innovation, that an increase in the number of countries participating in a tariff reduction increases the likelihood of the effect on growth being positive.

4 Heterogenous Learning Dynamics

In this section, we formulate adaptive learning dynamics to augment the description of the long run steady states in (21)-(24).[19]

4.1 Stability under Heterogenous Learning:

The process of learning dynamics comprises two components: i) the mapping from expectations to a temporary equilibrium, and ii) the learning rule that describes the updating of expectations based on the observed past.[20] Of these, the mapping from expectations to a temporary equilibrium is of the form

$$T(g^e_{it}) = g_t, \quad i = 1, ..., N, \tag{38}$$

where individual expectations for time period t, $g^e_{i,t}$, determine a temporary equilibrium value of growth, g_t.[21] The mapping T is defined using the behavioral relationships contained in the model (21)-(24).

Expectations are adjusted according to the learning rule

$$g^e_{i(t+1)} = g^e_{it} + \gamma_{it}(g_t - g^e_{i(t+1)}), \quad i = 1, ..., N. \tag{39}$$

By (39), decisionmakers apply similar learning rules to update expectations except that the gain parameters, γ_{it}, that indicate responsiveness expectations to earlier errors are allowed to vary by

[18] As shown by Rivera-Batiz and Romer (1991) and Rivera-Batiz and Xie (1992), in a knowledge driven model of innovation, the rate of growth is not necessarily a monotonic function of a tariff distortion

[19] This discussion applies the structurally symmetric model of Section 3.2.

[20] See Evans and Honkapohja (2001) and Honkapohja and Mitra (2006).

[21] All individuals within a country are assumed to hold the same expectations; thus, $g^e_{i,t}$ in (38) are indexed by country i.

individual and over time.[22] We assume that the gain sequences satisfy the regularity Assumption A in Honkapohja and Mitra (2006).[23] Together, equations (38) and (39) define a dynamic system for which the long run equilibria are the fixed points. Stability of the steady states under adaptive learning is determined by local stability of the system (38)-(39) near each fixed point.

In our present context, there are two representative learners: a country that is a member of a customs union U (country 1, say) and a country that remains outside the union (country 2, say). While it is natural to assume that countries in each group apply similar learning rules, learning heterogeity in the gain parameters between the country groups arises quite naturally as a feature of the open economy solutions. As shown by Pfajfar (2007), individuals differ in their ability to forecast economic conditions, and the speed of adjustment in expectations can significantly vary. Within our model, we interpret differences in the gain parameters, γ_{it}, as reflecting the level differences in technology attainment that characterize steady state solutions when policy or other asymmetries are present (the gain parameter is taken to be the lower the lower the level of innovation (θ) in a country group).

For the present case the mapping (38) is constructed as follows. At time period t, producers in each country hold expectations about growth in the world economy; we denote these expectations by g_{1t}^e (for U) and g_{2t}^e (for the rest of the world). Based on expectations, firms project profitability of investment and make plans to invest in innovation and production of capital goods. Investment decisions are affected by expected productivity of aggregate production as well as this productivity affects return to investments and depends on future availability of capital goods. The investment plans of producers interact in the financial markets where the demand for aggregate capital determines a temporary equilibrium interest rate; this interest rate is consistent with expected zero profitability for innovation in all locations. In addition to the interest rate, the zero profit conditions also determine the temporary equilibrium solution for the technology level θ in the rest of the world (we maintain the assumption that $\theta_1 \equiv 1$). Modifying equation (23) to reflect expectations, we obtain two technology arbitrage conditions

$$g_{it}^e = \left[1 + r_t - \Omega \widehat{k}_i(\theta, \tau) r_t^{\frac{\alpha}{\alpha-1}} (\Gamma'(g_{it}^e - 1))^{\frac{1}{\alpha-1}}\right]^{\frac{1+\xi}{\xi}} \equiv F_i(g_{it}^e, r, \theta, \tau), \quad i = 1, 2. \tag{40}$$

The growth multipliers $\widehat{k}_i$ in (40) attain equal values only at steady state solutions or if $g_{1t}^e = g_{2t}^e$. Given growth expectations, g_{1t}^e and g_{2t}^e, and trade policy of each representative country, equations (40) yield a unique temporary equilibrium solution for the interest rate, $r_t(g_{1t}^e, g_{2t}^e, \tau)$ and the technology level, $\theta(g_{1t}^e, g_{2t}^e, \tau)$.

Consistent with the temporary value of the interest rate and using equation

$$g_t = [\beta(1 + r_t(g_{1t}^e, g_{2t}^e, \tau)]^{1/\sigma} \equiv T(g_{1t}^e, g_{2t}^e, \tau), \tag{41}$$

consumers choose how much to consume and the amount of consumption they forego; this determines the rate of growth at a temporary equilibrium in time period t, namely g_t. Combining the

[22] Learning rules (39) allow individuals to apply different econometric learning algorithms in real time. Within the present model, equation (39) includes both the recursive least squares and stochastic gradient updating rules (Honkapohja and Mitra (2006)). More generally, the gain sequence could include stochastic variation and random inertia by period; these extensions would not alter the results below.

[23] These regularity conditions require that i) $\gamma_{i,t} \le K_i \gamma_t$ for some constant $K_i > 0$, ii) $\sum_{t=1}^{\infty} \gamma_t = \infty$, $\sum_{t=1}^{\infty} \gamma_t^2 < \infty$, and iii) $\lim\sup(1/\gamma_{t+1} - 1/\gamma_t) < \infty$. Thus, the gain parameters of individual learners are majored by a gain sequence γ_t that places less weight on observations farther in the past.

behavior of producers and consumers as specified in (40) and (41), we obtain the mapping T from expectations to realized growth.

Substitution of the T mapping (41) into the learning rule (39) yields a dynamic system that describes the adjustment in expectations over time:

$$g^e_{1(t+1)} = g^e_{1t} + \gamma_{1t}(T(g^e_{1t}, g^e_{2t}, \tau) - g^e_{1t}), \tag{42}$$

$$g^e_{2(t+1)} = g^e_{2t} + \gamma_{2t}(T(g^e_{1t}, g^e_{2t}, \tau) - g^e_{2t}). \tag{43}$$

From a set of initial expectations, the dynamics specified by equations (42)-(43) approach over time certain long run steady states and drift away from the vicinity of others. This classifies steady states as either stable or unstable under adaptive learning.

Proposition 9 *A steady state equilibrium, (r^*, g^*_Z), that solves the model (21)-(24), is stable under adaptive learning if*

$$\mathcal{B} \equiv \left(\frac{\partial r_t(g^*, g^*)}{\partial g^e_{1t}} + \frac{\partial r_t(g^*, g^*)}{\partial g^e_{2t}} \right) - \frac{\partial r_{cons}(g^*)}{\partial g_t} < 0, \tag{44}$$

where the partial derivatives $\partial r_t(g^*, g^*)/\partial g^e_{1t}$ and $\partial r_t(g^*, g^*)/\partial g^e_{2t}$ are obtained from (40) (technology arbitrage conditions) and the partial derivative $\partial r_{cons}(g^*)/\partial g_t$ from (41) (the CC curve), respectively. A steady state equilibrium is unstable if $\mathcal{B} > 0$.

Stability condition (44) requires that the aggregate reaction of the market interest rate to expected growth in the two countries is smaller than the interest rate response to a change in growth that arises from consumer behavior. As in the case of autarky, the last term in (44) that corresponds to the consumer response can be intereted as the slope of the CC curve at a steady state. However, the first bracketed term that gives the interest rate reaction to growth, taking into account the adjustment in the technology level (θ), is not equal to the slope of a TT curve such as in Figure 1 at a steady state because the short run responses of the growth multipliers $\widehat{k}_1$ and $\widehat{k}_2$ to a change in θ can differ even at a long run equilibrium. Accordingly, the bracketed expression in (44) must be regarded as the slope of a short run aggregateTT curve that gives the aggregate short run reaction in the interest rate to all growth expectations; by (44), for stability under adaptive learning, the slope of the aggregate TT curve must be smaller than the slope of the CC curve.

4.2 Short Run Adjustment and Growth:

The nature of the short run adjustment process from one steady state toward another is also of interest. Figure 5a illustrates the short run adjustment following a *reduction* in the tariff imposed by the customs union (country 1).[24] This change of trade policy is expansionary and so, given the initial steady state at (r_0, g_0) with $\tau_0 > 1$, the subsequent long run equilibrium will exhibit higher growth and, by Proposition 2, the technology level variable θ will also increase ($\theta_0 < 1$ at the initial equilibrium so that the customs union is the initially more technologically advanced country group). To analyze short run changes we refer to the various TT curves in the figure.

[24] We continue to assume, for simplicity, that the rest of the world remains passive.

Curves $TT_1(\theta_0, \tau)$ and $TT_2(\theta_0, \tau)$ show the effect of the tariff reduction on the profitability calculations of producers in each group of countries, keeping the technology level variable fixed at its initial value θ_0 (these curves are obtained from (40) after adjusting the tariff to its post-reform value, τ). Assuming that the initial growth expectation of producers in country 2 equals $g^e_{2t} = g_0$, we obtain the corresponding expected interest rate, r_{2t}, along $TT_2(\theta_0, \tau)$; similarly, given $g^e_{1t} > g_0$, we obtain r_{1t} using $TT_1(\theta_0, \tau)$. If the interest rate at the temporary equilibrium, r_t, takes the value shown in the figure, the adjustment of the technology level in time period t shifts the two TT curves to the positions labeled $TT_1(\theta_t, \tau)$ and $TT_2(\theta_t, \tau)$, respectively, and thus $\theta_t > \theta_0$. Then, following the adjustment of expectations to $g^e_{1(t+1)}$ and $g^e_{2(t+1)}$, say, we can apply curves $TT_1(\theta_t, \tau)$ and $TT_2(\theta_t, \tau)$ to attain the expected $r_{1(t+1)}$ and $r_{2(t+1)}$ (shown by points b and a in Figure 5a); thus the temporary equilibrium in time period $(t + 1)$ features $r_{t+1} > r_t$, $g_{t+1} > g_t$, and $\theta_{t+1} > \theta_t$.

The notion of a temporary equilibrium implies that the short run rates of technology growth differ for the two learners, i.e., even as the world capital stock grows at rates g_t and g_{t+1} over the two time periods, the temporary realized rates of growth for the two country groups differ. This occurs because the short run adjustment of the aggregate economy includes dynamic changes in technology level; e.g., in Figure 5a, the fact that $\theta_{t+1} > \theta_t > \theta_0$ implies that innovation must advance faster in countries outside the customs union than in those in it. In other words, in Figure 5a, there are *two growth clubs*, the customs union and the rest of the world; within each club, short run growth experience is shared but, across groups, short term growth performance varies (in Figure 5a, the rest of the world continues to catch up with the previously more advanced group).

The length of time that differences in relative growth persist between growth clubs depends in a complicated manner on the learning dynamics and the shifs in the producers' profit calculations that occur as the aggregate economy transitions toward a new steady state. In Figure 5a, the short run growth advantage of the rest of the world lasts for at least two time periods because the producers outside the customs union expect a large improvement in profitability as the tariff reduction is undertaken (the expected return for investment (r_{2t}) in country 2 is very high, given τ and g^e_{2t}) and their expectations for future growth sufficiently respond to a positive growth surprise ($g_t > g^e_{2t}$) to maintain the growth momentum. If learning dynamics in the rest of the world are slower (the gain parameter γ_{2t} is low), the same outcome may not occur. If, for example, producers in country 2 exhibit complete inertia by which $g^e_{2(t+1)} = g^e_{2t} = g_0$, then, in Figure 5a, $r_{2(t+1)} = r_t < r_{1(t+1)}$ and thus $\theta_{t+1} < \theta_t$, i.e., the period of faster technological growth in the rest of the world is interrupted by at least one period of relatively slow growth during which the technological lead of the customs union widens. Slow growth could last several time periods if the adjustment in expectations is sufficiently asymmetric in the two country groups.

Periods of relatively slow growth can also arise because of changes in the producer's expected return calculations. As shown in Figure 5c, the immediate reaction to a tariff reduction in the customs union may be a slowdown (not an increase) in innovation in the rest of the world. The initial growth expectations in Figure 5c are similar to those in Figure 5a, but the relative increase in expected returns to investment in country 2 is smaller (curve $TT_2(\theta_0, \tau)$ is located above, but rather close to, curve $TT_1(\theta_0, \tau)$). Then, given g^e_{1t} and g^e_{2t}, $r_{1t} > r_{2t}$ and so the temporary equilibrium in period t must be such that $\theta_t < \theta_0$, i.e., the pace of innovation in the customs union exceeds that of the rest of the world. Expressions for the growth multipliers $\widehat{k}_1$ and $\widehat{k}_2$ in (40) indicate that the paradoxical outcome of Figure 5c is more likely to occur if the initial tariff is low so that the initial technological lead of the customs union is not very wide. Conversely, Figure 5a is more likely when the initial tariff of the customs union is high and the initial technology level of the rest of the world is low (see Appendix 4).

Several observations can be obtained using Figures 5. First, *even if, in this model, countries experience a common rate of growth at each long run steady state, their growth experiences in the short run need not be the same* and, second*, transition dynamics do not necessarily yield a monotonic sequence of changes toward a new steady state.*

Third, *heterogeneities in structural parameters (here, tariff policy) are a cause of both long run income clubs and short run growth clubs.* In the above examples, asymmetry of tariff policy identifies the members of the customs union and the rest of the world, and these policy induced country groups form the two subsequent income and growth clubs. While the growth clubs are transitory in the sense that the relative growth performance of the two groups may vary over time, monotonic sequences of persistent growth differences may also appear. Also, because of the transitory nature of short run growth clubs, *there is no fixed correlation between the technology levels of two clubs and their relative short run growth rates:* the countries in which the relative level of innovation in low (above, the rest of the world) do not necessarily experience faster growth in innovation in the short run.

Fourth, *heterogeneity in initial expectations and learning can be sources of growth surprises.* As shown above, a slow pace of learning can cause of further innovation set-backs in the short run even as a country's long run growth prospects are positive (Figure 5a); in Figure 5c, low initial expectations ($g^e_{2t} < g^e_{1t}$) sufficiently reduce the expected return for investment ($r_{2t} < r_{1t}$) so that the rate of short run growth in the rest of the world is lower than in the customs union. Further, *heterogeneity in expectations and learning may be consequences of economic policy.* Specifically, if low levels of technological attainment negatively impact the speed of learning as suggested by empirical evidence, then the inertia in expectations that slows down growth in the rest of the world (Figure 5a) may well reflect the relative technology position of this country group induced by the establishment of the customs union ($\theta_0 < 1$ because $\tau_0 > 1$).

Analogous conclusions apply in the case of a tariff increase (establishment of a customs union or an increase in the union tariff). In the long run, the customs union will attain a technological lead position and steady state growth will be slower. But, the customs union is also likely to experience a short run boost in growth relative to the rest of the world (assuming that the temporary equilibria involve a decreasing θ sequence) and, depending on growth expectations and changes in firms' expected profitability, such asymmetric growth may persist for a considerable time period. Other details of the union policy may also be important because the shifts in the profitability of investment (the TT curves) are determined by the growth multipliers $\widehat{k}$ and these depend on the height of the tariff wall and the size of the customs union (the number fo countries in the union and outside it).

References

[1] Alesina, A. and F. Giavazzi (2006), *The Future of Europe: Reform or Decline*, MIT Press.

[2] Baldwin, R., A. Venables (1995), "Regional Economic Integration", in *Handbook of International Economics*, Vol. III, G. Grossman and K. Rogoff (eds.), Elsevier.

[3] Baldwin, R., P. Martin and G. Ottaviano (2001), "Global Income Divergence, Trade, and Industrialization: The Geography of Growth Take-Offs", *Journal of Economic Growth* 6, 5-37.

[4] Bhagwati, J., D. Greenaway and A. Panagariya (1998), "Trading Preferentially: Theory and Policy", *Economic Journal* 108, 1128-1148.

[5] Berthelon, M. (2004), "Growth Effects of Regional Integration Agreements", Central Bank of Chile Working Paper No. 278, December.

[6] Caselli, F. and W. Coleman II (2006), "The World Technology Frontier", *American Economic Review* 96, 3, 499-522.

[7] Evans, G., S. Honkapohja and P. Romer (1998), "Growth Cycles", *American Economic Review* 88, 495-515.

[8] Evans, G. and S. Honkapohja (2001), *Learning and Expectations in Macroeconomics*, Princeton University Press.

[9] Grossman, G. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, MIT Press.

[10] Honkapohja, S. and A. Turunen-Red (2002), "Complementarity, Growth, and Trade", *Canadian Journal of Economics* 35, 3, 495-516.

[11] Honkapohja, S. and K. Mitra (2006), Learning Stability in Economies with Heterogenous Agents", *Review of Economic Dynamics* 9, 284-309.

[12] Kali, R., F. Mendez and J. Reyes (2007), "Trade Structure and Economic Development", *Journal of International Trade and Economic Development* 16, 2, 245-269.

[13] Limao, N. (2006), "Preferential Trade Agreements as Stumbling Blocks for Multilateral Trade Liberalization: Evidence for the United States", *American Economic Review* 96, 3, 896-914.

[14] Pfajfar, D. (2007), "Asymmetries in Inflation Expectation Formation Across Demographic Groups", Department of Economics, Cambridge University.

[15] Prittchet, L. (2000), "Understanding Patterns of Economic Growth: Searching for Hills among Plateaus, Mountains, and Plains", *World Bank Economic Review* 14, 2, 221-250.

[16] Prittchet, L. (2006), "The Quest Continues", *Finance and Development* 43, 1, March.

[17] Rivera-Batiz, L. and P. Romer (1991), "International Trade with Endogenous Technological Change", *European Economic Review* 35, 971-1004.

[18] Rivera-Batiz, L. and D. Xie (1992), "GATT, Trade, and Growth", *American Economic Review* 82, 2, 422-427.

[19] Walz, U. (1997), "Growth and Deeper Regional Integration in a Three-Country Model", *Review of International Economics* 5,4, 492-507.

[20] Walz, U. (1999), *Dynamics of Regional Integration*, Physica-Verlag (Springer).

[21] Waugh, M. (2007), "International Trade and Income Differences", Department of Economics, University of Iowa.

[22] The World Bank (The International Bank for Reconstruction and Development) (2005), *Global Economic Prospects: Trade, Regionalism and Development*, Washington, D.C.

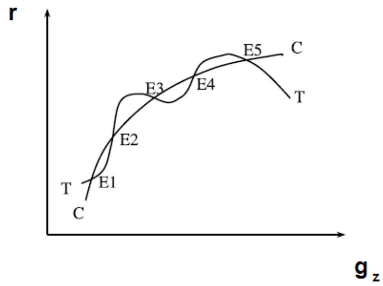


FIGURE 1 Steady states

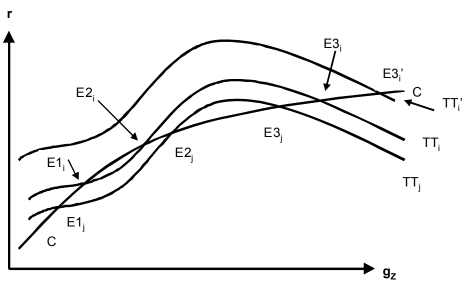


FIGURE 2b: Autarky Steady States

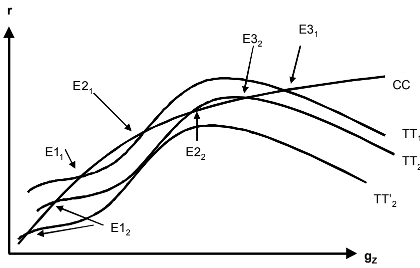
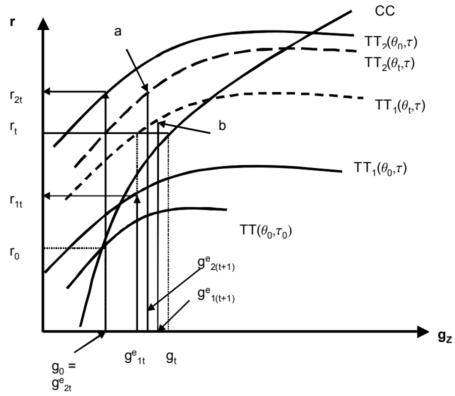
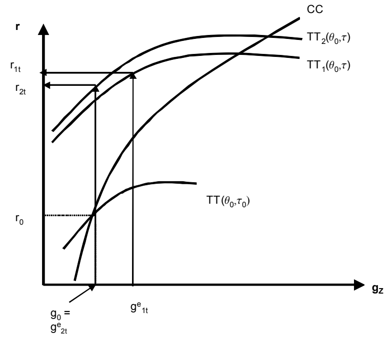


FIGURE 3: Unilateral Tariff and Growth



Short run adjustment with a Owo-Period Growth Club (τ reduction)



Short run adjustment (τ reduction)