# AN EXAMINATION OF SYSTEMATIC RISK IN A SCALED MARKET MODEL: A CONCEPTUAL CRITIQUE OF THE CAPM USING A CLOSED INDEX SYSTEM

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## ABSTRACT

The theoretical assumptions and practical limitations of the CAPM have been subjected to early scrutiny, however, causal relationships embedded in these data, and the index management practices directly affecting the characteristics of "market" models, have been ignored in the empirical literature.

Stock market indexes are effectively "closed systems" comprising relatively few *ex post variables* (i.e. security size and performance). Without invoking the CAPM's theoretical framework, its conjectures about security-market relationships can be scrutinised axiomatically. Two immediate conceptual queries arise. First, since stock market indexes capture the market contribution of individual securities, why is it necessary to rely upon statistical processes to estimate the "market" risk contribution of index constituents? Second, since index performance *ex post* derives from the constituents and a controlled experiment is conducted within a closed market model excluding "exogenous" factors, how can "non-market" risk arise?

This paper analyses index constituent changes occurring within the S&P/ASX50 Index – a leading institutional equity index of Australia's largest and most liquid stocks – between 1994 and 2002. This identifies index turnover as an important source of statistical anomalies not previously documented in the literature. In addition, the efficacy of beta values is scrutinised using a "scaled" market model not afflicted by constituent turnover: this permits direct evaluation of security-market relationships in accordance with Markowitz's original portfolio analysis approach. It is argued that the CAPM's theoretical specification of "systematic" risk is mistaken and that security betas are unreliable measures for their stated purpose.

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## 1. Introduction

Markowitz's (1952, 1959) pioneering mean-variance optimisation model transformed previously-accepted notions of judgement and diversification within the "portfolio selection" problem: it provided a scientific decision-making process for investment which contrasted with the inherently subjective/arbitrary approach of the intrinsic value paradigm which had become commercially indefensible early in the twentieth century (Gold, 2007).

Subsequent positive economic applications of Markowitz's normative equilibrium model – exemplified by Sharpe's CAPM (1963, 1964, 1970) – employed stock market indexes as proxies of mean-variance efficient "market" portfolios. Importantly, these models have not provided any theoretical advancement *per se* on the normative portfolio theory precepts advanced by Markowitz: instead, they have enabled a drastic simplification of Markowitz's pairs optimisation method, offering scholars and practitioners alike, the alluring promise of predictive capabilities for pricing risk in investment and corporate finance applications.

The CAPM has been invalidated scientifically according to its innate methodological benchmarks: even early empirical testing of a single-index factor model found that it was unable to provide reliable *ex ante* security forecasts (Black, Jensen and Scholes, 1972). Despite continuing intellectual skirmishes about its scientific validity, the CAPM endures as a principal exemplar of investment theory and modern portfolio theory precepts have been codified in legislation governing pension funds in major Anglo-Saxon jurisdictions such as Australia, the UK, and the US (Ali, Stapledon and Gold, 2003; Bines and Thel, 2004).

Although the CAPM's theoretical assumptions and practical limitations have been subjected to early scrutiny (e.g. Roll, 1977), the empirical literature has ignored the causal relationships embedded in these data and index management practices which directly affect the performance characteristics of market models. Stock market indexes are effectively "closed systems" which comprise relatively few *ex post* variables (i.e. security size and performance). Accordingly, without invoking the CAPM's theoretical framework, its conjectures about security-market relationships can be scrutinised axiomatically. Two immediate conceptual queries therefore arise. First, since the computation of stock market indexes captures the contribution of individual securities, the "systematic" return/risk contribution of index constituents can be measured without econometric techniques. Second, since the *ex post* performance characteristics of an index are derived from the constituents, how can "non-market" risk arise (literally, there are no exogenous factors)?

The first part of this paper provides a brief overview of the extant literature which traces the efforts to verify and refine the CAPM. Recognising that day-to-day management of indexes can create truncations between index constituents (and by extension, all securities) and the "market", the second part provides an empirical study which examines the performance effects resulting from index turnover. This analysis uses a unique dataset which captures the index constituent changes which occurred between January 1994 to June 2002 within the S&P/ASX50 Index (a leading Australian institutional equity index). Index turnover is revealed as an important source of statistical anomalies has not previously documented in the empirical literature. Further, it is argued that these anomalies have been mistakenly accorded deterministic meaning under the CAPM's theoretical framework (namely, supporting the conjecture that a dichotomy between "systematic" and "idiosyncratic" risk actually exists).

In light of this evidence, the third part uses a scaled market model – the Closed Market Index (CMI) – which unlike "branded" stock market indexes, is not afflicted by constituent turnover. This permits scrutiny of security-market relationships in accordance with the Markowitz's portfolio analysis approach (which considers the *portfolio* rather than *market* risk contribution of each security). With cognizance of the *actual* performance contributions from each of the CMI constituents, an examination of CAPM's "beta factors" is possible according to two separate perspectives. First, security betas are estimated using value and equally weighted market reference indexes and compared over the analysis period. Second, a comparison is made between the actual risk contribution from each of the CMI constituents and their "standard" security betas (estimated using value-weighted indexes). The results from this scaled model suggest that the CAPM's theoretical specification of systematic risk is mistaken and it is argued that security betas are unreliable measures for their stated purpose. The final part of the paper summarises the main implications arising from these findings and suggests avenues for future research.

## 2. Background and previous literature

### *2.1 Methodological considerations*

Friedman's (1953) persuasive methodological argument that a theory should not be invalidated by virtue of the unreasonableness of its assumptions, has meant that efforts to verify the CAPM's predictive ability are predicated upon underlying assumptions about the performance characteristics of financial markets which are themselves of dubious validity. Further, his assertion that the existence of "disturbing influences" (Friedman, 1953: 10, 18) in real world financial markets would, *ipso facto*, undermine a model's capacity to reliably explain performance outcomes, has lent credence to a view that controlled testing of positive economic models is impossible. However, it is important to note that in *ex post* terms, the CAPM *should* be internally consistent: i.e. empirical analyses of index models should explain the bulk of security-market relationships they engender, and they *can* be tested independently of theoretical assumptions.

Subject to these limitations, therefore, a condensed, closed market model, which simplifies relationships and mitigates against extraneous factors (which may interfere with the essential task of examining security-market relationships) is efficacious for developing inductive generalisations about the CAPM.

Friedman (1953) argued that the testing of a substantive theory or hypotheses need not occur within an overly simplified or abstract model of reality which would represent a 'retreat into a purely formal or tautological analysis...' (p. 11): he contended that positive science should reveal '...*meaningful* (i.e., not truistic) predictions about phenomena before they are observed' (p. 7, emphasis added). With this methodological justification stated, the remainder of this section provides a truncated review of the literature[1] which traces the development and implications of market models generally, and more specifically, the pivotal issues of beta estimation and the CAPM's risk dichotomy.

*2.2 Market models and early empirical testing*

The development of market models in the 1960's (Sharpe, 1963, 1964; Lintner, 1965; Mossin, 1966) greatly simplified the mean-variance optimisation process originally propounded by Markowitz (1952, 1959). Rather than employing an analytically cumbersome process of calculating covariances via a "pairs" analysis of relationships between pairs of securities, these models measured covariance of security returns relative to a single factor: the broad market. Thus, the significant analytical advancement afforded by the single-period mean-variance portfolio model was a drastic reduction in the number of inputs and computing time compared with the Markowitz method, which at that time, was practically infeasible for larger portfolios.

Sharpe (1963, 1964), Lintner (1965), Mossin (1966) and Treynor (1965) posited a single factor model using the S&P500 Index as a proxy of broad market returns as being a sufficient model of covariance, despite the fact that this index did not include the returns from dividends and re-invested income (i.e. it was a price – rather than total return – index). As Black, Jensen and Scholes (1972) have noted, despite the widespread attention these models received in the academic literature, initial attempts to verify the model empirically were limited to

[1] For a more comprehensive review of the literature, see Gold (2007).

performance evaluation systems for mutual funds, rather then direct tests or improvements to the specification of the CAPM.

Subsequent empirical testing of the single index model by Black, Jensen and Scholes (1972) studying portfolios rather than individual stocks, used a more detailed two-factor CAPM and provided affirmative evidence of a positive linear relationship existing between beta values and average rates of return. Fama and MacBeth (1973) expanded the security market line to include the square of the beta coefficient using 20 portfolios between January 1935 and June 1968. Ball, Brown and Officer (1976) analysed an equally-weighted average of 651 Australian industrial stocks over the period 1958-1970 and concluded that a linear relationship existed between the expected rates of return for these stocks and the market aggregate. Subsequent testing of more convoluted models, however, found that the predictive capacity of single index models was no worse than the Markowitz model, and no better than multi-index models (Elton and Gruber, 1973).

Despite the emergence of apparently affirmative empirical evidence, the CAPM's predictive capacity (and security betas as valid measures of a firm's systematic risk) soon came under renewed attack. Roll (1977) criticised index models conceptually as an over-simplification: he argued that the opportunity set represented by stock market indexes did not include the full universe all risky assets available to investors. Accordingly, citing the incompleteness (or misspecification) of the available market benchmarks, he concluded that the CAPM was not testable:

> The two-parameter asset pricing theory is testable in principle; but arguments are given here that: (a) no correct or unambiguous test of the theory has appeared in the literature, and (b) there is practically no possibility that such a test can be accomplished in the future (Roll, 1977: 129-30).

Further, Roll noted that the CAPM relied upon an assumption of market mean-variance efficiency and the presumption that the only relevant criteria for investors' decision making were expected returns and the dispersion of those returns. The questionable nature of these features has been confirmed by subsequent empirical studies which have found varying levels of mean-variance efficiency in both offshore and Australian equity markets (e.g. Grinold, 1992; Wood, 1991).

Other studies have focused on the pivotal role of market indexes within the CAPM (Frankfurter, Phillips and Seagle, 1976; Frankfurter and Phillips, 1979). Frankfurter, Phillips and Seagle (1976) constructed different market indexes with narrow and broad constituent populations, and used arithmetic and geometric rates of returns (and including dividends and adjustments for corporate actions excluded from Sharpe's earlier analysis). They concluded: '…[e]ven the *a priori* assumed best [value weighted] index explains only 53.2% of variation…and this puts to test the reality of the "common factor" assumption and supports arguments of misspecification already entertained in the literature' (p. 952).

Noting that early studies used arithmetic averages of returns, Jensen (1972) acknowledged that '…the correct definition of these returns is the continuously compounded rate…' (p. 33) but concluded '…it is evident that the data does not fit the model. Every coefficient is significantly different from its theoretical value' (p. 36). More recently, Fitzherbert (2001) demonstrated that the use of [more appropriate] geometric – rather than arithmetic calculation – produced significantly different mean rates of return.

In the CAPM, residual variance (denoted by $\varepsilon_i$) is commonly labelled "idiosyncratic risk", "unsystematic risk" and "extra-market" covariance, and is considered to be unrewarded and is diversifiable. This variance is the statistical by-product of the regression between the returns of a security and those of the reference index. Rosenberg (1974) used various micro-economic risk factors including industry, growth orientation, firm size, and financial risk. Elton, Gruber and Padberg (1979) also constructed multi-factor models in attempts to more accurately attribute residual variances according to extra-market influences on the assumption that residual risk was company-specific risk which was '…not uncorrelated but rather can be explained by industry influences or other broad economic influences' (p. 7).

In direct contrast to conventional assumptions that residual risk was irrelevant, more recent research has assigned it a deterministic role. Malkiel and Xu (1997, 2000) have introduced controversial evidence suggesting that idiosyncratic risk was associated with firm size and plays a significant role in explaining the variation of returns. Further, Xu and Malkiel (2003) have attributed increasing levels of idiosyncratic risk amongst stocks to the growing participation in financial markets (stock ownership levels and trading volumes) of institutional investors: the authors have argued that these participants receive their research signals simultaneously and react in a more organised manner (compared with personal investors) resulting in increased price responsiveness.

*2.3 Beta estimation techniques and efforts to produce "better" betas*

Despite on-going queries regarding the CAPM's validity, a large body of research literature has focused on the process of beta estimation. Early research efforts confirmed that adjusted betas offered greater predictive power than purely historical (or "raw") regression betas (Blume, 1971; Vasicek, 1973). More recently, Rosenberg (1985) noted that historical (or "raw") beta values are not "true" betas because they only measure the relationship between a stock and the broad market over a specific measurement window.

The hypothesis that trading liquidity could impact on the frequency and speed of stock price adjustments and the underlying assets, with general result that standard OLS regression beta estimates would tend to be downward biased for thinly traded stocks and upward biased for frequently traded stocks, led to the extension of the OLS estimation methods (Scholes and Williams, 1977; Dimson, 1979; Hawawami, 1983).

*2.4. Portfolio management applications*

In terms of security selection, the CAPM's primary conjecture is that in equilibrium, a positive linear relationship is expected to exist between risk and return for risky assets, and the slope of the security market line (the "beta factor" for individual assets) which denotes the expected return that can be obtained if extra risk is accepted, can alternatively be viewed as the price of risk reduction (Sharpe, 1970: 84)).

In relation to diversification, a number of empirical studies (Evans and Archer, 1968; Statman, 1987; de Vassal, 2001) have considered how many stocks are required to achieve "efficient" portfolio diversification assuming that residual risk is uncorrelated and irrelevant. These studies have offered contradictory findings. Whereas Evans and Archer (1968) stated their results '[r]aise doubts concerning the economic justification of increasing portfolio sizes beyond 10 or so securities…' (p. 767), Statman (1987) concluded that '…a well-diversified stock portfolio must include, at the very least, 30 stocks for a borrowing investor…' (p. 362).

**2.3 Index constituent turnover as a source of residual errors**

This part addresses the second conceptual query posited in the introduction: how can residual risk (as specified in the CAPM) arise within the confines of a market index, which, in effect, is a closed system? Logically, this phenomenon *must* arise from unidentified "leakage" within the market model. Market indexes are a relatively simple mathematical instrument: the performance of an index is calculated using constituent weightings and their returns. The *actual* market risk contribution of index constituents can therefore be directly attributed to these inputs when evaluated exclusively in *ex post* terms. Therefore, according to the CAPM's theoretical prescription, beta values *should* explain *most* – if not *all* – of each index constituent's systematic risk, assuming that a continuous relationship exists between the index constituent and the market index.[2]

[2] This causal relationship is a restrictive condition which is relied upon in this part: all constituents are assumed to be continuous members of the "market" which itself is assumed to have comprised a stable population over the analysis period.

Since security returns are not changed retrospectively in published stock market indexes, it can be deduced that discrepancies arising from the econometric methods which estimated security betas (the index constituent-index relationship) *must* be attributable to index turnover (i.e. additions and deletions to the reference index) and changes in the constituents' index *weighting* which have created discontinuities/truncations in security-market relationships.

Within the extant literature, the "intra-model" effects of index constituent turnover have not explicitly taken into consideration, even though this phenomenon directly impacts upon index constituent-index relationships, and thus, exacerbates errors arising from statistical regressions and associated descriptive data.

*2.3.1 A description of index events and resultant constituent turnover*

By way of background, an explanation of index management events is required. Index publishers which aim to measure the performance characteristics of financial markets are subject to the sometimes competing objectives of representation, investability, and minimization of constituent turnover. Changes to index constituents, and to a lesser degree, alterations to the index weightings of constituents, are discouraged because they increase transactions costs for index funds which are managed mechanistically using replication and partial replication techniques. Consistently high levels of turnover negate the principal economic rationale of indexing strategies as a low-cost, "unmanaged" passive investment. Indexes, therefore, even where they are carefully managed to minimize turnover, are not the "buy-and-hold" investment strategies which would be familiar to many investors. Although index-tracking strategies are typically compelled to alter portfolio holdings in accordance with changes announced by index publishers, stocks may be removed from indexes but continue to trade on the market. Also, stocks may be de-listed from the market but retain economic value.

While index publishers may alter both the constituents of an index, and their respective weightings, it is important to not that these changes may be made independently of security returns. Table 1 summarises the two main categories of index events and the types of modelling errors which they can potentially create. The most dramatic changes to indexes occur from constituent substitutions; however, more subtle effects flow from weighting changes which are associated with free-float assessments, corporate actions, and *ad hoc* changes to index policy which are made at the publisher's prerogative.

**Table 1 – Index events and error types**

| Index event/ change type | Specific description | Modelling impact |
|---|---|---|
| Constituent | Addition<br>Deletion | Intra-model |
| Weighting | Up weighting<br>Down weighting | Relative performance |

Because beta estimation typically relies upon a regression of security returns and the reference index, *all* index events are potentially sources of statistical artefacts. However, it is the performance effects stemming from the first category (index constituent turnover) which are the most important to identify, because, according to industry convention, market indexes are not back-filled retrospectively include the historical performance of the new constituents, nor are the historical performance contributions of deleted constituents retrospectively removed from an index.

This methodology also introduces the potential for series of performance effects which impact upon security betas and other statistics which are generated from comparisons between securities and the continuing "market". Therefore, even if a firm is a continuous constituent of the market over an analysis period, intra-model changes will degrade the actual security-index relationships and introduce spurious statistical artefacts if explicit adjustments are not made index constituent change.

The second category of index turnover occurs from constituent weighting changes. These changes do not create errors *within* the model itself because the performance effects of weighting changes are simultaneously reflected in the index contribution of constituents (in the calculation of the index return). However, this category of index turnover is critically important where relative performance comparisons are made between investor portfolios and a "market" benchmark subject to continual change: without adjustment for the performance effects from weighting changes, there is potential for spurious results in performance measurement.

### *2.3.2 Quantifying index constituent turnover in a real market index*

The following section provides a quantification of the performance impact wrought by index constituent changes and the resultant data truncations occurring within S&P/ASX50 Index (a representative basket of Australia's largest and most liquid stocks) between 1 January 1994 and 30 June 2002 (the analysis period). Although this index comprises a relatively few stocks by number, it describes 72.9 per cent of the Australian stock market's total capitalisation, and thus, it can be considered as being representative. In addition, this index is particularly relevant because it covers only the largest and most liquid stocks in the Australian equity market and allays concerns about non-synchronous trading, and its performance characteristics of this index are highly correlated with other broader indexes.

#### *2.3.2.1 Data and methodology*

The data for used for this study were historical month-end snapshots of index constituents and returns of the S&P/ASX50 Index extracted from the Australian Securities Exchange's (ASX) historical database. Constituent changes within this index were identified by reviewing the index constituents in each month of the analysis period. The *actual* (not theoretical) total return (i.e. capital appreciation and income) contributions for each constituent to the index were isolated in each discrete month of the analysis period.

In contrast to approach outlined by Malkiel (2001) (that survivorship can be approximated by comparing constituent populations at the start and end of analysis periods) this study measures all stocks which have entered and exited the index throughout the analysis period. In order to determine an index of the "underlying" market return (rather than the "published" ASX50 Index whose returns are distorted by index management processes) the performance contributions of index constituent additions (deletions) were deleted from (added back to) the published index return. Table 2 reveals the annual performance effects of index turnover throughout the analysis period, and shows how the pro forma (or underlying) market return is constructed from the data.

**Table 2: Performance effects of constituent turnover in the S&P/ASX50 Index**

| Year ended 30 June* | S&P/ ASX50 Index return | Pro-forma "market" return | Performance impact of index turnover <…………………………………………..> | | | Cumulative effect of turnover |
|---|---|---|---|---|---|---|
| | | | *Deletes* | *Adds* | *Net* | |
| 1994 | -7.2% | -3.1% | -4.1% | 0.0% | -4.1% | -4.1% |
| 1995 | 9.2% | 9.3% | -0.3% | 0.2% | -0.2% | -4.3% |
| 1996 | 11.8% | 8.7% | 2.5% | 0.7% | 3.2% | -1.1% |
| 1997 | 28.3% | 22.9% | 4.1% | 1.4% | 5.5% | 4.3% |
| 1998 | 7.1% | 6.8% | -1.9% | 2.2% | 0.3% | 4.7% |
| 1999 | 14.9% | 7.9% | 0.2% | 6.8% | 6.9% | 11.6% |
| 2000 | 18.9% | 15.5% | 3.9% | -0.5% | 3.4% | 15.0% |
| 2001 | 10.2% | 8.9% | -1.2% | 2.5% | 1.3% | 16.4% |
| 2002 | -6.8% | -4.7% | 0.3% | -2.5% | -2.1% | 14.2% |
| | | | | | | |
| Total return | 118.5% | 95.5% | | | | |
| Annualised return | 14.8% | 11.9% | | | | |
| Differential | 2.9% | | | | | |

Notes: * data availability limited by the database: performance figures for 1994 incorporates six months (1 January 1994 to 30 June 1994)
Sources: ASX/Sirca ;IRESS

The magnitude of performance effects from index events is highly changeable over the analysis period. Figure 1 provides a more detailed view of these effects: this shows the pattern of differential performance effects as they have emerged in each quarter throughout the analysis period. Figure 2 uses data from each year to illustrate the index turnover-related performance discrepancies arising between the published and underlying indexes throughout the analysis period.

**Figure 1: The performance effects of index turnover in the ASX50 Index**

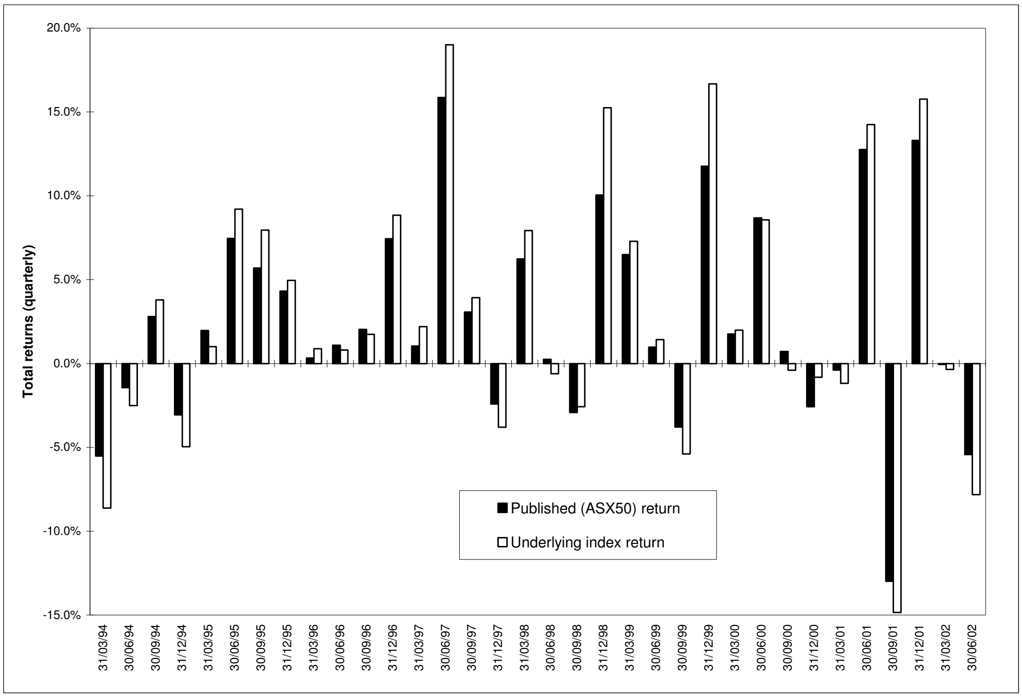
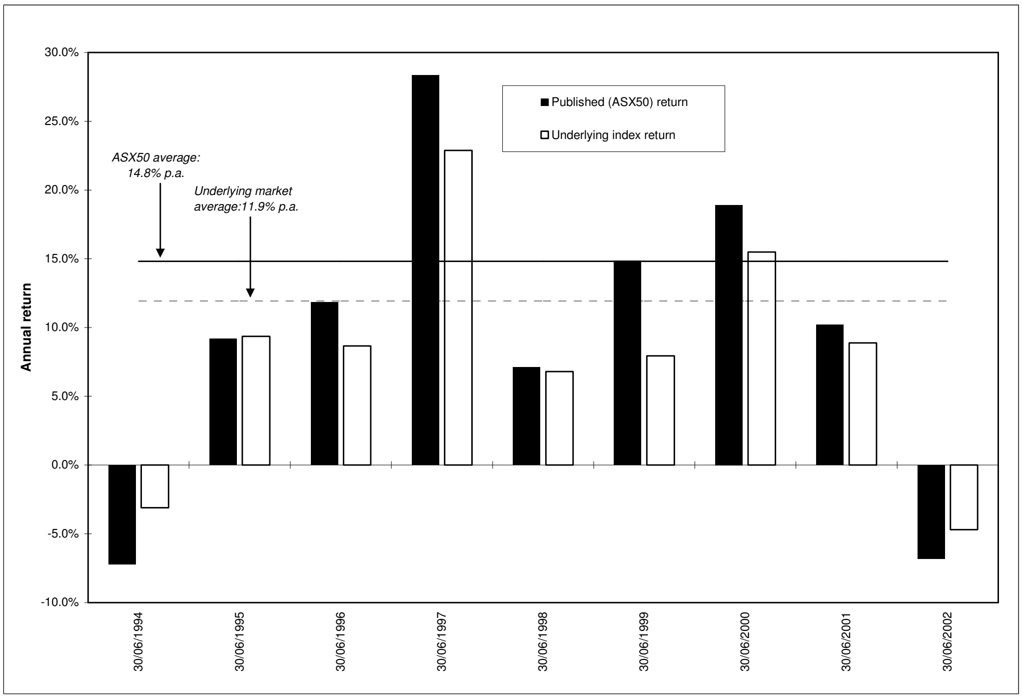



**Figure 2: Pro forma and published index performance**




In summary, it is observed that the average performance impact of index turnover was approximately 3 per cent per annum (equivalent to approximately 20 per cent of the total return from the published index) over the analysis period. If the conventional "forward-looking" calculation basis of the market index was adjusted to take into account this phenomenon, index reconstitutions (and the associated performance legacies of constituents) *per se* artificially and materially boosted the performance of the "continuing market". Prima facie, whilst these effects appear relatively modest – especially in the context of long-run equity market returns – the significant issue is that the *magnitude* of these performance impacts was both highly material and volatile relative to the published index returns. Inspection of these results – albeit over the relatively limited time frame of this study – indicates that without explicit adjustment for the performance effects of index turnover require the econometric methods would introduce spurious associations between securities (whether or not they were index constituents) and the "market".

**2.4 An examination of systematic risk in a scaled market model**

Without invoking the CAPM – but with cognisance of the performance effects of intra-model changes introduced within a real world index – the preceding analysis illustrated the importance of quarantining the effects of index turnover for econometric applications. The purpose of this part is to address the first conceptual query outlined in the introduction: to consider the usefulness of the CAPM's security betas as valid descriptor of constituent-index return relationships – namely "systematic risk" – from a purely *ex post* context.

This part therefore uses a scaled market model comprising a fixed basket of ten large capitalisation stocks which were continuously listed on the Australian stock market index. This model uses an extended longitudinal study which allows for "controlled testing" of the theoretical specification of systematic risk. It deliberately captures direct security-market index relationships without the intrusions of the "disturbing influences" created by index turnover, to enable a direct scrutiny of beta values in this market system.

Security betas are estimated using both equally-weighted and value-weighted reference indexes; the former is considered to be a more appropriate benchmark because it captures the *ex post* characteristics of *individual* securities.[3]

*2.4.1 Data and methodology*

Because a security's beta is determined by its index weighting and its returns, *ceteris paribus*, a security with a larger index weighting *should* have a larger impact upon the market's overall risk because the market's beta is a weighted sum of individual stock betas. To evaluate security-market relationships without introducing the econometric limitations caused by index turnover (as discussed in the previous part) a model which enables direct scrutiny each security's actual index weighting and performance contributions over the long-term is necessary. We construct a Constant Market Index (CMI) comprising a fixed basket of 10 large stocks and its performance history extends longitudinally across 22 years providing 264 monthly observations between 1 January 1981 and 31 December 2002.

The constituents for the model were selected from S&P/ASX50 Index constituents existing at both the start and end of the analysis period. Accordingly, this model captures a basket of large stocks held by investors in Australia throughout an extended period of more than two decades. No adjustments are made to individual security sizes or industry exposures (i.e. the model is computed "as is" to capture the historical experience of real world investors over this period). The combined value of the CMI constituents over the full analysis period represented a minimum of 18.5 per cent, maximum of 39.5 per cent and mean of 30.0 per cent, respectively, of the total market value of all stocks listed on the ASX (including stocks of foreign corporations not included in the published indexes) (RBA, 2006). Moreover, these constituents represented a minimum of 45.0 per cent of the ASX50 Index's total market capitalisation over this period (RBA, 2006).

[3] Markowitz's (1952) mean-variance optimisation procedure did not explicitly consider stock *size* (i.e. the market capitalisation) as a deterministic factor in the mean-variance portfolio optimisation process. Rather, it was assumed that rational investors were expected to combine securities within portfolios according to their respective *investment* potentialities when assembling efficient portfolios.

Total security returns were obtained (including all income redistributions) in each month from Datastream. Two weighting schemes were used to construct reference indexes: a value-weighted Constant Market Index (CMI-VW) and an equally weighted Constant Market Index (CMI-EW). The constituents of the CMI-VW were weighted according to their market values existing at the commencement of the analysis period, and all subsequent corporate actions/redistributions/recapitalisations were incorporated into the market capitalisation data (via the Datastream data-type "MV"). The CMI-EW used the same security returns data as the CMI-VW; however, its returns were computed by summing the total of security returns in each month and dividing them by a fixed denominator of 10 (providing an equal proportionate index weighting for each constituent of 10 per cent).

The longitudinal data was separated into four sub-periods of 5.5 years (1 January 1994 to 30 June 1998 and 1 July 1998 to 30 June 2002). Security betas were estimated from a standard OLS regression procedure for each of the sub-periods and for the total analysis period. Table 3 shows the beta values estimated using the two reference indexes in the four discrete analysis periods between 1 January 1981 and December 2002.

**Table 3: Comparative betas for constituents of the CMI-VW model**

| Period | Reference Index | ANZ | BHP | BIL | CML | FGL | NAB | NCP | RIO | WBC | WMC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/1/81 to 30/6/86 | VW | 0.745 | *1.230* | 0.605 | 0.612 | 0.664 | 0.569 | 1.183 | *1.020* | 0.699 | 1.186 |
| | EW | *0.906* | 1.123 | *0.833* | *0.784* | *0.830* | *0.823* | *1.571* | 1.016 | *0.885* | *1.228* |
| 1/7/86 to 31/12/91 | VW | 0.794 | *0.810* | *0.783* | *0.881* | *0.988* | 0.710 | 1.517 | *1.370* | *0.906* | *1.353* |
| | EW | *0.809* | 0.710 | 0.758 | 0.865 | 0.982 | *0.721* | *1.545* | 1.359 | 0.904 | 1.346 |
| 1/1/92 to 30/6/97 | VW | 1.011 | *1.229* | 0.815 | 0.519 | 0.828 | *0.804* | 0.869 | 0.974 | 1.105 | 1.076 |
| | EW | *1.172* | 1.158 | *0.964* | *0.686* | *0.935* | 0.802 | *0.876* | *0.984* | *1.235* | *1.187* |
| 1/7/97 to 31/12/02 | VW | 0.924 | 1.084 | 0.555 | 0.354 | 0.300 | 1.048 | *1.536* | 1.093 | 0.729 | 1.219 |
| | EW | *1.044* | *1.174* | *0.809* | *0.531* | *0.373* | *1.052* | 1.327 | *1.376* | *0.785* | *1.527* |
| Full period | VW | 0.823 | *1.039* | 0.699 | 0.673 | 0.756 | 0.728 | 1.335 | 1.167 | 0.834 | 1.248 |
| | EW | *0.908* | 0.947 | *0.818* | *0.779* | *0.849* | *0.804* | *1.448* | *1.214* | *0.916* | *1.316* |

Notes: Italicised values indicate higher beta factors for the respective stocks in the different analysis periods.
Source: Datastream

There are two principal observations which can be made regarding these data. First, the comparison of security betas estimated using the "Markowitz-appropriate" reference index (the CMI-EW) and "standard" value-weighted (CMI-VW) reference indexes, revealed several significant differences. In most cases, standard betas were consistently lower than their equally-weighted counterparts. The main implication of these discrepancies (according the CAPM's specification that security betas reflect the security's contribution to the *market's* overall risk) is that standard betas systematically understated the risk of those constituents if they were included in an investor's *portfolio* in equal dollar amounts. The second observation is that over the full analysis period, and within the sub-periods, significant convergence occurred between the two types of security betas. This phenomenon can be attributed to the CMI's changing capitalisation structure over the analysis period. As noted above, over the analysis period the capitalisation structure of the CMI (and, the broad Australian stock market), was highly skewed to the largest stocks.

Figure 3 shows the capitalisation structure and table 4 provides four-firm concentration ratios and Herfindahl indexes (a measure of concentration which gives more weighting to the largest stocks) of the CMI-VW: this market system experienced moderate to high levels of concentration, and the changing relative performance of constituents (and thus index weightings) resulted in transition amongst the larger index constituents, resulting in significant convergences between the EW and VW betas.

**Figure 3: Capitalisation structure of the CMI-VW model**

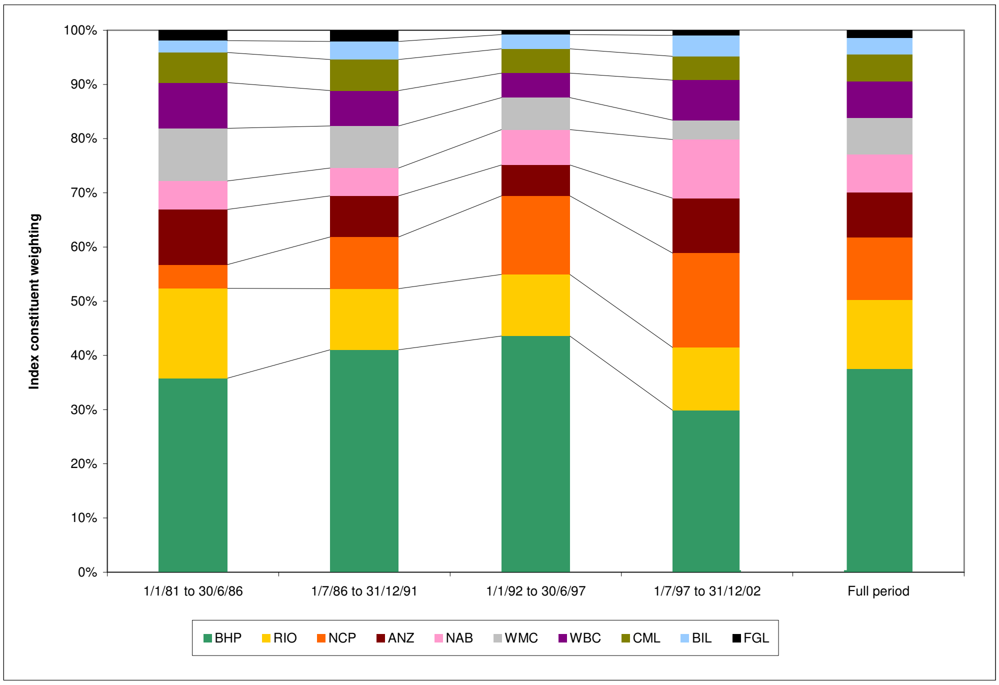



**Table 4: Market concentration characteristics of the CMI-VW model**

| Security | 1/1/81 to 30/6/86 | 1/7/86 to 31/12/91 | 1/1/92 to 30/6/97 | 1/7/97 to 31/12/02 | Full period |
|---|---|---|---|---|---|
| BHP | 35.8% | 41.1% | 43.6% | 29.8% | 37.5% |
| RIO | 16.6% | 11.3% | 11.4% | 11.6% | 12.7% |
| NCP | 4.4% | 9.5% | 14.5% | 17.4% | 11.5% |
| ANZ | 10.2% | 7.6% | 5.7% | 10.1% | 8.4% |
| NAB | 5.3% | 5.1% | 6.5% | 10.9% | 6.9% |
| WMC | 9.7% | 7.8% | 5.9% | 3.6% | 6.7% |
| WBC | 8.4% | 6.5% | 4.5% | 7.4% | 6.7% |
| CML | 5.6% | 5.7% | 4.5% | 4.4% | 5.0% |
| BIL | 2.2% | 3.3% | 2.6% | 3.9% | 3.0% |
| FGL | 1.9% | 2.1% | 0.8% | 1.0% | 1.5% |
| Total | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
| *Ratios:* | | | | | |
| Concentration | 72.3% | 69.6% | 76.0% | 69.7% | 70.1% |
| Herfindahl index | 0.12 | 0.22 | 0.24 | 0.17 | 0.21 |

Notes: The concentration ratio shown is calculated using the index weights of the four largest constituents. The Herfindahl Index is obtained by summing the squares of the respective index constituent weightings: an index of between 0.10 and 0.18 indicates moderate concentration, and above 0.18 indicates high concentration.
Source: Datastream

These data reveal that the estimation of conventional beta values is affected by individual stock capitalisation, levels of concentration in the market overall, and the changes occurring in the market capitalisation structure. Even though the CMI experienced moderate-high concentration levels, closer attention to the changes in market prominence (and thus performance) of individual stocks would significantly affect sizes, and thus, beta values of the market constituents.

A remaining aspect which needs to be addressed is whether CAPM beta values are appropriate proxies of "systematic" risk. This requires an evaluation of the *actual* proportionate market risk contribution of the CMI constituents. Figure 4 juxtaposes standard security betas and the actual proportionate market risk contribution of the CMI constituents (table 5 shows the underlying data used to construct this figure).

**Figure 4: A comparison of actual market risk contributions and security betas**

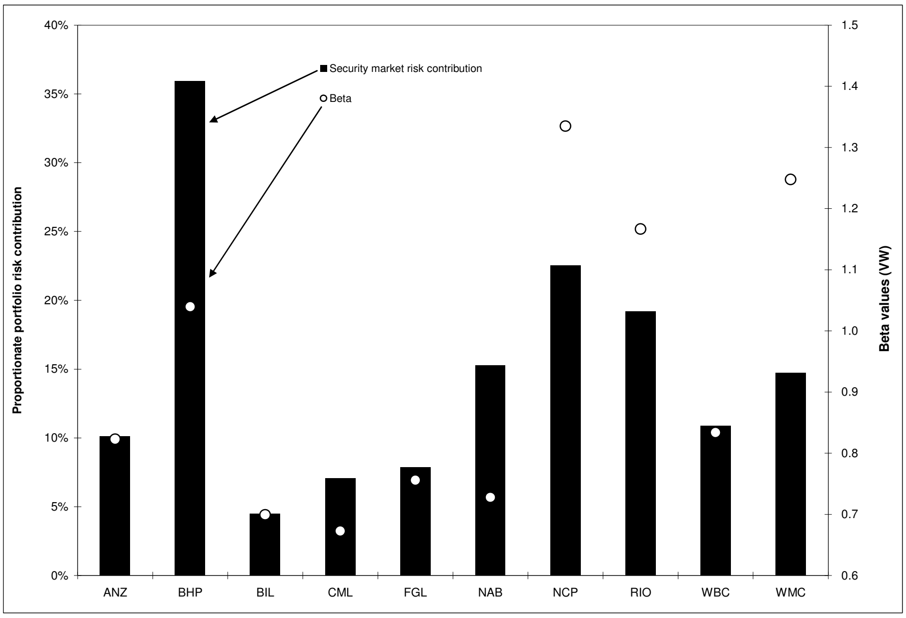


**Table 5: Market risk contributions of the CMI-VW model constituents**

| Stock | 1/1/81 to 30/6/86 | 1/7/86 to 31/12/91 | 1/1/92 to 30/6/97 | 1/7/97 to 31/12/02 | Market risk contribution | Beta |
|---|---|---|---|---|---|---|
| BHP | 46.1% | 27.2% | 39.0% | 27.3% | 35.9% | 1.04 |
| NCP | 8.2% | 19.1% | 23.4% | 42.0% | 22.5% | 1.33 |
| RIO | 23.7% | 18.3% | 13.0% | 14.5% | 19.2% | 1.17 |
| NAB | 7.4% | 7.7% | 17.3% | 31.6% | 15.3% | 0.73 |
| WMC | 15.6% | 16.4% | 11.8% | 8.9% | 14.7% | 1.25 |
| WBC | 10.7% | 9.4% | 12.7% | 13.1% | 10.9% | 0.83 |
| ANZ | 10.1% | 7.5% | 10.3% | 14.9% | 10.1% | 0.82 |
| FGL | 3.1% | 11.2% | 6.8% | 4.4% | 7.9% | 0.76 |
| CML | 5.9% | 8.0% | 7.5% | 6.0% | 7.1% | 0.67 |
| BIL | 2.4% | 3.8% | 5.2% | 7.7% | 4.5% | 0.70 |
| CMI-VW | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 1.00 |

Source: Datastream

On casual inspection, figure 4 shows several *coincidences* between the actual market risk contributions and security beta factors. In actuality, the CAPM is fundamentally (and formally) a tautology: tellingly, therefore, this figure reveals that econometric estimation techniques used for the CAPM actually misrepresent systematic risk (a mathematical analysis is provided in the appendix). Referring to the methodology of positive economics, CAPM security betas are not useful scientific measures because they do not express anything which cannot be observed instantaneously (and more accurately) from the index data.

### 5.5 Conclusions and avenues for further research

This paper has examined the CAPM from two related aspects. First, by acknowledging that index performance, when measured in *ex post* terms, *should* be directly attributable to the performance and index weightings of its constituents, it documented index turnover as a source of idiosyncratic risk (in addition to other statistical phenomena which has been previously documented in the literature). Second, this paper employed a condensed market model to evaluate the *actual* market risk contribution of constituents directly without resorting to econometric estimation methods, thus permitting the efficacy of security betas (as specified by the CAPM) to evaluated.

The examination of index turnover identified intra-model errors which can introduce spurious security-market associations; albeit within a relatively small index basket. It therefore represents a "pilot" study and further research is desirable to quantify these effects within broader market benchmarks (both in Australia and offshore) which, by virtue of their expanded breadth of constituents and/or selective construction rules, are likely to experience higher frequency of constituent turnover. Additional research should be undertaken to quantify the more subtle effects of constituent weighting changes (including free float adjustment procedures) which introduce "permanent" differences between the economic returns of investors' portfolios and market benchmarks. While these errors are not accurately quantified and their causes remain unattributed, they will continue to mistaken as "un-priced" residual risk or "performance anomalies".

As alluded to in the introduction of this paper, the purpose of employing the scaled market of the Australian stock market in this paper was to evaluate axiomatically the efficacy of security betas without directly invoking the CAPM or its assumptions. Security betas were estimated using both value and equally-weighted reference indexes. A comparison of these betas revealed significant discrepancies: those estimated from an equally-weighted market index were typically higher than their value weighted counterparts, suggesting that conventional betas may understate systematic risk of constituents. A corollary of this observation is that those relying upon conventional CAPM security betas for deterministic purposes, may receive inadequate risk premia.

The CMI model was also used to evaluate security betas in accordance with the CAPM's theoretical prescription of systematic risk. A comparison between the *actual* market risk contribution of securities and standard security betas revealed that the latter, at best, only approximate actual historical performance relationships when index turnover and performance momentum/capitalisation features were taken into account. As proxies of market risk, conventional security betas did not describe anything which could not be obtained directly from the actual/historical risk contribution of the stocks.

The evidence in this paper queries the idiosyncratic-market risk dichotomy of the CAPM. Concomitantly, residual variance should be more accurately considered a statistical artefact rather than "security-specific" risk, and accordingly, it is not necessarily diversifiable. This realisation calls into question the previously-accepted notions of "efficient" and "purposeful" diversification (e.g. Statman, 1987). Indeed, diversification relying upon CAPM precepts may result in sub-optimal portfolio decisions and the misallocation of capital amongst the issuers in financial markets.

Finally, these findings have direct implications for portfolio performance evaluation systems: if beta values are not accurate proxies of "systematic" risk, then the selection and remuneration of fund managers using the CAPM (and associated ratios) may be spurious and economically suboptimal.

## Appendix: Mathematical analysis of the CAPM

*(a) The CAPM specification of "systematic" risk as a disguised tautology*

Consider a full Market index $M_t$ based on $n$ stock prices $X_{i,t}$ where $i$ = 1, 2, … , $n$, and $t$ denotes time. The volume of stock $i$ is denoted by $v_i$, and the initial value of the index is denoted by $M_0$, so that

$$M_t = M_0 \frac{\sum_{i=1}^{n} v_i X_{i,t}}{\sum_{i=1}^{n} v_i X_{i,0}}$$

Let $R_{i,t} = (X_{i,t} - X_{i,t-1}) / X_{i,t-1}$ be the rate of return per unit time for the $i$th stock.

Alternatively, the following theory can be formulated in terms of the continuously compounded rate $R_{i,t} = \ln X_{i,t} - \ln X_{i,t-1}$; for short time increments there is very little numerical difference between the two types of rate.

The rate of return for the market index is given by

$$R_{M,t} = \frac{M_t - M_{t-1}}{M_{t-1}} = \frac{\sum_{i=1}^{n} v_i X_{i,t-1} R_{i,t}}{\sum_{i=1}^{n} v_i X_{i,t-1}} = \sum_{i=1}^{n} w_{i,t} R_{i,t}$$

where $w_{j,t} = v_j X_{j,t-1} / \sum_{i=1}^{n} v_i X_{i,t-1}$ is the capitalisation weight of stock $j$ at time $t$ −1, and $\sum_{i=1}^{n} w_{i,t} = 1$.

Let $\mu_{i,t}$ and $\sigma_{i,t}$ denote the conditional mean and conditional standard deviation of $R_{i,t}$ respectively, given publicly available data relating to stock prices up to and including time $t$ – 1. In mathematical finance literature, conditional expectation given past history is often formally expressed as $\mu_{i,t} = E(R_{i,t} \mid \mathcal{F}_{t-1})$. According to the efficient market hypothesis, future returns are inherently unpredictable given the past history of the stock market.

*(b) Mathematical analysis of the CAPM's "market risk" specification assuming constant volatility and zero correlations between returns*

One possibly over-simplistic statistical model which encapsulates the concept of informationally efficient markets is the case when $R_{1,t}$, $R_{2,t}$, …, $R_{n,t}$ are conditionally independent, with constant mean $\mu_{i,t} = \mu$ and volatility $\sigma_{i,t} = \sigma$. The reason for starting with this restrictive model is to emphasize the point that artefacts can arise when estimating betas by fitting regression lines, even in a situation when prices evolve in a completely random manner and all investments are in fact *equally* risky. This scenario, referred to as "Model I", has the following statistical properties:

$$E(R_{M,t} \mid \mathcal{F}_{t-1}) = \mu_{M,t} = \sum_{i=1}^{n} w_{i,t}\mu = \mu$$

$$Var(R_{M,t} \mid \mathcal{F}_{t-1}) = \sigma^2_{M,t} = \sigma^2 \sum_{i=1}^{n} w^2_{i,t}$$

$$Cov(R_{j,t}, R_{M,t} \mid \mathcal{F}_{t-1}) = w_{j,t}\sigma^2$$

From standard statistical linear model theory, the theoretical regression equation relating expected individual stock return to market return involves the ratio of the previous covariance and variance terms:

$$\begin{aligned} E(R_{j,t} \mid R_{M,t}, \mathcal{F}_{t-1}) &= E(R_{j,t} \mid \mathcal{F}_{t-1}) + \frac{Cov(R_{j,t}, R_{M,t} \mid \mathcal{F}_{t-1})}{Var(R_{M,t} \mid \mathcal{F}_{t-1})}\left(R_{M,t} - E(R_{M,t} \mid \mathcal{F}_{t-1})\right) \\ &= \mu + \frac{w_{j,t}}{\sum_{i=1}^{n} w^2_{i,t}}(R_{M,t} - \mu) \end{aligned} \quad (*)$$

Note that equation (*) has the same form as the classic CAPM model, where $\mu$ is interpreted as the risk-free rate and $w_{j,t} / \sum_{i=1}^{n} w^2_{i,t}$ is interpreted as the instantaneous beta coefficient of stock $j$ at time $t$.

An empirical analysis of data generated by Model I would lend apparent support to the CAPM, especially if the capitalisation weights of the constituents have not drifted too far from their initial values in time window selected. In other words, a least squares regression analysis would reveal an approximately linear relationship existing between any individual stock return and the market return. The CAPM's security beta estimated retrospectively as the slope of the least squares line, would be shown in hindsight to describe the sensitivity of a particular stock to changes in the market index.

On further reflection, however, this empirical exercise can be seen to have very little practical or academic significance. First, for index constituents there is no need to resort to historical data for estimation of beta given that the capitalisation weights at time *t* are readily available and provide an up-to-date estimate $w_{j,t} / \sum_{i=1}^{n} w^2_{i,t}$, without imposing the assumption of time-invariance within [a suitably chosen] time window. Second, the existence of a regression equation does not imply a causal relationship: non-zero beta coefficients do not necessarily demonstrate that stock prices are being influenced by a common factor.

For Model I, the relationship between the returns of securities and the market exists only because the index is calculated directly from the stock prices. The assumptions of constant volatility and conditional independence between stocks implies that all stocks are equally risky. Apparent differences between betas are due simply to differences in index capitalisation weights, which in turn, are driven by volumes and prices. According to this model, the perception that it is less risky to invest in high volume, high priced stocks is just an illusion.

*(c) Mathematical analysis of the CAPM's "market risk" specification acknowledging individual security volatilities but no correlations between returns*

The fact that it is possible to construct one statistical model which apparently conforms to CAPM from an empirical perspective, yet refutes its theoretical value, could be argued as providing insufficient grounds for dismissal. Accordingly, Model II which is slightly more sophisticated – in which the volatilities $\sigma_{i,t} = \sigma_i$ vary between stocks but not over time, and the means $\mu_{i,t} = \mu + \kappa\sigma_i$ also vary between stocks but are related to volatility – is examined (the constant $\kappa$ represents a risk premium, with positive values indicating risk aversion).

Model II has the following statistical properties.

$$E(R_{M,t} \mid \mathcal{F}_{t-1}) = \mu_{M,t} = \sum_{i=1}^{n} w_{i,t}(\mu + \kappa\sigma_i) = \mu + \kappa \sum_{i=1}^{n} w_{i,t}\sigma_i$$

$$Var(R_{M,t} \mid \mathcal{F}_{t-1}) = \sigma_{M,t} = \sum_{i=1}^{n} w_{i,t}^2 \sigma_i^2$$

$$Cov(R_{j,t}, R_{M,t} \mid \mathcal{F}_{t-1}) = w_{j,t}\sigma_j^2$$

$$E(R_{j,t} \mid R_{M,t}, \mathcal{F}_{t-1}) = \mu + \kappa\sigma_j \left( 1 - \frac{w_{j,t}\sigma_j \sum_{i=1}^{n} w_{i,t}\sigma_i}{\sum_{i=1}^{n} w_{i,t}^2\sigma_i^2} \right) + \frac{w_{j,t}\sigma_j^2}{\sum_{i=1}^{n} w_{i,t}^2\sigma_i^2}(R_{M,t} - \mu) \quad (**)$$

Equation (**) has the form of the extended CAPM model which involves alphas (stock-specific intercepts) as well as betas. More specifically,

$$\beta_{j,t} = \frac{w_{j,t}\sigma_j^2}{\sum_{i=1}^{n} w_{i,t}^2\sigma_i^2}$$

$$\alpha_{j,t} = \kappa\sigma_j \left( 1 - \frac{w_{j,t}\sigma_j \sum_{i=1}^{n} w_{i,t}\sigma_i}{\sum_{i=1}^{n} w_{i,t}^2\sigma_i^2} \right)$$

Once again, an empirical analysis of data generated by Model II would result in an approximately linear relationship between an individual stock and the market return, provided that the time interval over which the analysis is conducted is sufficiently short to ensure that neither capitalisation weights nor volatility change too dramatically within the interval. Again, the estimation of alpha and beta from fitting a least squares line is misguided, as an up-to-date estimate is provided by capitalisation weights and volatilities. However, unlike Model I, Model II requires the estimation of volatilities of individual securities from the historical data. Once again, an empirical verification using regression analysis does not establish that security prices are causally influenced by a common factor. Stocks with higher capitalisation weights have higher betas, but beta is also driven by volatility. Rather than using beta (historic or instantaneous) as a measure of risk, volatility is a more meaningful measure for this model. The capitalisation weights only appear to be relevant because of the way in which the market index is calculated.

*(d) Mathematical analysis of the CAPM's "market risk" specification acknowledging individual security volatilities and non-zero covariances between securities*

Markowitz's portfolio theory is based on the premise of non-zero covariance between anticipated future returns, so it is also of interest to consider Model III with non-independent stock returns. If Model II is generalised to include non-zero conditional covariance $\sigma_{ij}$ between stocks $i$ and $j$, then the resulting expression for $\beta_{j.t}$ becomes

$$\beta_{j,t} = \frac{w_{j,t}\sigma_j^2 + \sum_{k \neq j} w_{k,t}\sigma_{jk}}{\sum_{i=1}^{n} w_{i,t}^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{k \neq i} w_{i,t}w_{k,t}\sigma_{ik}}$$

Empirical analysis of real data sets typically shows that covariances between stocks tend to be small and erratic relative to volatilities calculated over the same time window. Also many stock markets are dominated by a small number of stocks which account for most of the total capitalisation weight. So the extra terms in the numerator and denominator of $\beta_{j,t}$ for Model III relative to Model II will often not make material numerical impact.

The same general comments made for Models I and II continue to hold. In other words, empirical confirmation of the CAPM's regression equation does not prove a causal relationship, and only provides spurious historic estimates rather than instantaneous beta estimates. Although beta estimates are easy to calculate from the available historical data, more meaningful measures of market risk can be obtained directly from the volatilities of individual stocks.

Models II and III both include the precept of higher prices for less risky stocks, which in turn will result in capitalisation weights with some information content about risk. However, this information content will be clouded by many other sources. The calculation and interpretation of betas based on least squares regression lines overemphasises the relevance of capitalisation weights, due to the index construction methodologies which are typically used.

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