Posterior Distributions for Welfare Changes in Agricultural Commodity Markets

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A formal Bayesian-based methodology is presented for evaluating the welfare effects of economic changes in agricultural commodity markets. The procedure is applied to an empirical example, demonstrating how posterior densities may be obtained for estimated welfare changes. These posterior densities provide an intuitive and rigorous method for illustrating the robustness of the results from an applied welfare analysis to the effects of parameter uncertainty. The procedure is used to explore the implications of stochastic error terms in supply and demand curves for the measurement of welfare changes.

Keywords: Bayesian, welfare analysis, sensitivity analysis

1. Introduction

Applied welfare analyses are frequently conducted by agricultural and resource economists in order to examine a wide variety of issues and problems which are relevant to the agricultural and resource sector. For instance, following the example set by Griliches (1958), they have been used extensively to estimate the rates of return to agricultural research and promotional activities. Similarly, Wallace (1962) illustrated how applied welfare analyses may be used in a benefit cost framework to differentiate between alternative policies which have been designed to achieve the same goal. Subsequent to these early examples, many articles have appeared in the literature which are either advances, refinements or applications of the concepts used in these studies.

One such stream of research has been concerned with developing methodologies for incorporating the effects of parameter uncertainty into applied welfare analyses. This work has been motivated by the acknowledgement that estimates of the various measures which may be used in such studies are entirely dependent upon the parameter values use to calculate them. For instance, estimates for the commonly used Marshallian welfare measures of consumer and producer surplus are dependent on the parameters of demand and supply functions. Irrespective of whether these parameter values have been obtained from econometrically estimated structural models, previously published estimates, or guided by economic theory and individual expert opinion, there is uncertainty regarding their true values. Consequently, these estimates, and by extension, the results and conclusions which may be drawn from them, are also subject to uncertainty.

This uncertainty has been incorporated into previous studies in different ways. The most common method is to propose a set of plausible, alternative parameter values, and then use these values compute the corresponding series of welfare changes. Examples can be found in Wallace (1962), Harrison and Vinod (1992) and Mullen, Alston and Wholgenant (1989). This approach provides a quick and simple method for determining whether apparently small changes in parameter values may lead to qualitatively different results or conclusions. This basic methodology may be given a more rigorous footing by specifying subjective probability distributions for the parameters based on the knowledge of experts, and then using simulation techniques to trace out the implied subjective probability distributions for the welfare changes themselves as has been done by Abler, Rodrieguez and Shortle (1999), Davis and Espinoza (1998) and Zhao *et al* (2000).

Where data has been available, other studies have been able to appeal to the sampling theory properties of econometrically obtained parameter values to find estimates for the moments of a particular welfare change estimate. This may be achieved through the use Taylor's series approximations to approximate the variance of an estimated welfare change, which is often a complex nonlinear function of the parameters in the model (Alston and Larson, 1993; Chotikapanich and Griffiths, 1998). Alternatively, bootstrapping techniques may be used to determine the statistical properties associated with an estimate (Kling and Sexton, 1990). Further studies have treated the sampling distributions of the estimated parameters like a posterior distribution, from which draws are simulated to estimate welfare changes (Adamowicz, *et al.*, 1989); Creel and Loomis, 1991).

Bayesian inference provides an alternative methodology for accommodating the effects of parameter uncertainty in applied welfare analyses. It may be considered to be an advance on previous approaches in that it explicitly allows for the incorporation of both sample and non-sample. Consequently, Bayesian inference has been promoted as providing an ideal framework for accommodating the impacts of parameter uncertainty in applied welfare analysis (Zhao *et al.*, 2000); Pannell, 1997). Despite such recommendations however, no such methodology has been thus far been developed which is suitable for investigating the types of examples relevant to the agricultural sector. Hence in this paper we present a formal Bayesian-based methodology for evaluating the welfare effects of economic changes in agricultural commodity markets.

This methodology may also be extended to allow for the effects of other sources of uncertainty. For instance, the proposed methodology is dependent upon the availability of econometrically obtained parameter estimates. Associated with every econometric model is a stochastic error term which may be interpreted as representing the predictive uncertainty of the model. Traditional approaches ignore this source of uncertainty by only considering the deterministic component of an econometric model when calculating estimates of consumer and producer surplus. However, Bockstael and Strand (1987) provided some results illustrating that the results from an applied welfare analysis may be affected by the convention taken regarding the error term.

To help illustrate the various elements of this procedure, a simple empirical example will be used in which the welfare impacts associated with an exogenous demand shift in the domestic demand for Australian lamb are evaluated. In Section 2 we describe a 2-equation dynamic model for the demand and supply of lamb. Because the model is a dynamic one, and because we are considering welfare changes that occur when moving from one equilibrium position to another, we derive expressions for equilibrium price and quantity. In Section 3 we introduce an exogenous permanent shock that increases the demand for lamb and we derive expressions for the welfare changes that result from this increase in demand. In Section 4 we describe how to estimate posterior densities for the welfare changes given in Section 3. The results from an empirical illustration are presented in Section 5 and some concluding remarks given in Section 6.

2. Model

One of the characteristics of the Australian agricultural sector is the various interrelationships which exist between different commodities in either demand or supply. In general this has led to the use of multi-equation modelling frameworks, which explicitly incorporate as many of these relationships as is feasible, when describing agricultural commodity markets, e.g., Vere, Griffith and Jones (2000). When conducting an applied welfare analyses, this approach also has the advantage of providing information on the distribution of welfare effects amongst different groups following an economic change. To help demonstrate the practicality of the proposed Bayesian approach, the econometric model of the Australian lamb industry used in the example to demonstrate this procedure will follow this convention.

The model of the Australian lamb industry in the example consists of two equations linked in a recursive relationship with a third equation imposing a market clearing condition. It is given by

(1) $$P_t^L = \alpha_1 + \alpha_2 Q_{D,t}^L + \alpha_3 Y_t + \alpha_4 Q_t^B + \alpha_5 P_t^C + \varepsilon d_t$$

(2) $$Q_{S,t}^L = \beta_1 + \beta_2 P_{t-1}^L + \beta_3 Q_{S,t-1}^L + \beta_4 DT_{t-1} + \varepsilon s_t$$

(3) $$Q^L_{D,t} = Q^L_{S,t}$$

where

$Q^L_{D,t}$ = Australian per-capita consumption of lamb in kg

P^L_t = The deflated Australian retail price for lamb in c/kg

Q^B_t = Australian per-capita consumption of beef in kg

Y_t = The deflated Australian per capita disposable income in dollars

P^C_t = deflated Australian retail price for chicken in c/kg.

$Q^L_{S,t}$ = Australian per-capita supply of lamb in kg

DT_t = Incidence of drought: A dummy variable denoting a drought in period *t*.

t = index for time

εd_t & εs_t = error terms where $(\varepsilon d_t, \varepsilon s_t)' \sim N(0, \Sigma)$

Given the market clearing condition in (3), in what follows we will not distinguish between quantity demanded and quantity supplied. We will simply write Q^L_t, dropping the *S* and *D* subscripts as well as equation (3).

In equation (2) quantity supplied depends only on previously determined values of price and quantity as well as the drought incidence variable. It can be viewed as a partial adjustment model where quantity supplied cannot adjust fully to desired quantity within one period and where lagged price is a proxy for price expectations. Then, given quantity supplied depends only on predetermined variables, the demand curve can be viewed as one where price adjusts to clear the market. Thus, the demand equation in (1) is written as a price dependent one. Also writing it in this way proves convenient for later estimation.

The inclusion of income in the demand equation is natural, but the presence of the quantity of beef and the price of chicken needs more explanation. An increase in the demand (or supply) of beef will lead to a decrease in the price of lamb. An increase in the price of chicken will lead to increase in the demand for lamb and hence a higher lamb price. While it would have been more consistent to include the quantities of both substitute meats instead of the price of chicken, data on the price of chicken was more readily available, and if one views the equations as a subset of a more complete set of simultaneous equations, there are a variety of ways in which the effects of price-quantity changes in the other markets, treated as exogenous for the purpose of this example, can be represented.

The welfare changes, for which the posterior densities will be obtained, are to be generated by an exogenous shift in the demand for lamb. Given the dynamic nature of the supply equation, the effects of such a shift will be felt for several periods and it will take time to reach a new equilibrium. This observation raises questions about how consumer and producer surplus change in each time period, about the aggregate changes over all periods and about the changes when one goes from equilibrium position to the next. The dynamic welfare changes, how they are defined and estimated, are described in Bialowas (2007) and will be the subject of another paper. At this time we focus on a comparison of consumer and producer surpluses at initial and final equilibrium points.

The first step in this direction is to define equilibrium prices and quantities. To do so we need to set specific values for the exogenous variables and assume that these values are constant at these particular values. Using an over-bar to denote these values, a notation consistent with setting them at the sample means, it is convenient to redefine the intercepts in the model to include the constant exogenous variables. Thus we have

(4) $$\alpha_1^* = \alpha_1 + \alpha_3 \bar{Y} + \alpha_4 \bar{Q}^B + \alpha_5 \bar{P}_C$$

(5) $$\beta_1^* = \beta_1 + \beta_4 \overline{DT}$$

Then, the model can be written as

(6) $$\begin{bmatrix} 1 & -\alpha_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_t^L \\ Q_t^L \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ \beta_2 & \beta_3 \end{bmatrix} \begin{bmatrix} P_{t-1}^L \\ Q_{t-1}^L \end{bmatrix} = \begin{bmatrix} \alpha_1^* \\ \beta_1^* \end{bmatrix} + \begin{bmatrix} \varepsilon d_t \\ \varepsilon s_t \end{bmatrix}$$

Letting

$$A_0 = \begin{bmatrix} 1 & -\alpha_2 \\ 0 & 1 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 0 & 0 \\ \beta_2 & \beta_3 \end{bmatrix}$$

and multiplying through by A_0^{-1}, equation (6) can be rewritten as

(7) $$\begin{bmatrix} P_t^L \\ Q_t^L \end{bmatrix} - A_0^{-1}A_1 \begin{bmatrix} P_{t-1}^L \\ Q_{t-1}^L \end{bmatrix} = A_0^{-1} \begin{bmatrix} \alpha_1^* \\ \beta_1^* \end{bmatrix} + A_0^{-1} \begin{bmatrix} \varepsilon d_t \\ \varepsilon s_t \end{bmatrix}$$

or

(8) $$(I - A_0^{-1}A_1 L) \begin{bmatrix} P_t^L \\ Q_t^L \end{bmatrix} = A_0^{-1} \begin{bmatrix} \alpha_1^* \\ \beta_1^* \end{bmatrix} + A_0^{-1} \begin{bmatrix} \varepsilon d_t \\ \varepsilon s_t \end{bmatrix}$$

where L is the lag operator. Solving for price and quantity yields

(9) $$\begin{bmatrix} P_t^L \\ Q_t^L \end{bmatrix} = (I - A_0^{-1}A_1)^{-1} A_0^{-1} \begin{bmatrix} \alpha_1^* \\ \beta_1^* \end{bmatrix} + (I - A_0^{-1}A_1 L)^{-1} A_0^{-1} \begin{bmatrix} \varepsilon d_t \\ \varepsilon s_t \end{bmatrix}$$

where the lag operator drops out of the first right-hand-side term because (α_1^*, β_1^*) is constant. We use (9) to distinguish between a stochastic equilibrium and a deterministic equilibrium. The deterministic equilibrium is obtained by ignoring the error terms. Thus we have

(10) $$\begin{bmatrix} P_{DIE}^L \\ Q_{DIE}^L \end{bmatrix} = (I - A_0^{-1}A_1)^{-1} A_0^{-1} \begin{bmatrix} \alpha_1^* \\ \beta_1^* \end{bmatrix}$$

where *DIE* refers to deterministic initial equilibrium, the deterministic equilibrium before the shift in demand. Letting

(11) $$\begin{bmatrix} v_{1E} \\ v_{2E} \end{bmatrix} = (I - A_0^{-1}A_1 L)^{-1} A_0^{-1} \begin{bmatrix} \varepsilon d_t \\ \varepsilon s_t \end{bmatrix}$$

be a realization of the equilibrium error terms, the stochastic initial equilibrium is given by

(12) $$\begin{bmatrix} P_{SIE}^L \\ Q_{SIE}^L \end{bmatrix} = \begin{bmatrix} P_{DIE}^L \\ Q_{DIE}^L \end{bmatrix} + \begin{bmatrix} v_{1E} \\ v_{2E} \end{bmatrix}$$

To complete the specification of this stochastic equilibrium, we need the distribution of the errors. Given that $(\varepsilon d_t, \varepsilon s_t)' \sim N(0, \Sigma)$, it can be shown that $(v_{1E}, v_{2E})' \sim N(0, \Sigma_v)$ where, given that conditions necessary for the stability of the model are satisfied, the covariance matrix Σ_v is obtained from

$$\text{vec}(\Sigma_v) = (I - B \otimes B)^{-1}\,\text{vec}(\Sigma) \qquad \text{where} \qquad B = A_0^{-1}A_1$$

See, for example, Lütkepohl (1991, p.21).

To be able to define consumer and producer surplus at equilibrium prices and quantities, it is necessary to define equilibrium demand and supply curves consistent with the equilibrium prices and quantities in equations (10) and (12). Using the subscript E to denote equilibrium, the equilibrium equations are given by

(13) $$P_E^L = \alpha_1^* + \alpha_2 Q_E^L + \varepsilon d_E$$

(14) $$Q_E^L = \delta_1^* + \delta_2 P_E^L + w_E$$

where $\delta_1^* = \beta_1^* /(1-\beta_3)$ and $\delta_2 = \beta_2 /(1-\beta_3)$. The subscript E on the error terms denotes a realized error at equilibrium. The error w_E is a realization from the distribution of $\beta_2(1-\beta_3 L)^{-1}\varepsilon s_t$. The deterministic equilibrium values P_{DIE}^L and Q_{DIE}^L in (10) are given by the simultaneous solution of (13) and (14) with the error terms ignored. The stochastic equilibrium values P_{SIE}^L and Q_{SIE}^L in (12) are given by the simultaneous solution of (13) and (14), with appropriate recognition given to the bivariate distribution for $(\varepsilon d_E, w_E)'$.

We are now in a position to examine the welfare changes that occur when going from an initial equilibrium point to a final equilibrium point following a shift in the demand curve.

3. Welfare effects

The shift in the demand function is represented by a change in the intercept of equation (2) by an amount equal to k ($k > 0$) units. Thus, the new demand curve becomes

(15) $$P_t^L = \alpha_1 + k + \alpha_2 Q_{D,t}^L + \alpha_3 Y_t + \alpha_4 Q_t^B + \alpha_5 P_t^C + \varepsilon d_t$$

The resulting new deterministic and stochastic equilibrium values, subscripted as *DFE* and *SFE* to denote a final equilibrium, are given by

(16) $$\begin{bmatrix} P_{DFE}^L \\ Q_{DFE}^L \end{bmatrix} = (I - A_0^{-1}A_1)^{-1} A_0^{-1} \begin{bmatrix} \alpha_1^* + k \\ \beta_1^* \end{bmatrix}$$

and

(17) $$\begin{bmatrix} P^L_{SFE} \\ Q^L_{SFE} \end{bmatrix} = \begin{bmatrix} P^L_{DFE} \\ Q^L_{DFE} \end{bmatrix} + \begin{bmatrix} v_{1E} \\ v_{2E} \end{bmatrix}$$

The errors in equations (12) and (17) are assumed to be the same. Thus, we are comparing two hypothetical equilibrium points where the demand is greater in one than the other, but the realized error terms are the same for each scenario. The new equilibrium demand equation obtained by modifying (13) is

(18) $$P^L_E = \alpha^*_1 + k + \alpha_2 Q^L_E + \varepsilon d_E$$

The equilibrium supply equation remains the same as (14). The deterministic equilibrium values P^L_{DFE} and Q^L_{DFE} in (16) are given by the simultaneous solution of (18) and (14) with the error terms ignored. The stochastic equilibrium values P^L_{SFE} and Q^L_{SFE} in (17) are given by the simultaneous solution of (18) and (14) with due recognition of the error terms.

The effects of the proposed demand shock may be illustrated diagrammatically using Figure 1. The demand equations in (13) and (18) are represented by D_0 and D_1, respectively; the supply equation in (14) is denoted by S_0. Abstracting for the moment from the question of deterministic versus stochastic equilibrium, the effect of the demand shock is to shift the demand function vertically along the price axis by an amount equal to *k*, to increase equilibrium quantity from Q^L_{IE} to Q^L_{FE} and to increase equilibrium price from P^L_{IE} to P^L_{FE}.

When defining the welfare changes caused by the demand shift the Marshallian welfare measures of consumer and producer surplus will be used. These welfare measures are frequently conceptualised as geometric areas behind demand and supply curves. For instance, in the current example, the change in consumer surplus resulting from the demand shift may be represented geometrically as the area $P^L_0 ab P^L_{FE}$ behind the demand curve D_1, where $P^L_0 = P^0_{FE} + k$. Similarly, the change in producer surplus resulting from the demand shift is represented by the trapezoidal area $P^L_{FE} bc P^L_{IE}$ behind the supply function S_0. In terms of equilibrium prices and quantities, these quantities are

(19) $$\Delta CS = 0.5(P^L_{IE} + k - P^L_{FE})\left(Q^L_{FE} + Q^L_{IE}\right)$$

(20) $$\Delta PS = 0.5\left(P_{FE}^{L} - P_{IE}^{L}\right)\left(Q_{FE}^{L} + Q_{IE}^{L}\right)$$

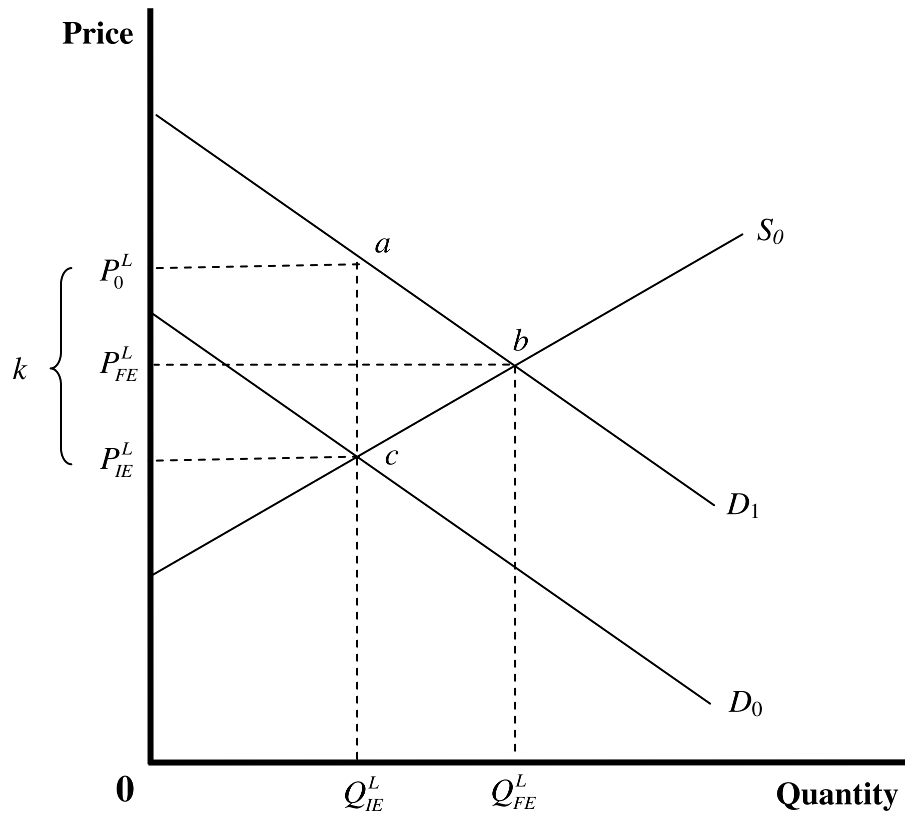

Figure 1

There is a need to consider how the existence of the stochastic error terms in the demand and supply functions will be accommodated. In an econometric model the error term is used to explain any differences which may exist between the predicted and the observed values of a dependent variable. Consequently, a different supply or demand curve exists for every realised value of the error terms in these models. This raises questions about what is considered to be the relevant demand or supply function. Does the term "relevant" refer to the demand or supply function as traced using only the deterministic component of a demand or supply function? Or, is it more appropriate to base our inferences upon the observed demand and supply function which requires consideration of both the deterministic component as well as a stochastic component? This is an important issue that affects the stochastic properties of the estimated welfare changes (Bockstael and Strand 1987; Kling, 1992).

In the first approach that we adopt only the deterministic components of the demand and supply functions are considered when deriving algebraic expressions for the changes in consumer and producer surplus. The premise for this approach is that, should the same exogenous shift to be repeated at different times, there would be different realisations for the errors. On average however, these realisations will average out to zero and only the welfare estimate which corresponds to this value for the error is considered to be of interest. For this reason, under this approach the welfare changes may be interpreted as a long-run effect.

Expressions derived under this approach will be called deterministic changes in consumer and producer surplus. Including the subscript *D* to denote deterministic, these changes are given by

(21) $$\Delta CS_D = 0.5(P_{DIE}^L + k - P_{DFE}^L)\left(Q_{DFE}^L + Q_{DIE}^L\right)$$

(22) $$\Delta PS_D = 0.5\left(P_{DFE}^L - P_{DIE}^L\right)\left(Q_{DFE}^L + Q_{DIE}^L\right)$$

The objective of this study is to show how to derive posterior densities for these changes and hence provide a means for expressing the uncertainty associated with estimating these quantities. Assuming the model specification is correct, when deterministic quantities are used, the only source of uncertainty in (21) and (22) will be the unknown parameters in the supply and demand curves. However, realized prices and quantities depend on realized error terms. Thus, it seems reasonable to also consider the uncertainty associated with the error terms when deriving the posterior densities of the changes in consumer and producer surplus. To describe the surplus changes that include error uncertainty we use the subscript *S*, recognizing that these changes are stochastic. They are given by

(23) $$\begin{aligned}\Delta CS_S &= 0.5\left(P_{SIE}^L + k - P_{SFE}^L\right)\left(Q_{SFE}^L + Q_{SIE}^L\right)\\ &= 0.5\left(P_{DIE}^L + k - P_{DFE}^L\right)\left(Q_{DFE}^L + Q_{DIE}^L\right) + \left(P_{DIE}^L + k - P_{DFE}^L\right)v_{2E}\\ &= \Delta CS_D + \left(P_{DIE}^L + k - P_{DFE}^L\right)v_{2E}\end{aligned}$$

(24) $$\begin{aligned}\Delta PS_S &= 0.5\left(P_{SFE}^L - P_{SIE}^L\right)\left(Q_{SFE}^L + Q_{SIE}^L\right)\\ &= 0.5\left(P_{DFE}^L - P_{DIE}^L\right) + \left(P_{DFE}^L - P_{DIE}^L\right)v_{2E}\\ &= \Delta PS_D + \left(P_{DFE}^L - P_{DIE}^L\right)v_{2E}\end{aligned}$$

Having derived algebraic expressions for the welfare changes resulting from the demand shift, the next step in the procedure involves deriving Bayesian posterior densities for the parameters in the model. The deterministic changes in welfare depend on the equilibrium prices and quantities that depend in turn on the parameters through the expression $(P^L_{DIE}, Q^L_{DIE})' = (I - A_0^{-1}A_1)^{-1}A_0^{-1}(\alpha_1^*, \beta_1^*)'$. Once posterior densities for the parameters have been obtained they imply a particular density for the welfare changes. The stochastic welfare changes also depend on v_{2E} which in turn is a function of the errors εd and εs; thus, to derive posterior densities for these welfare changes, we need the predictive densities for the errors.

4. Bayesian estimation

To proceed with Bayesian estimate we begin by writing the demand and supply equations as

(25)
$$\begin{aligned} P^L &= \alpha_1 j + \alpha_2 Q^L + \alpha_3 Y + \alpha_4 Q^B + \alpha_5 P^C + \varepsilon d \\ &= \alpha_1 j + \alpha_2 Q^L + Z_1\lambda + \varepsilon d \end{aligned}$$

(26)
$$\begin{aligned} Q^L &= \beta_1 j + \beta_2 P^L_{-1} + \beta_3 Q^L_{-1} + \beta_4 DT + \varepsilon s \\ &= \beta_1 j + Z_2\delta + \varepsilon s \end{aligned}$$

where j is a $T \times 1$ vector of ones, P^L and Q^L are $T \times 1$ vectors containing observations on the endogenous variables, $Z_1 = (Y, Q^B, P^C)$ is a $T \times 3$ matrix of observations on the exogenous parameters exclusive to the demand function, $Z_2 = (P^L_{-1}, Q^B_{-1}, DT)$ is a $T \times 3$ matrix of observations on the exogenous and predetermined variables which are not included in the demand function, εd and εs are $T \times 1$ vectors of random disturbances where it is assumed that $(\varepsilon d, \varepsilon s)' \sim N(0, \Sigma \otimes I_T)$. The coefficients of the variables in Z_1 and Z_2 are $\lambda = (\alpha_3, \alpha_4, \alpha_5)'$ and $\delta = (\beta_2, \beta_3, \beta_4)'$ respectively. Allowing for the covariance matrix Σ to be nondiagonal means we are allowing for contemporaneous correlation in the errors of the supply and demand equations.

It is convenient to write the model in terms of its reduced form and to make this form the basis for estimation. Working in this direction we have

(27) $$P^L = (\alpha_1 + \alpha_2\beta_1)\, j + \alpha_2 Z_2\delta + Z_1\lambda + \alpha_2\varepsilon s + \varepsilon d$$

(28) $$Q^L = \beta_1 j + Z_2\delta + \varepsilon s$$

Also, let the reduced form errors in these equations be given by $u_P = \alpha_2\varepsilon s + \varepsilon d$ and $u_Q = \varepsilon s$ and let Σ_u be to covariance matrix for (u_{Pt}, u_{Qt}) so that

(29) $$u = \begin{pmatrix} u_P \\ u_Q \end{pmatrix} \sim N(0, \Sigma_u \otimes I)$$

We will estimate the model in terms of the parameters $\theta' = (\alpha_1, \alpha_2, \beta_1, \delta', \lambda')$ and Σ_u. The first task is to set up a prior density for these parameters. In doing so, we seek a prior that (i) is relatively uninformative and hence is not subject to the criticism of incorporating too much personal subjectivity , (ii) includes generally acceptable information from economic theory, and (iii) does not suffer from a local nonidentification problem that can exist in simultaneous equation models when noninformative priors are used. Considering the last issue first, it can be seen from the reduced form that the parameter α_2 is identified from its product with the vector δ. Thus, if $\delta = 0$, which is equivalent to saying there are no predetermined variables excluded from the demand equation, α_2 is not identified. Thus if $\delta = 0$, which is equivalent to saying there are no predetermined variables excluded from the demand equation, α_2 is not identified. This property can cause problems if a uniform noninformative prior is used for the parameters. The posterior density function can become non-integrable because it approaches infinity at the point $\delta = 0$. This characteristic has led Kliebergen and Van Dijk (1994, 1988), and Chao and Phillips (1998) to explore other alternatives. In line with Chao and Phillips, we include a term in the prior, one that comes from a Jeffrey's prior, that places zero weight on the point $\delta = 0$ and a small weight in the neighbourhood of $\delta = 0$. Our prior density is

(30) $$p(\theta, \Sigma_u) \propto |\Sigma_u|^{-(m+1)/2} \left|\delta' Z_2' Q_{Z_1} Z_2' \delta\right|^{1/2} I_R(\theta)$$

where $m = 2$ is the number of equations and $Q_{Z_1} = I - Z_1 (Z_1' Z_1)^{-1} Z_1'$. The term $\left|\delta' Z_2' Q_{Z_1} Z_2' \delta\right|^{1/2}$ overcomes the problem at $\delta = 0$. The term $|\Sigma_u|^{-(m+1)/2}$ is the conventional multivariate non-informative prior. The remaining term is the indicator function

(31) $$I_R(\theta) = \begin{cases} 1 & \text{if} \quad \theta \in R \\ 0 & \text{if} \quad \text{otherwise} \end{cases}$$

where R is a set of restrictions implied by economic theory. Specifically,

$$R = \left\{ \theta \middle| \alpha_2 \leq 0, \alpha_3 \geq 0, \alpha_4 \leq 0, \alpha_5 \geq 0, \beta_2 \geq 0, 0 \leq \beta_3 < 1, \frac{\alpha_3}{\alpha_2}\bar{Q}_L + \frac{1}{\alpha_2} \leq 0, \beta_3 + \alpha_2\beta_2 < 1 \right\}$$

The inequalities in R that involve single parameters are sign expectations from economic theory. The condition $\frac{\alpha_3}{\alpha_2}\bar{Q}_L + \frac{1}{\alpha_2} \leq 0$ is the integrability condition evaluated at the mean quantity of lamb. This condition makes the analysis consistent with utility maximising behaviour. It is equivalent to saying the substitution effect from a price change is negative. The last restriction $\beta_3 + \alpha_2\beta_2 < 1$ is a stability one required for the system to converge to a new equilibrium following an exogenous shift.

Under the normal distribution the likelihood function is

(32) $$p(y|\theta, \Sigma_u) \propto |\Sigma_u|^{-T/2} \exp\left\{ -\frac{1}{2} \operatorname{tr}\left(\Sigma_u^{-1} S\right) \right\}$$

where $S = \begin{pmatrix} u_P' \\ u_Q' \end{pmatrix} \begin{pmatrix} u_P & u_Q \end{pmatrix}$ and y is used to denote all observations on P^L and Q^L. In this likelihood we have not explicitly accommodated the fact that the lagged price and lagged quantity appear as explanatory variables. To do so we need to assume the initial values P_1^L and Q_1^L are fixed and to view $p(y|\theta, \Sigma_u)$ as being derived from $p(y|\theta, \Sigma_u) = \prod_{t=2}^{T} p(y_t | y_{t-1}, \theta, \Sigma_u)$. Assuming P_1^L and Q_1^L fixed may seem contrary to the assumptions made to find stochastic equilibrium price and quantity where the distribution of errors into the infinite past was used to find the distribution of the equilibrium errors. It is, however, a convenient assumption and not one likely to have a big impact on estimation.

Combining the prior and the likelihood yields the joint posterior density

(33)
$$\begin{aligned} p(\theta,\Sigma_u|y) &\propto p(\theta,\Sigma_u)\,p(y|\theta,\Sigma_u) \\ &\propto |\Sigma_u|^{-(m+T+1)/2}\left|\delta' Z_2' Q_{Z_1} Z_2'\delta\right|^{1/2}\exp\left\{-\frac{1}{2}\mathrm{tr}\left(\Sigma_u^{-1}S\right)\right\} I_R(\theta) \end{aligned}$$

Given we are interested in θ and the deterministic welfare changes that are functions of θ, it is useful to obtain the marginal posterior density for θ, which is given by

(34)
$$\begin{aligned} p(\theta|y) &= \int p(\theta,\Sigma_u|y)\,d\Sigma_u \\ &\propto |S|^{-T/2}\left|\delta' Z_2' Q_{Z_1} Z_2'\delta\right|^{1/2} I_R(\theta) \end{aligned}$$

Also, for the stochastic welfare changes, we need the conditional posterior density for Σ_u given θ. It is the inverted Wishart distribution

(35)
$$p(\Sigma_u|\theta,y) \propto |\Sigma_u|^{-(m+T+1)/2}\exp\left\{-\frac{1}{2}\mathrm{tr}\left(\Sigma_u^{-1}S\right)\right\}$$

The posterior density $p(\theta|y)$ is an intractable one, but one from which a random-walk Metropolis algorithm can be used to draw observations $\theta^{(1)},\theta^{(2)},\ldots,\theta^{(M)}$ which can then be used to estimate the posterior means and variances of the parameters as well as plot marginal posterior densities of individual parameters. In turn the θ-draws can be used in the expressions for the deterministic welfare changes to obtain their posterior means and variances and to plot their posterior densities.

One can also proceed to include the effect of the error term, necessary for the stochastic welfare changes. A full Bayesian analysis of this effect is more complicated. We need to obtain draws from the predictive density $p(v|y)$ where $v=(v_{1E},v_{2E})'$ is a (2×1) vector of errors from the equilibrium form of the model. Taking draws from $p(v|y)$ is equivalent to obtaining draws from the joint density

(36)
$$p(v,\Sigma_v,\theta|y) = p(\theta|y)\,p(\Sigma_v|\theta,y)\,p(v|\Sigma_v,\theta,y)$$

Now, for each draw of θ from $p(\theta\,|\,y)$ obtained using equation (34), we can obtain a draw of Σ_u from $p(\Sigma_u|\theta,y)$ given in equation (35). This draw for Σ_u can be transformed to a draw Σ_v from $p\left(\Sigma_v \middle| \theta, y\right)$, the second term on the left side of (36), using the relationship between Σ_u and Σ_v. To define this relationship, note that

$$u_t = \begin{pmatrix} u_{Pt} \\ u_{Qt} \end{pmatrix} = C \begin{pmatrix} \varepsilon d_t \\ \varepsilon s_t \end{pmatrix} = C\varepsilon_t \qquad \text{where} \qquad C = \begin{pmatrix} \alpha_2 & 1 \\ 0 & 1 \end{pmatrix}$$

Thus, $\varepsilon_t = C^{-1}u_t$ and $\Sigma = C^{-1}\Sigma_u C'^{-1}$. Given draws for θ and Σ_u, we can compute a value for Σ from $\Sigma = C^{-1}\Sigma_u C'^{-1}$ followed then by a value for Σ_v from the expression $\text{vec}(\Sigma_v) = (I - B \otimes B)^{-1}\,\text{vec}(\Sigma)$. Using this value for Σ_v we can draw v from the distribution $(v_{1E}, v_{2E})' \sim N(0, \Sigma_v)$ which, given the sequence of draws we have described, will be a draw from $p\left(v \middle| \Sigma_v, \theta, y\right)$, the third density on the left side of (36). We then have all the values needed to compute the value for a draw from the posterior density of the stochastic welfare changes.

5. Empirical illustration

In this section, the investigation culminates in an empirical example which ties all of the various elements of the procedure together to assess the welfare impacts of an exogenous 1% demand shock in the domestic demand for lamb. Within the context of this example the demand shift may be interpreted as being the result of a successful marketing and advertising campaign for lamb such as may be undertaken funded by a producer funded organisation. Thus the results may be interpreted as evaluating the welfare impact of such a campaign and the likely distribution of benefits amongst consumers and producers. The data used in the analysis consists of 30 annual time series observations obtained from the NSW Department of Agriculture.

The random walk Metropolis-Hastings algorithm was used to generate 110,000 observations on θ from its marginal posterior density function, with the first 10,000 being discarded as a burn-in, leaving an effective sample of 100,000 observations. Summary statistics for the marginal posterior densities for the parameters in the demand function are presented in Table 1 while summary statistics for the marginal posterior densities for the parameters in the supply function are presented in Table 2.

Name	Mean	Median	S.D.	95% PI	
α_1	381.9900	384.9534	185.0100	12.2610	740.9950
α_2	-52.9630	-53.0556	6.9730	-66.4186	-38.9354
α_3	0.0304	0.0302	0.0103	0.0103	0.0509
α_4	-6.8443	-6.8594	1.1717	-9.1115	-4.5004
α_5	2.3950	2.3944	0.2958	1.8158	2.9864

Table 1: Summary statistics for the demand equation

Name	Mean	Median	St.Dev	95% PI	
β_1	-3.0487	-2.9164	2.3053	-7.9425	0.9289
β_2	0.0056	0.0053	0.0032	0.0006	0.0126
β_3	0.9311	0.9392	0.0482	0.8187	0.9968
β_4	0.6090	0.5999	0.4199	-0.1964	1.4793

Table 2: Summary statistics for the supply equation

The sample means are typically taken as the Bayesian estimators of the coefficients. Not only these values but the complete range of the parameter draws over their entire distributions is consistent with economic theory because of the prior restrictions. The effect of this prior can be most easily seen in the marginal posterior densities for the slope coefficient and the coefficient of the lagged dependent variable in the supply equation. For example, examination of the mean and standard deviation for β_2 suggests that, without the prior density, the posterior density for this parameter would assign positive density to a negative region. In addition to such simple restrictions upon individual parameters, the prior was used to impose other restrictions which are necessary in order to be able to obtain meaningful results. Particularly, care was taken in order to ensure that the econometric model satisfied the integrability conditions at all points so that valid values for welfare changes were obtained.

The 95% probability intervals are obtained by taking the 0.025 and 0.975 empirical quantiles of the generated observations. They allow us to make probability statements about the likely value for each parameter. For example, for the coefficient of quantity of lamb in the demand equation $\Pr(-66.4 < \alpha_2 < -38.9) = 0.95$.

Draws of the parameters were used to obtain draws from the posterior densities for the welfare changes using the procedure describe in Section 4. This information may be presented using tables of summary statistics or graphically using histograms as estimates of the posterior densities. The summary statistics are presented in Table 3. Estimates of the posterior densities are presented diagrammatically in Figures 2, 3 and 4.

Name	Mean	Median	S.D	95% PI	
ΔCS_D	18.1920	18.7201	5.9317	14.8874	22.2448
ΔCS_S	18.1990	18.6071	6.4600	14.4044	22.5119
ΔPS_D	5.1668	4.1288	4.2562	2.0706	7.0271
ΔPS_S	5.1715	4.0710	4.4299	2.0243	6.9740
ΔTS_D	23.3590	23.6778	5.0170	20.5143	26.5871
ΔTS_S	23.3710	23.5811	6.0572	19.8347	27.2275

Table 1 Summary Statistics for Posterior Densities of Welfare Changes

The posterior distributions for the deterministic welfare changes indicate a domestic 1% increase in the demand for lamb will results in an unambiguous increase in social welfare. The magnitude of the increase is given by the posterior density for the change in deterministic total surplus, $\Delta TS_D = \Delta CS_D + \Delta PS_D$. From this density we see that the magnitude of the gain is expected to lie between \$20.5 million to \$26.6 million with a point estimate given by the mean of \$23.3 million. The distribution of this welfare gain amongst consumer and producer groups indicates that consumers are the primary beneficiaries, appropriating the majority of the gain. The posterior density for the deterministic change in consumer surplus, ΔCS_D, indicates that consumers' welfare increases by between \$14.9 million and \$22.2 million and has a mean of \$18.2 million. Producers are able to appropriate only a relatively small proportion of the benefits from the demand shift. The posterior density for the deterministic change in producer surplus, ΔPS_D, indicates that the total benefit accruing to producers lies between \$2.1 million and \$7 million and has a mean of \$5.2 million.

The posterior densities pertaining to the stochastic versions of the alternative welfare measures produce results which are almost identical to those for the deterministic specifications. It can be seen from Table 3 that summary statistics used to describe the posterior densities for both the deterministic and stochastic welfare measures differ only in the third or later digit. What this result implies is that, in this particular example, the measurement of consumer and producer surplus changes are not sensitive to the way in which the error terms have been introduced. Although the difference is small, the stochastic changes do have higher standard deviations, reflecting the additional uncertainty that arises from the error terms and the additional uncertainty from estimation of the error covariance matrix. This additional uncertainty can also be seen from the comparison of the posterior densities for the deterministic and stochastic changes that appears in the figures. Those for the stochastic changes have a slightly greater spread.

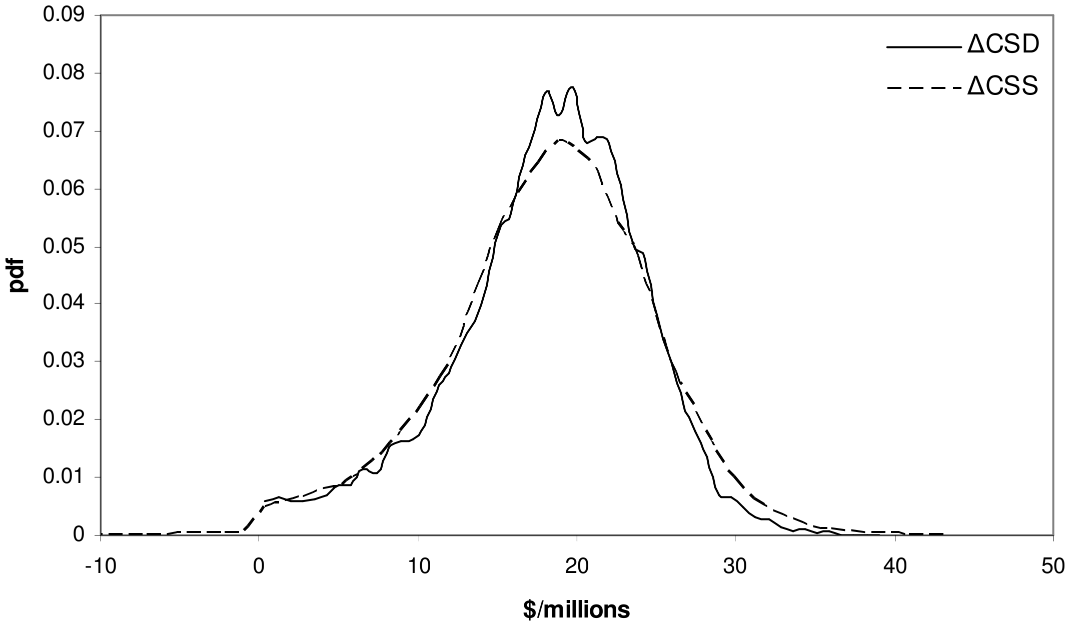

Figure 2: Change in Consumer Surplus

There is another observation that can be made from the figures. Economic theory implies that both consumers and producer should gain from the demand shift. Once the new equilibrium has been reached consumers gain because they are able to consumer more lamb at a lower price. Similarly, once the demand shift has occurred producers gain because they are able to sell more lamb at a higher price. These results imply the histograms should assign zero weight to negative regions of the welfare changes. The histograms describing the deterministic welfare changes can be seen to conform to this expectation as they are all truncated at zero. This is most obviously the case for the posterior density for ΔPS_D, although it is also present in the posteriors for the other welfare measures. It is most noticeable for ΔPS_D because the truncation occurs in a region of high density as opposed to the tails. In contrast, the posterior densities for the stochastic welfare changes all assign a small amount of weight to regions of negative welfare changes. The result reflects the influence of explicitly including the error terms.

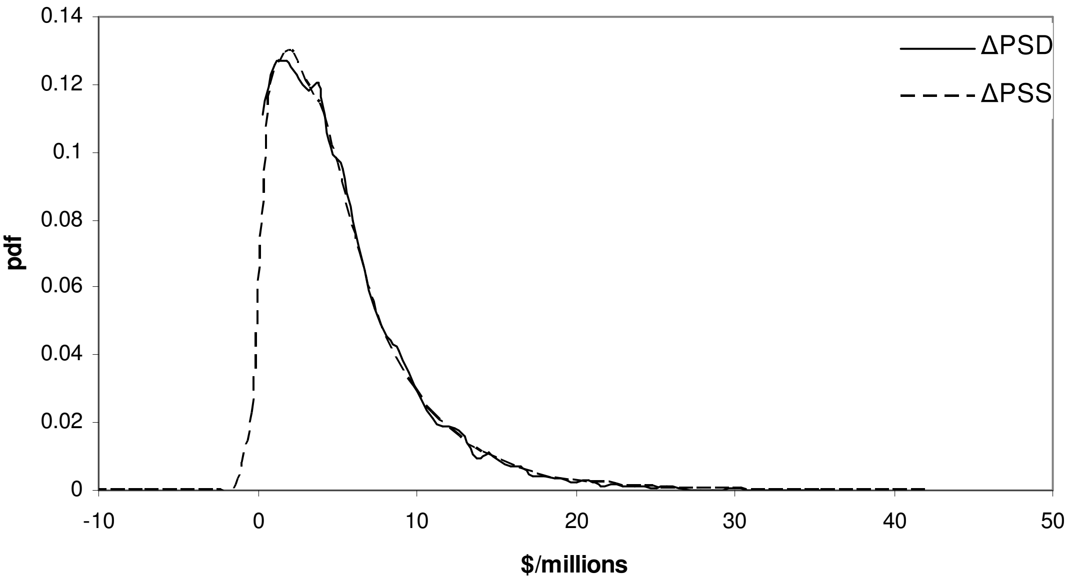

Figure 3: Change in Producer Surplus

The small difference between the stochastic and deterministic results is perhaps a surprising one. It is most likely attributable to the assumption that the same error is realized at the initial and final equilibrium points. This assumption is the appropriate one if we are comparing two hypothetical scenarios, assumed to be at the same time, one of which has a higher level of demand than the other. There are other scenarios that could be examined where treatment of the error would be different. For example, if we envisage moving from one uncertain equilibrium to another uncertain equilibrium where the level of demand in the second case is greater, then it would be reasonable to generate two different errors, one each for the initial and final equilibriums. Alternatively, one could assume the initial equilibrium is a deterministic one and that we are moving to a stochastic final equilibrium. Another possibility is to assume the last sample observation is the current period and to measure welfare changes from this point. Proceeding in this way would mean deriving the posterior density for the last realized error in the sample. It would also introduce other complications because the last sample point could not be assumed to be an equilibrium one and because the values of the exogenous variables would be changing.

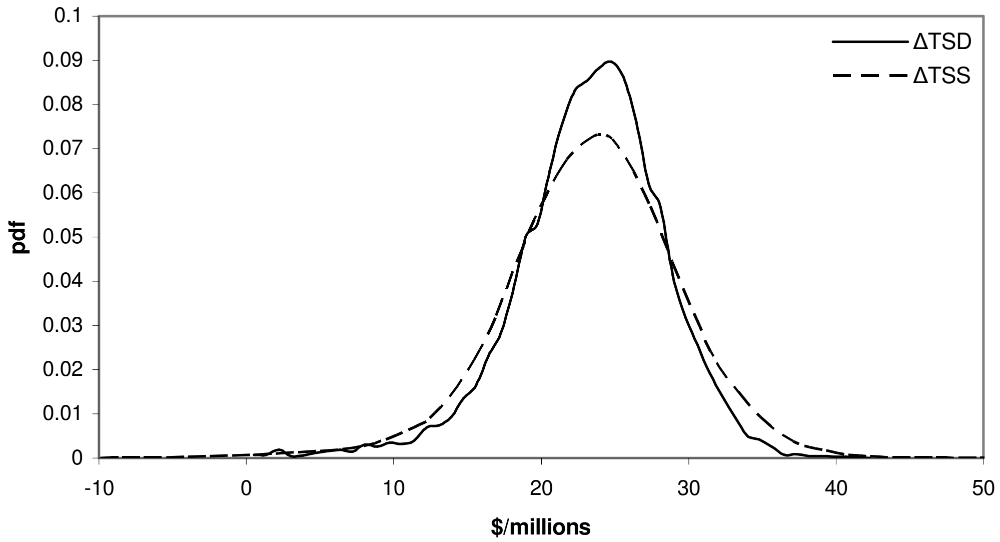

Figure 4: Change in Total Surplus

6. Conclusion

Measurement of changes in consumer and producer welfare is an important issue considered frequently in the policy arena. Uncertainty about such changes inevitably exists because of uncertainty about key parameters that affect the magnitude and direction of the changes. Bayesian prior or posterior densities provide a natural instrument for expressing knowledge and the degree of uncertainty about that knowledge when reporting likely changes in welfare. Previous studies have examined how prior densities, influenced by past studies and expert opinion can be used. In this paper we have provided a framework for using posterior densities that incorporate information from a sample of data. We show how to obtain Bayesian estimates of a 2-equation dynamic model and then how to subsequently use those estimates to find posterior densities for surplus changes that occur when we move from one equilibrium to the next. While our framework is couched in terms of the example considered, it readily generalizes to other examples. Also, although we have focused on changes at equilibrium, it is possible to consider dynamic changes that occur as one moves towards equilibrium. Results from this research will be reported in the near future.

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