BEHAVIOURAL AND DYNAMICAL SCENARIOS FOR CONTINGENT CLAIMS VALUATION IN INCOMPLETE MARKETS

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Abstract. We describe three different but related scenarios for determination of asset prices in an incomplete market: one scenario uses a market game approach whereas the other two are based on risk sharing or regret minimizing considerations. Furthermore, we point out some new dynamical schemes modeling the convergence of the buyer's and of the seller's prices of a given asset to a unique price.

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1. Derivative pricing in incomplete markets

Realistic markets are incomplete, due to either the existence of frictions or to the non existence of the necessary assets needed to achieve the complete replication of any contingent claim by linear combinations of available (traded) assets. Incomplete markets is a very interesting field of economic theory and finance which through seminal studies (see e.g. [1, 2, 3]) has led to important results that have helped the community to reach a deeper understanding of the function of financial markets.

While there is a rich literature on this field, the majority of these works focuses on the determination of bounds on the prices that are consistent with general equilibrium considerations. It is well known that in an incomplete market set up there is no longer a unique pricing kernel (martingale measure) and the existence of more than one pricing kernel may at best point out a whole band of prices that do not allow for arbitrage opportunities. The determination of a single price, at which an asset will eventually be traded in this market, out of this whole band, is still an open problem. There exists an extensive and very interesting literature on the subject, focusing on the determination of the upper and lower hedging prices (see e.g. [2]) as well as a number of suggestions on the price selected by the market (e.g. Kuhlback–Leibner or related entropy minimization criteria [4]) which lead to interesting implications, some of which are testable from real market data, however, a complete theory of price selection in incomplete markets is still missing.

The aim of the papers [5, 6] is to contribute to this literature by providing types of scenarios on price selection in incomplete markets. The scenarios are based on behavioural considerations and lead to interesting results. We deliberately use a simple one period discrete model in order to reveal and clarify the concepts and ideas behind price formation rather than linger on laborious technical details that would be of interest to a more specialized audience. Then the passage to a multiperiod model should not cause any vital difficulties and we plan to present it in future work together with the case of the continuous model.

2. Three scenarios for price selection

It is well known that in an incomplete markets setting, if equivalent martingale measures exist, they are not unique. Therefore, this leads to more than one possible price, all of which are consistent with the absence of arbitrage arguments. Other criteria will therefore be needed in order to select the price at which a particular asset is traded in an incomplete market. Several criteria have been proposed in the literature for the selection of the measure chosen for the price of a particular asset in an incomplete market, the majority of which is based, to the best of our knowledge, on the minimization of entropy related functions. Such functions quantify the "distance" between the true statistical measure of the market and the equivalent martingale measure chosen by the agents in the market.

In [5, 6] an alternative route is taken where three different, but ultimately related, scenarios are proposed for the price selection in incomplete markets. All scenarios assume that the participating agents have some initial beliefs about the distribution of the future states of the world. Based on these beliefs, each of them has in mind an initial non arbitrage valuation of the derivative security, according to which no risk is assumed utility-wise. The first approach is a market game approach, the second is a risk sharing approach whereas the third approach is one in which the agents update their beliefs about the possible prices of the states of the world, in a way which is consistent with the minimization of total regret.

2.1. **Market games approach.** The first scenario is a market game where the buyer and the seller bargain on the price of the derivative and choose the bargaining strategy that minimizes maximum regret. Given their initial valuations, this mechanism offers a unique bargaining strategy, that will lead to at most one unique price (depending on their initial valuations).

We assume that the two agents will eventually reach a price agreement and we explore the basic scenarios under which this can be achieved. We also assume that the two agents are impatient. Impatience leads them to act (even momentarily) as if they are entering a one bid sealed-auction and within this framework we consider that their objective is minimization of maximum regret. According to Linhart [7] in the minimax regret case there exists unique strategy (in contrast to the case of maximization of profit objective) which is a generalization of the linear equilibrium of Chatterjee-Samuelson [8].

2.2. **The risk sharing approach.** The second scenario which leads to a unique price for the asset is based on the concept of risk sharing price for the asset. In this scenario we assume that each of the agents has firm beliefs about the future prices of the world but deliberately undertakes some risk so that the transaction will be made possible. The unique price of the asset is defined by the solution of an optimization problem, in which the risk undertaken by each agent is chosen so that a convex combination of the risks undertaken by the agents is minimized, under the constraint that the transaction is made possible, i.e. under the constraint that the buyer's price is greater or equal than the seller's price.

2.3. **Optimal choice of the agents market price of risk.** The third scenario models the situation where the two agents do not have firm beliefs about the future states of the world but are willing to update their beliefs as part of the bargaining procedure. Their quoted prices thus do not entail any risk but there is some potential loss, which we call regret. The potential loss for agent 1 comes about from not being able to persuade agent 2 to accept her original belief (that would lead to the best possible price for her) and similarly for agent 2. A unique price is then chosen by the solution of an optimization problem in which the beliefs are chosen so that the convex combination of the regrets of the two agents is minimized under the constraint that the transaction eventually takes place.

3. Dynamic mechanisms for price selection

In [6] we introduce dynamic mechanisms that lead the agents to the price at which the derivative is traded. Such dynamic mechanisms lead to a common price between buyer and seller, which may or may not be the one proposed in the three scenarios of the previous section. These dynamical mechanisms are reminiscent of the Walras tattônement scenario in general equilibrium considerations and add to the general literature on how markets are led to their "equilibrium" states. We study the stability of these mechanisms as well as their robustness with respect to random perturbations.

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