Value-at-Risk evaluations in Malaysian Stock Exchange: heavy-tailed and long-memory-ARCH approaches

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Abstract
This study investigates the value-at-risk (VaR) using nonlinear time-varying volatility and heavy-tailed distribution. Our results evidenced that the predicted VaR under the generalized extreme-value (GEV) distribution exhibited similar results with the symmetric heavy-tailed long-memory ARCH model. However, it is found that only the GEV distribution is able to provide a convenient framework for asymmetric properties in both the lower and upper tails.

Keywords: Value-at-risk, long-persistence volatility, ARCH model, heavy-tailed.

1.0 Introduction

The stock market normally consisted of investors who made money during both bearish and bullish. For long position traders, they invested as buying a stock, holding it while it appreciated in price, and eventually sell it for profit. They encountered risk when the price of the stock decreased. On the other hand, the short trading position traders reacted exact opposite where they firstly sell the stock with the intention to later buy it back at a lower price. Therefore, the risk come from a rise in the price of the stock. Both the trading positions relied strongly to the extreme movements that governed the tails behaviour at the upper and lower tails. Besides the heavy-tailed issue, asymmetry distribution also often observed in financial time series. Studies by Barndorff (1997) and Giot and Laurent (2004) implemented skewed distributions that allowed upper and lower tails to have dissimilar behaviours.

Risk management is a crucial issue for financial institutes because billions of dollars can be lost due to failure of supervising and control the financial risks. The early pioneering work by Markowitz(1959) suggested that the portfolio selection is depended on the definition and measurement of risk. Value-at-Risk (VaR) is one of the famous indicator (Morgan,1996;Jarion,1997) that widely implemented by financial institutions and banks in their risk management.

This study focused on the Kuala Lumpur Stock Exchange (KLSE) indices which consisted of composite index (CI) and finance (FIN) index. As an emerging stock market, KLSE has received great attentions (Kok and Lee,1994;Lim et al.,2003;Cajueiro and Tabak,2005;Chin et al.,2007) from researchers and investors as the source of case studies and potential investment alternatives. The VaR is evaluated by ARCH-type models and quantile estimations using GEV distribution. Our empirical result evidenced that the GEV distribution provided a convenient framework for asymmetric properties in both the lower and upper tails. This finding is important because the tail behaviours have the direct impact to the VaR for portfolios defined on long and short trading positions.

2.0 Data and Methodology

All the data are taken from *Datastream* from 25 Oct 1993 until 31 Jan 2007 with a total of 3569 observations for each series. According to the *Datastream*, both the selected sectoral indices are available during this period of time. This is important for us to investigate the possible similarities and divergences in their returns series. The percentage continuous compounded interday returns can be expressed as $r_t = 100(\ln P_{t,close} - \ln P_{t-1,close})$.

2.1 Parametric long memory GARCH

Ding and Granger (1996) and Engle and Lee (1999) introduced the two component GARCH that can capture the high persistence in volatilities. Specifically, the model is decomposed into two components with one component captures the short-run innovation impact and the other captures the long-run impact of an innovation as follow:

$$\sigma_t^2 = \sigma_{t,q}^2 + \sigma_{t,s}^2$$
$$\sigma_{t,q}^2 = \omega + \gamma_{1q}\sigma_{t-1,q}^2 + \gamma_{2q}(a_{t-1}^2 - \sigma_{t-1}^2) \quad (1)$$
$$\sigma_{t,s}^2 = \gamma_{1s}\sigma_{t-1,s}^2 + \gamma_{2s}(a_{t-1}^2 - \sigma_{t-1}^2)$$

The asymmetric two component GARCH(1,1) can be estimated by including the asymmetric parameter ϕ in the transitory equation as follow:

$$\sigma_{t,s}^2 = \gamma_{1s}\sigma_{t-1,s}^2 + \gamma_{2s}(a_{t-1}^2 - \sigma_{t-1}^2) + \phi(a_{t-1}^2 - \sigma_{t-1}^2)I_{t-1} \quad (2)$$

where *I* is the dummy variable indicating negative innovation. The news impact coefficient is able to inform the market participants on how the volatility responses to 'bad news' compare to 'good news'. The maximum likelihood estimation used the iterative optimization algorithm to determine the second derivatives (Hessian matrix) under the parametric normal and heavy-tailed student-t (with degree of freedom exceeds 2) distributions. Finally, the *q%* quantiles are defined as:

$\hat{\mu}_t + D_q\hat{\sigma}_t$ for long position trading; (3)

$\hat{\mu}_t + D_{1-q}\hat{\sigma}_t$ for short position trading; (4)

where $\hat{\mu}_t$, $\hat{\sigma}_t$ and D are estimated conditional mean, estimated conditional standard deviation and the parametric distributions respectively.

2.2 Generalized extreme value distribution

In this study, we have selected the GEV distribution as our framework to study the tail behaviour. The GEV distribution is related to extreme-value theory for Type I, II and III distribution (Samuel and Saralees,2000). The GEV distribution is parameterized by the location, scale and shape parameters. The value-at-risk analysis for long financial position can be formulized as follows:

Consider *m* returns, $\{r_1, r_2, \ldots, r_m\}$ with order statistics minimum, $r_{(1)}$ and maximum, $r_{(m)}$ respectively. For long financial position, we concentrated on minimum, $r_{(1)}$ for the VaR with the lower probability (left-tail) quantile with p* (small loss) and r_n^* as the p*th quantile of subperiod minimum $\{r_{n,i} | \ i=1,\ldots,h\}$ for the limiting GEV distribution (Longin,2000):

$$p^* = \begin{cases} 1 - e^{-\left(1+\frac{k_n(r_n^* - \mu_n)}{\alpha_m}\right)^{1/k_n}} & if\ k_n \neq 0 \\ 1 - e^{-e^{\left(\frac{r_n^* - \mu_n}{\alpha_n}\right)}} & if\ k_n = 0 \end{cases} \qquad (4)$$

Rearrange,

$$r_n^* = \begin{cases} \mu_n - \frac{\sigma_n}{k_n}\left[1 - \left(-\ln(1-p^*)\right)^{k_n}\right] & if\ k_n \neq 0 \\ \mu_n + \sigma_n \ln\left[-\ln(1-p^*)\right] & if\ k_n = 0 \end{cases} \qquad (5)$$

The relationship between the sub-period minima and r_t can be obtained as follows:

$$p^* = P(r_{n,j} \leq r_n^*)$$
$$= 1 - P(r_{n,j} > r_n^*)$$
$$= 1 - \prod_{h=1}^{m} P(r_h > r_n^*)$$
$$= 1 - \left[1 - F(r_m^*)\right]^n$$

Rearrange:

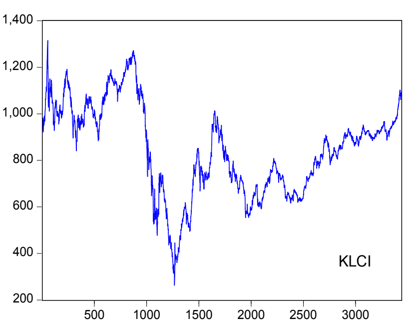
$$1 - p^* = 1 - \left[1 - F(r_n^*)\right]^n \qquad (6)$$

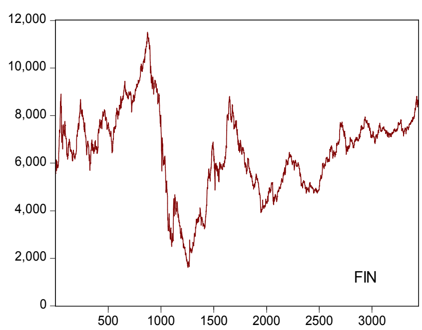
Substitute to eq x,

$$r_n^* = VaR_{Long} = \begin{cases} \mu_m - \frac{\sigma_n}{k_n}\left[1 - \left(-n\ln(1-p)\right)^{k_n}\right] & if\ k_n \neq 0 \\ \mu_n + \sigma_n \ln\left[-\ln(1-p)\right] & if\ k_n = 0 \end{cases} \qquad (7)$$

Under the maximum likelihood estimation, the subperiod minima $\{r_{n,i}\}$ where i=1,…,h, is assume to be followed a GEV distribution. Some statistical tests such as Q-Q Plots and Goodness of fit tests are evaluated for the empirical GEV distribution.

3.0 Empirical results





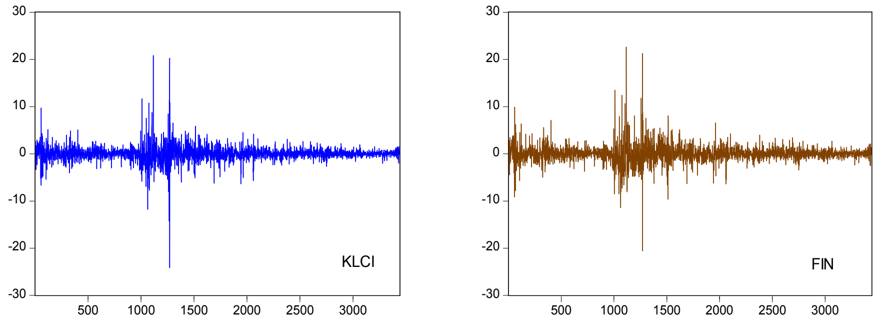


Figure 1. Price indices and percentage compounded returns for CI and PLN

Figure 1 illustrated the corresponding daily price indices and returns for CI and FIN indices. A clear relative high volatile price changes are exhibited around the observations 1000 to 1500 (year 1996 until 1998).

Table 1: Descriptive statistics

Statistics	KLCI	FIN
Mean	0.0048	0.0128
Maximum	20.8174	22.6276
Minimum	-24.1534	-20.5651
Std. Dev.	1.5759	1.7702
Skewness	0.5573* (13.5921)	1.2218* (29.7987)
Kurtosis	45.8857* (522.97)	30.7199* (338.03)
Autocorrelation		
lag 1	**0.053**	**0.140**
lag 2	0.036	0.078
lag 3	0.029	0.071
Normality test		
Jarque-Bera	263795*	110992*

Note: * represents the 5% significant level.

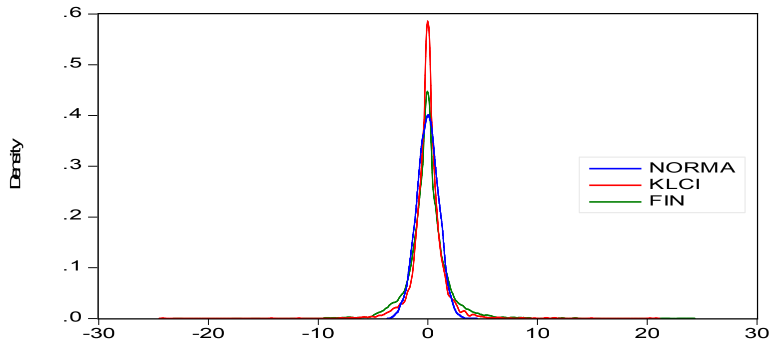
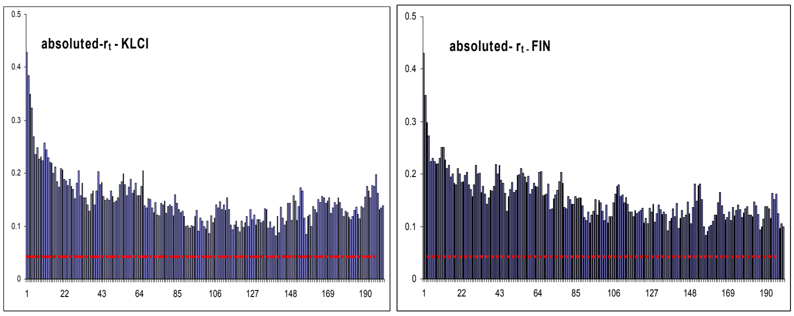


Figure 2. Kernel density plots

Table 1 summarized the descriptive statistics, test statistics corresponding to skewness, kurtosis and autocorrelation tests for all the standardized returns series. The null hypotheses of zero skewness and zero excess kurtosis are both rejected with highly significant values of test statistics. Excess kurtosis clearly indicated in the market indices with 45.88 (CI) and 30.71 (FIN) respectively. Overall, the first order autocorrelations indicated relatively higher values as compared to further lags autocorrelation. However, the daily returns especially in emerging market might cause by the growth (Urrutia,1995) of a particular stock market or possibility infrequent trading (Miller et al.,1994). According to Miller et al(1994), this spurious autocorrelation can be adjusted using a first order autoregressive or moving average. The ARCH-type modelling will include either AR(1) or ARMA(1,1) in the conditional mean equations. For normality test, the Jacque-Bera statistics indicated both the indices are significantly violated a normal distribution. Finally, the kernel density estimates in **Figure 2** indicated both the KLCI and FIN are heavier than a normal distribution.



Note: * the dotted line indicates the 5% level of significant ($1.96+ 1/\sqrt{T}$).

Figure 3: Autocorrelation function for KLCI and FIN

Table 2: Hurst's parameter estimation

$\lvert r_t \rvert$	KLCI	FIN	r_t^2	KLCI	FIN
VT plot*	0.815(0.990)	0.834(0.978)	VT plot*	0.708(0.992)	0.735(0.990)
R/S**	0.741(0.987)	0.738(0.980)	R/S**	0.703(0.995)	0.705(0.994)

Notes:

(1) *For the aggregated time series r(n) of a self-similar process, the variance obeys the following large sample property:

$V[r^{(n)}] \sim \frac{V[r]}{n^{b}}$, where $r_k^{(n)} = \frac{1}{n}\sum_{i=kn-(n-1)}^{kn} r_i$ and the self-similarity parameter H = 1- (β/2).

(2) **The ratio of R/S is defined as:

$$\frac{R}{S} = \frac{\max_{1\le j\le N}\left[\sum_{k=1}^{j}(r_k - M(L))\right] - \min_{1\le j\le N}\left[\sum_{k=1}^{j}(r_k - M(L))\right]}{\sqrt{\frac{1}{N}\left[\sum_{k=1}^{N}(r_k - M(L))^2\right]}}$$

where M(L) is the sample mean over the time period L. The Hurst's parameter is determined by: R/S ~ $(L/2)^{H}$,

(3) The values in the parentheses represent the R^2 obtained from the simple linear regression.

Besides the return series, absolute return and squared returns are the two commonly used volatility proxies in empirical financial time series analysis. In **Figure 3**, the first 200 autocorrelation function for both the squared and absolute returns are statistically significant after long lags especially for absolute returns. In addition, the Hurst's parameter (Hurst,1951) also indicated long memory volatility in all the series in **Table 2**. This result supported the long memory ARCH modelling in the VaR estimations.

3.1 Long memory ARCH maximum likelihood estimation

Table 3: ARCH maximum likelihood estimations and diagnostics

Estimation	KLCI		FIN	
	normal	Student-t	normal	Student-t
g_0	0.0352* (0.025)	0.0199(0.145)	0.0436* (0.039)	0.0226(0.197)
g_1	0.1474** (0.000)	0.1295** (0.000)	0.4893** (0.000)	0.3617** (0.006)
g_2			-0.3394** (0.001)	-0.2272(0.103)
a_0	0.0412* (0.029)	0.0643(0.138)	0.03138** (0.005)	0.0744* (0.045)
a_1	-0.1090(0.634)	-0.1517(0.758)	0.2749** (0.001)	0.1634 (0.493)

b_1	0.1135(0.665)	0.0108(0.984)	0.53838** (0.000)	0.3184(0.226)
d	0.3192** (0.000)	0.3229** (0.000)	0.3907** (0.000)	0.3386** (0.000)
v		4.6641** (0.000)		4.5762** (0.000)
L	-5000.76	-4829.24	-5570.43	-5390.97
AIC	2.9109	2.8117	3.2426	3.1389
SIC	2.9216	2.8242	3.2551	3.1532
Diagnostic				
$\tilde{a}_t$,Q(12)	18.230*(0.076)	22.634*(0.019)	18.874* (0.041)	28.446*(0.015)
LM(12)	0.4517(0.942)	0.4787 (0.928)	1.3127(0.203)	1.1420 (0.320)
Negative	0.3178(0.750)	0.4387 (0.660)	0.0997(0.920)	0.7560(0.449)
Positive	0.4963(0.619)	0.4844 (0.628)	0.9327(0.350)	0.1080(0.913)

Notes:
(1) $\tilde{a}_t$ represents the standardized residual;
(2) Ljung Box Serial Correlation Test(Q-statistics) on $\tilde{a}_t$: Null hypothesis – No serial correlation;
(3) LM ARCH test: Null hypothesis - No ARCH effect;
(4) Engle and Ng test (1993): News impact test based on the regression $\tilde{a}_t^2 = a_1 + a_2 S_t^- + a_3 S_t^- a_{t-1} + a_4 S_t^- a_{t-1}^2 + e_t$.
(5) * and ** represent 5% and 1% levels of significance. The values in the parentheses represent the p-value.

Table 3 reported the maximum likelihood estimated results for asymmetric two components GARCH(1,1) with the assumption of $z_t \sim N(0,1)$ and $z_t \sim student\text{-}t(v)$. In this model, the shock impacts on transitory and permanent components are represented by the lag shock component, g_{2q} and g_{2s} are significant across the studied indices. The coefficient, g_{2s}s, showed greater magnitude than g_{2q}s for both the indices over the periods. Meanwhile, the permanent components, g_{1q}s, are statistically significant with the average value of 0.9975 across the indices implied that the long persistence components converge slowly to the steady state. All the models exhibited heavy tails with the range of degree of freedom around 5. In **Table 3**, both the stock markets indicated news impact with the significant positive asymmetric coefficients(fs). The sectors implied that downward movements(shock) in the stock market are followed by a greater volatilities than upward movements of the same magnitude.

In **Table 3**, the diagnostic tests for the specifications in GARCH models indicated no significant serial correlations and ARCH effect in the variance equations at the 5% level respectively. In addition, the Engle and Ng (1993) tests shown that all the multi-sectors have no evidences of unexplained non-linearity and size bias in the negative side at 1% significance level.

3.2 GEV distribution estimations

Table 4: GEV estimators for lower and upper tail

	Lower tail						Upper tail					
	m	s	k	W^2	U^2	n	m	s	k	W^2	U^2	n
KLCI	-0.4270 (0.000)	1.2215 (0.000)	-0.2744 (0.000)	0.1307 (0.056)	0.0893 (0.052)	19	-0.4374 (0.000)	1.2335 (0.000)	-0.2931 (0.000)	0.0350 (0.087)	0.0343 (0.044)	19
FIN	-0.5057 (0.000)	1.4014 (0.000)	-0.2502 (0.000)	0.1665 (0.101)	0.1020 (0.087)	19	-0.4987 (0.000)	1.4519 (0.000)	-0.3179 (0.000)	0.0365 (0.075)	0.0264 (0.029)	19

Notes: The goodness-of-fit tests followed the null and alternative hypotheses as follows:
H_0: Both the empirical distribution and GEV distribution are identical;
H_1: H_0 is not true.

For empirical fitting tests, the Q-Q-plots fitted reasonably well between the empirical and estimated GEV distributions and the formal discrepancy tests also failed to reject the null hypothesis of no discrepancy between the two tails distributions at 10% significant level.

For thickness comparison of upper and lower tails, both KLCI and FIN indicated slightly heavier tails at the upper tails where the smaller the shape parameter (*k*), the heavier the density mass of the tail. This asymmetry property is further verified by the rejection of skewness test at 5% level in **Table 1**. These findings suggested that short trading (upper tail) might encounter higher risk as compared to long trading (lower tail) investments both the indices.

Table 5: Short-trading (upper tail) one-day-ahead loss VaR forecast (%)

index	quantile	0.05 (5%)	0.04(4%)	0.03(3%)	0.02(2%)	0.01(1%)	0.005(0.5%)	0.001(0.01%)
KLCI	ARCH-normal	1.0497	1.1153	1.1959	1.3031	1.4721	1.9456	2.0697
	ARCH-Student-t	1.3396	1.4642	1.6304	1.8771	2.3412	2.8728	4.4660
	GEV	0.4693	0.7628	1.1692	1.8009	3.0663	4.6128	9.6747
FIN	ARCH-normal	1.4823	1.5722	1.6827	1.8295	2.0609	2.7096	2.8796
	ARCH-Student-t	1.7637	1.9256	2.1414	2.4618	3.0646	3.7551	5.8243
	GEV	0.5362	0.8829	1.3660	2.1237	3.6626	5.5762	12.0300

Table 6: Long-trading (lower tail) one-day-ahead loss VaR forecast (%)

index	quantile	0.05 (5%)	0.04(4%)	0.03(3%)	0.02(2%)	0.01(1%)	0.005(0.5%)	0.001(0.01%)
KLCI	ARCH-normal	0.9895	1.0551	1.1358	1.2429	1.4119	1.8854	2.0096
	ARCH-Student-t	1.3047	1.4294	1.5956	1.8423	2.3064	2.8380	4.4312
	GEV	0.4585	0.7484	1.1478	1.7646	2.9872	4.4621	9.1821
FIN	ARCH-normal	1.3110	1.4008	1.5113	1.6581	1.8896	2.5382	2.7083
	ARCH-Student-t	1.6701	1.8320	2.0487	2.3701	2.9751	3.6549	5.6848
	GEV	0.5418	0.8733	1.3270	2.0218	3.3804	4.9918	10.0011

Table 5 and **Table 6** summarized the one-day-ahead VaR[1] forecasts for ARCH and GEV distribution approaches. The underestimated VaR (relative smaller losses) using Gaussian distribution is unable to portray the fat-tails syndrome exhibited in both the studied indices. GEV distribution forecasts indicated similar heavier losses compared to ARCH (student-t) only in the 1% and smaller quantiles. However, the symmetric ARCH (student-t) failed to capture the dissimilar tails behaviour for the upper and lower tails. In **Table 5** and **Table 6**, it is noticed that the losses from 5% until 3% VaR are underestimated as compared to ARCH-normal and ARCH-t. This shortcoming might cause by the violation of independent assumption of daily returns. However, for smaller quantile (≤1%) estimations, the results shown larger VaR than ARCH approach.

Our result evidenced both indices of KLCI and FIN encountered higher risks (greater losses) at the short trading positions which are tally with the shape parameter estimations in GEV distribution. It is well-known that short position investors (short sellers) encountered more risks compared to ordinary long position investors. Short sellers are facing limited gains (price go down to zero) but unlimited losses. In addition, longer duration of short sellers may cause further interest costs and the short squeezed might restrict their activities to large traded stocks.

4.0 Conclusion

This paper investigated the tail behaviours of the innovation distributions for Malaysian stock indices. We estimated the upper and lower tails separately by the GEV

[1] The actual loss may depend on the amount of capital invested. For example, suppose an investor is long in RM100,000 in a particular stock, then the 5% VaR for 1-day horizon under the normal distribution (**Table 6**) is 0.010707% x RM100,000 or equivalent to RM1070 .

distribution of the data. Even though the ARCH-type student-t model is able to capture the heavy-tailed property, but it failed to take into account the asymmetric behaviour at the end of both tails. These findings provided non-trivial information to the investors who involve in long and short financial positions in Malaysian stock exchange.

Reference

Barndorff-Nielsen, O. E.,1997 Normal inverse Gaussian distributions and the modelling of stock returns, Scandinavian Journal of Statistics, 24, 1-13.

Ding Z., and C.W.J. Granger,1996. Modeling volatility persistence of speculative returns: a new approach, *Journal of econometrics*, 73, 185-215.

Engle R.F., and G.G.J. Lee,1999. *A long-run and sort-run component model of stock return volatility*, in cointegration, causality and forecasting, ed. By Engle, and White. Oxford University Press.

Engle, R.R., Ng, V.,1993. Measuring and testing the impact of news on volatility, Journal of Finance, 48, 1749-1778.

Giot, P. and Laurent, S.,2004. Modelling daily Value-at-Risk using realized volatility and ARCH type models, 11(3), 379-398.

Hurst H.E.,1951. Long term storage capacity of reservoirs, Transaction of the American Society of Civil Engineers, 116, 770-799.

Jorion, P., 1997, Value at Risk: The New Benchmark for Controlling Market Risk. The McGraw-hill Company: Chicago.

Kok, K.L. and F.F. Lee, 1994. Malaysian second board stock market and the efficient market hypothesis, Malaysian Journal of Economic Studies, 31(2): 1-13.

Lim, K.P., M.S. Habibullah and H.A. Lee, 2003. A BDS test of random walk in the Malaysian stock market. Labuan Bulletin of International Business and Finance, 1(1): 29-39.

Longin, F., 2000. From Value-at-Risk to Stress testing: The Extreme Value Approach, Journal of Banking and Finance, 24, 1097-1130.

Markowitz. H.M.,1959. Portfolio Selection: Efficient Diversification of Investments, John Wiley & Sons, New York.

Miller M. H., Muthuswamy, J. and Whaley, R. E.,1994. Mean reversion of Standard and Poor 500 index basis changes: Arbitrage-induced or statistical illusion? Journal of Finance 49,479-513.

Morgan J.P. (1996), RiskMetricsTM – Technical Document.

Samuel, K. and Saralees, N., 2000. Extreme value distributions: theory and applications, Imperial College Press, London.

Urrutia, J.L., 1995. Tests of random walk and market efficiency for Latin American emerging markets, Journal of Financial Research, 18,299-309.