National accounts:

An optimisation approach for balancing supply and use tables

Statistics New Zealand
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Abstract

Statistics New Zealand uses a supply and use framework within the national accounts to confront and reconcile the annual production and expenditure estimates of gross domestic product (GDP). The supply and use approach also provides the basis for checking the consistency of the measures of the supply and use of goods and services, which have been estimated from different statistical sources. An optimisation approach was developed to support the implementation of new product data in the framework.

1. The supply & use framework in the system of national accounts

GDP can be measured using a production, expenditure and/or income approach. Part of the New Zealand System of National Accounts (NZSNA) is a supply and use framework, in which the annual production and expenditure measures of GDP are confronted and reconciled.

The supply and use framework is also a powerful statistical and analytical tool used to balance the flows of goods and services in the economy. The accounts are balanced when, for all industries, total inputs equal total outputs and, for products, total supply equals total demand. The balanced accounts provide a benchmark for the annual national accounts in following years, for quarterly estimates of GDP and for other publications like the Tourism Satellite Account.

Figure 1: A matrix representation of the supply & use framework

Total supply	=	Total use		
industries in columns products in rows **Domestic supply** **S** Total domestic supply by industry	+	**Imports** **M**	=	**Intermediate use** **U** Total intermediate use by industry	+	**Final expenditure** **F**

Production measure = **Expenditure measure**
GDP(P) = S - U GDP(E) = F – M

Value added (by industry)
= S – U (by column)
= compensation of employees + gross operating surplus and mixed income + taxes on production and imports less subsidies

Final expenditure
= final private and government expenditure + capital formation + exports

Note: S, U, F and M in purchasers' prices.
GDP is the sum of value added for all industries.

2. Data collection for product breakdowns

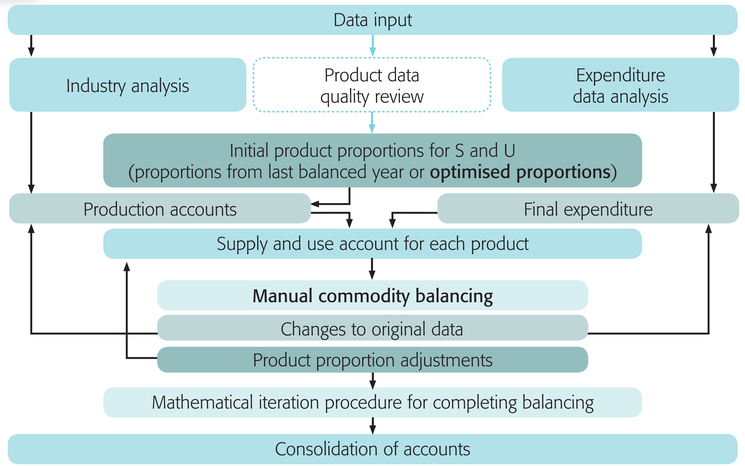
The framework requires the compilation of tables that detail the supply and use of goods and services by industry. There is no annual data source for detailed product breakdowns. The underlying product data for outputs and intermediate consumption by industry is largely based on data collections in the 1990s. Partial adjustments were made to reflect changes in the economy over time.

To address data deficiencies, the Commodity Data Collection (CDC) project was established. The primary vehicle for collecting product data is a CDC survey, tailored to the specifications of each individual industry. The collection period is from 2003 to 2008 and the approved sample size for the total of CDC surveys is 3,500 enterprises. The CDC information will also be used to establish weights for the business price indexes to be used as deflators in the production of chain-volume measures of GDP.

3. The supply and use balancing process

The balancing process is shown in the diagram below. In the regular process, product proportions from the last balanced year are applied to production accounts. This is combined with expenditure by product data, resulting in a supply and use account for each product, which is the starting point for manual commodity balancing. In the balancing process for the years 2003 to 2005, some of the available CDC results were used as additional data source in manual commodity balancing.

Figure 2: The supply & use balancing process



4. An optimisation approach for implementing new product data

To support the large-scale implementation of the new CDC product data for about 115 industries and 320 products in the future, an optimisation model was developed. The model is formulated as a linear program where, as far as possible, given some balancing constraints, the new product data is incorporated into the supply and use accounts for the last balanced year. The optimal solution from the model provides improved initial product proportions for manual commodity balancing of the revised estimates for the last balanced year and for subsequent years.

5. The optimisation model

The decision variables in the model represent domestic supply and intermediate use values by industry and product. They are bounded by values reflecting old and new product proportions. The objective function to be minimised is a weighted 'distance' to values reflecting new proportions. Explicit weights can be assigned for each variable to reflect reliability of the data. The linear constraints reflect fixed industry income and expenditure totals and supply and use balance for all products.

The optimisation model

Terminology

s_{ij} for $i = 1, \dots, m$ and $j = 1, \dots, n$ are variables reflecting domestic supply

u_{ij} for $i = 1, \dots, m$ and $j = 1, \dots, n$ are variables reflecting intermediate use

s^0_{ij}, u^0_{ij} and s^1_{ij}, u^1_{ij} are values reflecting old respectively new proportions

$\bar{S}_j$, $\bar{U}_j$ are fixed totals for domestic supply respectively intermediate use by industry

$\bar{E}_i = \bar{F}_i - \bar{M}_i$ are values for final expenditure minus imports by product

W^s_{ij} and W^u_{ij} are reliability weights (default is 1)

m = number of products
n = number of industries
$2 \cdot m \cdot n$ = number of variables

Linear program

$$\min_{s_{ij},u_{ij}} \sum_{i=1}^{m}\sum_{j=1}^{n} (W^s_{ij} \cdot (s^0_{ij} - s^1_{ij}) \cdot s_{ij} + W^u_{ij} \cdot (u^0_{ij} - u^1_{ij}) \cdot u_{ij})$$ weighted distance function to be minimised,

s.t. subject to linear constraints:

$\sum_{i=1}^{m} s_{ij} = \bar{S}_j$ for all $j = 1, \dots, n$ industry totals for domestic supply kept constant,

$\sum_{i=1}^{m} u_{ij} = \bar{U}_j$ for all $j = 1, \dots, n$ industry totals for intermediate use kept constant,

$\sum_{j=1}^{n} (s_{ij} - u_{ij}) = \bar{E}_i$ for all $i = 1, \dots, m$ supply and use balanced for each product,

values reflecting old and new proportions are:

$\min \{s^0_{ij}, s^1_{ij}\} \le s_{ij} \le \max \{s^0_{ij}, s^1_{ij}\}$ for all $i = 1, \dots, m$ and $j = 1, \dots, n$ bounds for supply variables,

$\min \{u^0_{ij}, u^1_{ij}\} \le u_{ij} \le \max \{u^0_{ij}, u^1_{ij}\}$ for all $i = 1, \dots, m$ and $j = 1, \dots, n$ bounds for use variables.

6. Limitations and extensions of the optimisation model

The objective function has a big impact on the results of the optimisation model. The chosen 'distance' function makes the model equivalent to a linear absolute value regression problem. The reliability weights are based on expert judgements and not on variances. Further research can be done on sensitivity analysis for the variable boundaries. An extension on the current model would be the inclusion of variables for final expenditure categories.

The optimisation model is a supporting tool for implementing new product data. Manual commodity balancing remains the main activity in the balancing process. Quality review of the newly observed proportions is an important part of the implementation, especially regarding the relatively small CDC sample size and data collection in different years. The optimisation model allows for a systematic approach to improve the relationships within the supply and use framework, although this presents a challenge to ensure that economic reality is well reflected and maintained.

7. Implementation date

The aim is to implement a comprehensive set of up-to-date input and output product proportions for the annual national accounts release in November 2009.

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