An Efficiency Wage – Imperfect Information Model of the Phillips Curve

Carl M. Campbell III
Department of Economics
Northern Illinois University
DeKalb, IL 60115
U.S.A.
Phone: 815-753-6974
E-mail: carlcamp@niu.edu

February 2008

Abstract

This study develops an efficiency wage model in which workers have imperfect information about wages elsewhere. Firms' profit-maximizing behavior results in a Phillips curve relationship between inflation (either of wages or prices), unemployment, and lagged inflation. The wage-wage Phillips curve is a reduced form relationship with the coefficient on lagged wage inflation equaling 1. The wage-price and the price-price Phillips curves are statistical relationships in which the coefficient on lagged inflation asymptotically approaches 1. In addition, this study derives an upward-sloping counterpart to the Phillips curve from profit-maximizing behavior and makes reasonable predictions about the cyclical behavior of real wages.

An Efficiency Wage – Imperfect Information Model of the Phillips Curve

I. Introduction

The Phillips curve was originally developed as a relationship between the unemployment rate and the rate of change in wages, based on Phillips' (1958) analysis of British data. Samuelson and Solow (1960) later extended the Phillips curve to also refer to the relationship between the unemployment rate and the rate of price inflation. Subsequent work by Friedman (1968) and Phelps (1968) argued that expected inflation should be included as an independent variable in a Phillips curve, with a predicted coefficient of 1. With this expectations-augmented Phillips curve, the economy is characterized by a natural rate of unemployment, to which it eventually returns following a shock.

In empirical estimation of the Phillips curve, expected inflation is generally proxied by lagged inflation. Researchers have found evidence for the expectations-augmented Phillips curve, as the coefficient on unemployment is generally negative and the sum of coefficients on lagged inflation is generally close to 1 in regressions of wage or price inflation on unemployment and lagged inflation.[1] However, while economists have found empirical support for the Phillips curve, it has been much more difficult to provide theoretical justification for it.[2] In addition, the Phillips curve does not have an upward-sloping counterpart in unemployment – inflation space. Thus, the Phillips curve shows the combinations of unemployment and inflation that are possible, but the Phillips curve framework does make predictions about the actual values of inflation and unemployment.

This study develops an efficiency wage model in which workers have incomplete information about wages at other firms. The wage and employment decisions of profit-maximizing firms result in a Phillips curve relationship at the aggregate level between inflation (either of wages or prices), unemployment, and lagged inflation. In addition, this model is used to derive an upward-sloping curve in unemployment – inflation space and to analyze the cyclical behavior of real wages.

Three Phillips curve specifications are considered: a regression of wage inflation on unemployment and lagged wage inflation (the wage-wage Phillips curve), a regression of wage inflation on unemployment and lagged price inflation (the wage-price Phillips curve), and a regression of price inflation on unemployment and lagged price inflation (the price-price Phillips curve).

The wage-wage Phillips curve is a reduced-form relationship that is derived directly from the profit-maximizing behavior of firms. In this equation, the sum of coefficients on lagged wage inflation equals 1, and the coefficient on unemployment is negative and depends on just four parameters.

The wage-price Phillips curve and price-price Phillips curve are not reduced-form equations. Rather, they are statistical relationships obtained from modeling stochastic shocks to the growth rate of demand. Modeling these shocks yields expressions for wages, prices, and unemployment in each period as functions of these shocks. These expressions are then treated as data in a regression in which the independent variables are unemployment and lagged price inflation and the dependent variable is the current value of either wage or price inflation. The equation $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ yields expressions for the coefficients on unemployment and lagged price inflation as functions of the model's microeconomic parameters. It is demonstrated that the coefficient on lagged price inflation asymptotically approaches 1 as the sample size increases. However, even when the sample size is small, the coefficient is very close to 1 when reasonable parameter values are chosen.

These Phillips curves are disequilibrium relationships determined from the response of wages, prices, and employment to exogenous demand shocks. Over time, these endogenous variables approach their equilibrium values, which means that the economy is characterized by a natural rate of unemployment.

The profit-maximizing behavior of firms also yields an upward-sloping curve (referred to as the Dynamic Labor Demand curve) in unemployment – wage inflation space. This curve is the counterpart to the Phillips curve, and the intersection of the Phillips curve and the Dynamic Labor Demand (DLD) curve determines both the rate of wage inflation and the unemployment rate. The Phillips curve – DLD curve framework is used to determine the paths followed by wage inflation and unemployment as they adjust from their initial equilibrium to their new equilibrium in response to a one-time disinflationary demand shock.

The framework developed in this study can be used to analyze technology shocks, as well as demand shocks. The dynamic effects of technology shocks depend on whether a constant velocity or an IS-LM specification for demand is assumed. In either case, the economy returns to its natural rate after a shock to technology.

The model also makes predictions about cyclical behavior of real wages. When there are technology shocks, real wages are strictly procyclical. However, real wages can be procyclical or countercyclical in response to shocks to aggregate demand. The model's prediction that real wages can be either procyclical or countercyclical can explain why real wages appear to behave differently in different time periods.

Most of the analysis in this study assumes that workers' efficiency depends on the ratio between their wages and their expectations of average wages. However, it is possible that their efficiency also depends partly on their expectations of the price level. When the model is modified to make efficiency a function of workers' wages relative to their expectations of the price level, a reduced-form equation for the price-price Phillips curve is derived.

II. Relationship of Present Study to Past Work on the Phillips Curve

Two models of the Phillips curve that have been developed in recent years are the sticky price Phillips curve and the sticky information Phillips curve. The sticky price Phillips curve (also referred to as the New Keynesian Phillips curve) is discussed in Roberts (1995), who shows that a Phillips curve relationship can be derived from the staggered contract models of Taylor (1979, 1980) and Calvo (1983) and from the quadratic adjustment cost model of Rotemberg (1982). Roberts demonstrates that these models all yield the prediction that inflation depends on expectations of future inflation and on the output gap. While the sticky price model is widely used in policy analysis,[3] it has been criticized on several grounds. Fuhrer and Moore (1995) find that it cannot explain why inflation is so persistent, and Ball (1994) shows that this model predicts that announced, credible disinflations may cause booms instead of recessions.

Another variant of the sticky price Phillips curve is the model of Galí and Gertler (1999), who develop a Phillips curve model in which price inflation depends on expectations of future marginal cost. They measure marginal cost by labor's share of national income and demonstrate that their model outperforms a conventional sticky price model in which inflation depends on the output gap. However, while Galí and Gertler show that price inflation depends on the behavior of wages, their study does not analyze the factors that determine wages.

The sticky information model is developed by Mankiw and Reis (2002). In their model, a fraction of firms receives information in each period that enables them to compute optimal prices, while the remaining firms set prices based on out-of-date information. They demonstrate that their model explains output and inflation dynamics better than a sticky price model. The present model is similar to Mankiw and Reis's model in that economic fluctuations result from imperfect information. However, it differs from Mankiw and Reis by assuming a different type of imperfect information. In the Mankiw-Reis model, firms have imperfect information about the optimal price of their products and this imperfect information affects their pricing and output decisions. In contrast, the present study assumes that workers have imperfect information about average wages in making decisions related to their effort and on-the-job search and that this imperfect information affects the wage decisions (and hence employment decisions) of firms.

There are good reasons to believe that, in explaining economic fluctuations, workers' imperfect information about average wages is more important than firms' imperfect information about optimal prices. Optimal prices in Mankiw and Reis' model depend on the aggregate price level and on aggregate output. Given the ease of accessing statistics on GDP and the price level on the internet, it is not obvious why some firms (particularly large firms) would operate with information on optimal prices that is out of date. Even if some firms do operate with out-of-date information, it seems unlikely that these informational errors would be large enough to cause large fluctuations in output. On the other hand, what matters for the effort and quit decisions of workers is their wages relative to average wages for workers who are employed in a similar narrow occupational group and who have similar qualifications (e.g., age, experience, and education), and this information is not easily obtainable. In fact, when Bewley (1999) interviewed employers about their labor relations, respondents indicated that they thought their workers did not have a very precise idea about the wages offered at other firms.

In addition, the connection between imperfect information and unemployment fluctuations is clearer in a model with a model with imperfect information about average wages than in a model with imperfect information about optimal prices. If workers have imperfect information about average wages, this imperfect information would affect firms' wage-setting behavior and would thus affect unemployment. In contrast, it is not obvious how firms' imperfect information about optimal prices would affect unemployment, as there is nothing in the sticky information model that would prevent the labor market from clearing.

There are other ways in which this study differs from the sticky price and sticky information models of the Phillips curve. First, this study considers both the labor market and the product market, whereas some previous studies (e.g., Galí and Gertler (1999) and Mankiw and Reis (2002)) do not consider the labor market and thus do not consider unemployment. Second, unlike previous Phillips curve models, this study derives expressions for both the wage-price and price-price Phillips curves (as well as the wage-wage Phillips curve) in the context of a single model. Third, this study derives explicit expressions for the paths of wages, prices, and unemployment (which are endogenously determined as functions of underlying demand shocks to the economy). Because expressions are derived for these variables, this study is able to analyze the cyclical behavior of real wages, an issue that is not examined in other Phillips curve studies.

III. Assumptions

In deriving the model, the following assumptions are made:

1. Workers' efficiency (e) depends on the ratio of their current wage to their expectations of wages at other firms and on the unemployment rate, so that

$$e = e[W_t / \overline{W}_t^{\,e}, u_t], \qquad \text{with } e_W > 0, \quad e_u > 0, \quad e_{WW} < 0, \quad \text{and } e_{Wu} < 0,$$ [4]

   where W_t is a worker's current wage, $\overline{W}_t^{\,e}$ denotes workers' expectations of average wages (to be defined below), and u_t is the unemployment rate. Explanations for a positive dependency of productivity on wages and unemployment include the shirking model of Shapiro and Stiglitz (1984); the gift-exchange/fair wage models of Akerlof (1982, 1984) and Akerlof and Yellen (1990); the labor turnover models of Stiglitz (1974), Schlicht (1978), and Salop (1979); and the adverse selection model of Weiss (1980). The function $e[W_t / \overline{W}_t^{\,e}, u_t]$ can be viewed as incorporating all of these explanations.[5]

2. In the short run, workers may have incomplete information on current wages at other firms and may use information on lagged average wages to help predict the current average wage rate. Note that this assumption means that wages must vary across firms, so that workers cannot infer the average wage from their own wage. For example, it could be assumed that firms make random errors in setting wages, but that the profit-maximizing wage is set on average.[6] The fact that workers use information on lagged average wages to predict current average wages means that their expectations of average wages can be viewed as a mixture of rational and adaptive expectations. Mathematically, it will be assumed that $\overline{W}_t^{\,e}$ is given by the equation

$$\overline{W}_t^{\,e} = \left(\overline{W}_t\right)^{\omega} \left(\widetilde{W}_t^{\,L}\right)^{1-\omega},$$

where $\widetilde{W}_t^{\,L}$ represents workers' expectations of current average wages based on their information on lagged wages, and ω measures the degree to which expectations are unbiased. Justification for the assumption that workers' wage expectations are a mixture of rational and adaptive expectations is discussed in Campbell (2008a, 2008b).

3. Firms produce output (Q) with the Cobb-Douglas production function,

$$Q_t = A_t^{\phi} L_t^{\phi} K_0^{1-\phi} e\left[W_t / \overline{W}_t^{\,e}, u_t\right]^{\phi},$$

in which L represents labor input, A represents technology (assumed to be exogenous), and K is the capital stock (assumed to be fixed at K_0)

4. Each firm faces a downward-sloping demand curve in the product market of the following form:

$$Q_t^D = Y_t \left(\frac{P_t}{\overline{P}_t}\right)^{-\gamma},$$

where Y is real aggregate demand, P is the firm's price, $\overline{P}$ is the aggregate price level, and γ is the price elasticity of demand. Accordingly, the firm's price and total revenue can be expressed as

$$P_t = Y_t^{\frac{1}{\gamma}} Q_t^{-\frac{1}{\gamma}} \overline{P}_t, \qquad \text{and}$$

$$P_t Q_t = Y_t^{\frac{1}{\gamma}} Q_t^{\frac{\gamma-1}{\gamma}} \overline{P}_t.$$

It will be assumed that real aggregate demand is determined from a constant velocity specification, so that $Y_t = M_t / P_t$. However, demand shocks have similar effects if it is assumed that aggregate demand is determined from an IS-LM framework.

5. Labor supply is inelastic and equals *N* times the number of firms. Parameters are chosen so that there is excess supply of labor.[7] Since parameters are chosen so that firms maximize profits by paying efficiency wages, wages (*W*) and employment (*L*) are determined by differentiating the profit function with respect to both *W* and *L*.

Given the model's assumptions, profits in period *t* (net of capital costs) can be expressed as

(1) $$\Pi = Y_t^{\frac{1}{g}} \left[A_t^{f} L_t^{f} K_0^{1-f} e[W_t / \overline{W}_t^{e}, u_t]^{f} \right]^{\frac{g-1}{g}} \overline{P}_t - W_t L_t .$$

IV. Basic Model

The profits of the typical firm are given by equation (1). Differentiating this equation with respect to L_t and setting the derivative equal to 0 yields

$$\frac{d\Pi}{dL_t} = 0 = \frac{f(g-1)}{g} Y_t^{\frac{1}{g}} A_t^{\frac{f(g-1)}{g}} L_t^{\frac{f(g-1)}{g}-1} K_0^{\frac{(1-f)(g-1)}{g}} e[W_t / \overline{W}_t^{e}, u_t]^{\frac{f(g-1)}{g}} \overline{P}_t - W_t ,$$

so that

$$(2) \qquad L_t = W_t^{\frac{g}{f(g-1)-g}} \left(\frac{g}{f(g-1)} \right)^{\frac{g}{f(g-1)-g}} Y_t^{-\frac{1}{f(g-1)-g}} A_t^{-\frac{f(g-1)}{f(g-1)-g}} K_0^{-\frac{(1-f)(g-1)}{f(g-1)-g}}$$
$$\times e[W_t / \overline{W}_t^e, u_t]^{-\frac{f(g-1)}{f(g-1)-g}} \overline{P}_t^{-\frac{g}{f(g-1)-g}}.$$

The other first-order condition is

$$(3) \qquad \frac{d\Pi}{dW_t} = 0 = f \frac{g-1}{g} Y_t^{\frac{1}{g}} A_t^{\frac{f(g-1)}{g}} L_t^{\frac{f(g-1)}{g}} K_0^{\frac{(1-f)(g-1)}{g}} e[\bullet]^{\frac{f(g-1)}{g}-1} e_W[\bullet] \frac{1}{\overline{W}_t^e} \overline{P}_t - L_t.$$

If (2) is substituted into (3), the following condition, which is analogous to the Solow (1979) condition, is obtained:

$$(4) \qquad W_t e[W_t / \overline{W}_t^e, u_t]^{-1} e_W [W_t / \overline{W}_t^e, u_t] \frac{1}{\overline{W}_t^e} = 1.$$

Totally differentiating equation (4) and dividing it by the original equation yields

$$0 = \left[1 - e^{-1} e_W \frac{W_t}{\overline{W}_t^e} + \frac{e_{WW}}{e_W} \frac{W_t}{\overline{W}_t^e} \right] \hat{W}_t + \left[-1 + e^{-1} e_W \frac{W_t}{\overline{W}_t^e} - \frac{e_{WW}}{e_W} \frac{W_t}{\overline{W}_t^e} \right] \hat{\overline{W}}_t^e$$
$$+ \left[\frac{e_{Wu}}{e_W} - e^{-1} e_u \right] du_t,$$

where $\hat{W}_t = dW_t / W_t$ and $\hat{\overline{W}}_t^e = d\overline{W}_t^e / \overline{W}_t^e$. The above equation can be viewed as representing the relationship between the percentage deviations in W_t, the percentage deviations in $\overline{W}_t^e$, and absolute deviations in u_t from their initial equilibrium values. This equation can be further simplified by substituting $(W_t / \overline{W}_t^e) = e e_W^{-1}$ (from equation 4), yielding

(5) $$\hat{W}_t = \hat{\overline{W}}_t^e + \left[\frac{e_u e_W^2}{e^2 e_{WW}} - \frac{e_{Wu} e_W}{e e_{WW}}\right] du_t .$$

If we consider small deviations of W, $\overline{W}^e$, and u from their initial equilibrium values, we can treat the coefficient on du_t as a constant, with this constant determined by the initial equilibrium values of e, e_W, e_u, e_{WW}, and e_{Wu}. The fact that $W=\overline{W}^e$ in equilibrium means that (from equation 4) $e=e_W$. This substitution allows (5) to be expressed as

(6) $$\hat{W}_t = \hat{\overline{W}}_t^e + \frac{e_u - e_{Wu}}{e_{WW}} du_t .$$

The unemployment rate is given by the equation

$$u_t = \frac{N - L_t}{N} .$$

Letting $s_L = L^* / N$ (where L^* is the equilibrium value of L), du_t can be approximated by

(7) $$du_t = \frac{-dL_t}{N} = \frac{-dL_t}{\frac{L_t^*}{s_L}} \approx -s_L \hat{L}_t .$$

Appendix A demonstrates that expressing (2) as deviations from steady-state values, substituting the resulting expression into (7), and substituting (7) into (6) yields

(8) $$\hat{W}_t = \frac{e_{WW}}{e_{WW} - s_L(e_u - e_{Wu})} \hat{\overline{W}}_t^e - \frac{s_L(e_u - e_{Wu})}{e_{WW} - s_L(e_u - e_{Wu})} \hat{M}_t .$$

Appendix A also derives the following expressions for unemployment and the price level:

(9) $$du_t = -s_L(\hat{M}_t - \hat{W}_t)$$ and

(10) $$\hat{P}_t = [1 - \mathrm{f} + \mathrm{f}e^{-1}e_u s_L]\hat{M}_t - \mathrm{f}\hat{A}_t - \mathrm{f}e^{-1}e_u s_L \hat{W}_t + \mathrm{f}\hat{\overline{W}}_t^{\,e}.$$

V. Wage-Wage Phillips Curve

This section derives an equation for the wage-wage Phillips curve from equation (6). As previously discussed, workers' expectations of average wages are assumed to be a mixture of rational and adaptive expectations. We first consider a specification for $\overline{W}_t^{\,e}$ in which the adaptive component of workers' wage expectations depends on lagged wages and on wage inflation in the previous period. (A specification in which expectations depend on last period's wages and last period's wage inflation would be reasonable in an economy that has historically experienced positive wage growth.) In particular, it is assumed that

$$\widetilde{W}_t^L = \overline{W}_{t-1}\frac{\overline{W}_{t-1}}{\overline{W}_{t-2}}, \qquad \text{so that} \quad \overline{W}_t^{\,e} = \overline{W}_t^{\,\mathrm{w}}\left(\overline{W}_{t-1}\frac{\overline{W}_{t-1}}{\overline{W}_{t-2}}\right)^{1-\mathrm{w}}.$$

In terms of percentage deviations, the above equation can be expressed as

(11) $$\hat{\overline{W}}_t^{\,e} = \mathrm{w}\hat{W}_t + 2(1-\mathrm{w})\hat{W}_{t-1} - (1-\mathrm{w})\hat{W}_{t-2}.$$

If (11) is substituted into (6), the following equation is obtained:

(12) $$(\hat{W}_t - \hat{W}_{t-1}) = \frac{e_u - e_{Wu}}{(1-\mathrm{w})e_{WW}}du_t + (\hat{W}_{t-1} - \hat{W}_{t-2}).$$

Equation (12) is a reduced-form relationship between current wage inflation, unemployment, and lagged wage inflation, and thus is a reduced-form equation for the wage-wage Phillips curve.[8] Accordingly, a regression of the form, $(\hat{W}_t - \hat{W}_{t-1}) = \hat{\beta}_1 du_t + \hat{\beta}_2(\hat{W}_{t-1} - \hat{W}_{t-2})$, will yield coefficient values of

$$\hat{\beta}_1 = \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}} \qquad \text{and} \qquad \hat{\beta}_2 = 1.$$

The coefficient on the unemployment rate depends on just four parameters. In addition, since (12) is a reduced-form relationship, these values of $\hat{\beta}_1$ and $\hat{\beta}_2$ will be the estimated coefficients regardless of whether economic fluctuations result from demand shocks or technology shocks, and these will be the estimated coefficients for any process governing the shocks (e.g., whether shocks are stochastic or deterministic).

More generally, it could be assumed that workers' expectations of average wages depend on lagged wages and on a weighted average of wage inflation in several previous periods, so that

$$\overline{W}_t^e = \overline{W}_t^{\omega}\left[\overline{W}_{t-1}\left(\frac{\overline{W}_{t-1}}{\overline{W}_{t-2}}\right)^{\lambda_1}\left(\frac{\overline{W}_{t-2}}{\overline{W}_{t-3}}\right)^{\lambda_2}\ldots\left(\frac{\overline{W}_{t-T}}{\overline{W}_{t-T-1}}\right)^{\lambda_T}\right]^{1-\omega}, \text{ with } \lambda_1 + \lambda_2 + \ldots + \lambda_T = 1.$$

In this case, expressing $\overline{W}_t^e$ in terms of percentage deviation yields

$$\text{(13)} \qquad \hat{\overline{W}}_t^e = \omega\hat{W}_t + (1-\omega)[\hat{W}_{t-1} + \lambda_1(\hat{W}_{t-1} - \hat{W}_{t-2}) + \lambda_2(\hat{W}_{t-2} - \hat{W}_{t-3}) + \ldots + \lambda_T(\hat{W}_{t-T} - \hat{W}_{t-T-1})].$$

Substituting (13) into (6) results in the following equation for the wage-wage Phillips curve:

$$(\hat{W}_t - \hat{W}_{t-1}) = \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}} du_t + \lambda_1(\hat{W}_{t-1} - \hat{W}_{t-2})$$
$$+ \lambda_2(\hat{W}_{t-2} - \hat{W}_{t-3}) + \ldots + \lambda_T(\hat{W}_{t-T} - \hat{W}_{t-T-1}).$$

The coefficient on the unemployment rate is the same as in (12), and the coefficients on lagged wage inflation sum to 1.

VI. Wage-Price Phillips Curve and Price-Price Phillips Curve

Section V derives a reduced-form equation for the wage-wage Phillips curve. However, Phillips curves are generally estimated by regressing either wage inflation or price inflation on unemployment and lagged price inflation. For the wage-price Phillips curve and the price-price Phillips curve, the model does not yield reduced-form coefficients on unemployment or lagged price inflation. To obtain coefficients for these versions of the Phillips curve, a different approach is taken. Stochastic shocks to the growth rate of demand are modeled, yielding expressions for wage inflation, price inflation, and unemployment as functions of the underlying shocks. These expressions are then treated as data in regressions in which unemployment and lagged price inflation are the independent variables and current wage or price inflation is the dependent variable. From these regressions, expressions are obtained for the coefficients on unemployment and lagged inflation as functions of the model's microeconomic parameters and the number of time periods in the sample (T). It is demonstrated that as $T \to \infty$, the coefficient on lagged inflation approaches 1 in both the wage-price and the price-price Phillips curves. However, even if the sample size is small, the coefficients on lagged inflation are very close to 1 with reasonable parameter values.

In modeling these shocks, it is assumed that $\hat{W}_t^e$ is given by equation (11). Substituting (11) into (8) yields

$$\hat{W}_t = \frac{e_{WW}}{e_{WW} - s_L(e_u - e_{Wu})}[\omega\hat{W}_t + 2(1-\omega)\hat{W}_{t-1} - (1-\omega)\hat{W}_{t-2}] - \frac{s_L(e_u - e_{Wu})}{e_{WW} - s_L(e_u - e_{Wu})}\hat{M}_t,$$

which can be expressed as

(14) $$\hat{W}_t - 2a\hat{W}_{t-1} + a\hat{W}_{t-2} = (1-a)\hat{M}_t ,$$

where

$$a = \frac{(1-\omega)e_{WW}}{(1-\omega)e_{WW} - s_L(e_u - e_{Wu})} < 1 .$$

Equation (14) is a second-order difference equation, and its solution yields an expression for wages in each period as a function of current and lagged values of demand. We consider the effect of a series of stochastic shocks to the growth rate of demand. In particular, it is assumed that the growth rate of demand is 0 for t=-∞ to t=0, and then can be expressed as $\hat{M}_t - \hat{M}_{t-1} = \hat{M}_{t-1} - \hat{M}_{t-2} + \varepsilon_t$ for t=1 to t=∞, where ε_t is a random error with a mean of 0. Appendix B derives the following solutions for wage inflation, price inflation, and unemployment:

(15a) $$\hat{W}_t - \hat{W}_{t-1} = \sum_{k=1}^{t} \varepsilon_k \left[1 - \rho^{\,t-k+1} \cos[\psi\,(t-k+1)]\right],$$

(15b) $$\hat{P}_t - \hat{P}_{t-1} = \sum_{k=1}^{t} \varepsilon_k \left[1 - \phi\left((\omega - e^{-1}e_u s_L)\,\rho^2 + 1 - \omega\right)\rho^{\,t-k-1} \cos[\psi\,(t-k+1)]\right],$$ and

(15c) $$du_t = -\frac{s_L}{\sqrt{1-\rho^2}} \sum_{k=1}^{t} \varepsilon_k\, \rho^{\,t-k+2} \sin[\psi\,(t-k+1)],$$

where

$$\rho = \sqrt{a}, \qquad \text{and}$$

$$\psi = \arccos\sqrt{a} .$$

We first consider the wage-price Phillips curve. To obtain predicted coefficients for this relationship, the above expressions for wage inflation, unemployment, and lagged price inflation are used as data in the regression,

$$(\hat{W}_t - \hat{W}_{t-1}) = \hat{\beta}_1 du_t + \hat{\beta}_2(\hat{P}_{t-1} - \hat{P}_{t-2}) + \varepsilon_t .$$

It is assumed that the data start in period t_0 and end in period T. If we let $\hat{\boldsymbol{\beta}}$ represent a vector of the estimated β's, then values for $\hat{\beta}_1$ and $\hat{\beta}_2$ can be obtained from the equation $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$, where

$$\mathbf{y} = \begin{bmatrix} \hat{W}_{t_0} - \hat{W}_{t_0-1} \\ \hat{W}_{t_0+1} - \hat{W}_{t_0} \\ \vdots \\ \hat{W}_T - \hat{W}_{T-1} \end{bmatrix} \quad \text{and} \quad \mathbf{X} = \begin{bmatrix} du_{t_0} & \hat{P}_{t_0-1} - \hat{P}_{t_0-2} \\ du_{t_0+1} & \hat{P}_{t_0} - \hat{P}_{t_0-1} \\ \vdots & \vdots \\ du_T & \hat{P}_{T-1} - \hat{P}_{T-2} \end{bmatrix}.$$

Appendix C demonstrates that as $T \rightarrow \infty$, the asymptotic values of $\hat{\beta}_1$ and $\hat{\beta}_2$ are

(16a) $$\lim_{T\rightarrow\infty} \hat{\beta}_1 = \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}} + 3(1-a)\frac{a - \phi\left((\omega - e^{-1}e_u s_L)a + 1 - \omega\right)}{s_L(1+a)},$$

(16b) $$\lim_{T\rightarrow\infty} \hat{\beta}_2 = 1 .$$

The asymptotic coefficient on unemployment ($\hat{\beta}_1$) equals the coefficient on unemployment in the wage-wage Phillips curve (which is unambiguously negative) plus an additional term. It can be demonstrated that this additional term is negative if real wages are procyclical and is positive if real wages are countercyclical. (The equation for real wages is reported in Section IX.) Thus, if real wages are procyclical (countercyclical), the coefficient on unemployment is larger (smaller) in absolute value in the wage-price Phillips curve than in the wage-wage Phillips curve. In addition, the coefficient on lagged inflation ($\hat{b}_2$) asymptotically approaches 1. While $\hat{b}_2$ equals exactly 1 only in the asymptotic case, under reasonable parameter values it is close to 1 even when the number of observations is small. For example, when there are only 10 observations, $\hat{b}_2$=1.00033 with the baseline parameters (with the micro-based efficiency function) in Campbell (2008a).

We next consider the price-price Phillips curve, which can be expressed as

$$(\hat{P}_t - \hat{P}_{t-1}) = \hat{b}_1 du_t + \hat{b}_2 (\hat{P}_{t-1} - \hat{P}_{t-2}) + e_t .$$

Given the paths of unemployment and price inflation in (15b) and (15c), Appendix C demonstrates the asymptotic coefficients are

(17a) $$\lim_{T\to\infty} \hat{b}_1 = \frac{e_u - e_{Wu}}{(1-w)e_{WW}} - \frac{(1-3r^2)(1-r^2)\left[r^2 - f\left((w - e^{-1}e_u s_L) r^2 + 1 - w\right)\right]}{s_L r^2 (1+r^2)},$$

(17b) $$\lim_{T\to\infty} \hat{b}_2 = 1 .$$

For the price-price Phillips curve, the coefficient on unemployment equals the coefficient on unemployment in the wage-wage Phillips curve plus a term that is positive (negative) if real wages are procyclical (countercyclical). As in the case of the wage-price Phillips curve, the coefficient on lagged inflation asymptotically approaches 1 and is close to 1 even when the number of observations is small. For example, when there are 10 observations, $\hat{b}_2$=0.999105 with the baseline parameters in Campbell (2008a).

From equations (15a), (15b), and (15c) we can calculate the long-run response of wage inflation, price inflation, and unemployment to aggregate demand shocks. The long-run effect of aggregate demand shocks (i.e., shocks to e) is to raise wages and prices by the same amount of the shock. On the other hand, aggregate demand shocks have no long-run effect on unemployment. In addition, it can be demonstrated that technology shocks have no long-run effect on unemployment. Thus, the economy is characterized by a natural rate of unemployment. While the dynamic effect of aggregate demand shocks is to produce a Phillips curve relationship between inflation (either of wages or prices), unemployment, and lagged inflation, these shocks have no long-run effect on unemployment and output.

VII. The Dynamic Labor Demand Curve

As demonstrated in Section V, the profit-maximization problem of firms yields a downward-sloping Phillips curve in wage inflation – unemployment space. This profit-maximization problem also yields an additional relationship between wage inflation and unemployment. From (9), the unemployment rate is determined from the equation,

$$du_t = -s_L(\hat{M}_t - \hat{W}_t).$$

If the lag of this equation is subtracted from it and the resulting equation is solved for the growth rate of wages, the following equation is obtained:

(18) $$\hat{W}_t - \hat{W}_{t-1} = (\hat{M}_t - \hat{M}_{t-1}) + s_L^{-1}(du_t - du_{t-1}).$$

Equation (18) shows the relationship between wage inflation, the unemployment rate, and the growth in nominal demand. Graphing this relationship in wage inflation –

unemployment space yields an upward-sloping curve, since $s_L^{-1} > 0$. This relationship will be referred to as the Dynamic Labor Demand (DLD) curve, and it is the upward-sloping counterpart to the Phillips curve. The DLD curve intersects a vertical line at the natural rate of unemployment at the growth rate of demand minus s_L^{-1} times the difference between last period's unemployment rate and the natural rate. Variables that shift this curve are the growth rate of nominal demand and lagged unemployment. The intersection of the DLD curve and the Phillips curve determines both the rate of wage inflation and the unemployment rate.

To illustrate the Phillips curve – DLD framework, we analyze a one-time disinflationary demand shock. It is assumed that demand increases at a rate of g_1 from t=-∞ through t=0, and then increases at a rate of g_2 from t=1 through t=∞, where $g_2<g_1$. From (12), the Phillips curve is

$$\hat{W}_t - \hat{W}_{t-1} = \frac{e_u - e_{Wu}}{(1-\mathrm{W})e_{WW}} du_t + (\hat{W}_{t-1} - \hat{W}_{t-2})\,.$$

Assuming that the economy is in a steady-state equilibrium for $t \leq 0$, the DLD curve can be expressed as $\hat{W}_t - \hat{W}_{t-1} = g_1$ for $t \leq 0$ and $\hat{W}_t - \hat{W}_{t-1} = g_2 + s_L^{-1}(du_t - du_{t-1})$ for $t \geq 1$.

Figure 1 shows the shifts in the Phillips curve and the DLD curve that result from a decrease in the rate of demand growth from g_1 to g_2 (for three periods following this decrease). The intersections of these curves trace out the paths of wage inflation and unemployment over time in response to this disinflationary shock. DLD_0 and $PC_{0,1}$ represent the DLD and Phillips curves in the initial equilibrium. In response to the decrease in the growth rate of demand, the DLD curve shifts to DLD_1 in period 1, so that it intersects the vertical line at the natural rate (u*) at g_2. However, the Phillips curve does not shift since lagged wage inflation is the same as in the initial equilibrium. Then in periods 2 and 3, the Phillips curve shifts because of changes in lagged wage inflation, and the DLD curve shifts because of changes in lagged unemployment (relative to the natural rate).[9] In the long run, both the Phillips curve and the DLD curve intersect u^* at g_2, the rate of wage inflation equals g_2, and the unemployment rate returns to the natural rate.

In response to this one-time disinflationary demand shock, it can be demonstrated that the paths of wage inflation and unemployment are

$$\hat{W}_t - \hat{W}_{t-1} = g_2 + (g_1 - g_2)\rho^t \cos(\psi t), \qquad \text{and}$$

$$du_t = (g_1 - g_2)\frac{\sigma_L}{\sqrt{1-\rho^2}}\rho^{t+1}\sin(\psi t),$$

and that these expressions for wage inflation and unemployment are the same that are obtained from the intersections of the Phillips curve and DLD curve in each period.

VIII. Technology Shocks

Under the assumption that demand is given by the constant velocity specification, $Y_t = M_t - P_t$, shocks to technology (A) have no effect on nominal wages and no effect on employment and unemployment. Given this specification for demand, equation (14) shows that the only determinant of nominal wages is nominal demand. In addition, (A3) shows that employment depends only on nominal demand and nominal wages. Thus, if nominal demand is held constant, technology shocks would have no effect on either wages or employment. The reason for this prediction is that, with a constant velocity specification, the direct effect of a rise in technology on labor demand is exactly offset by the effect of a decline in prices on labor demand (since an increase in technology reduces prices). While technology shocks have no effect on wages or employment, equation (10) shows that positive technology shocks reduce prices and thus raise real wages.

The prediction that technology shocks reduce prices but do not affect employment and nominal wages is consistent with the findings of Liu and Phaneuf (2007). Using a structural vector autoregression model, they find that a positive technology shock may either raise or lower per capita hours worked, depending on the specification of the model (i.e., whether hours are expressed in terms of log-levels or log-differences). They also find that a positive technology shock slightly lowers nominal wage inflation, but that the effect is not significantly different from 0. Their results also indicate that technology shocks significantly reduce price inflation and significantly increase real wages.

While the predictions of the present study are consistent with the findings of Liu and Phaneuf, it seems unlikely that the effects of technology shocks on wages and employment are exactly 0. As discussed above, the assumption of constant velocity is the reason for these predictions. However, Campbell (2007), which develops a model of the AD-AS framework, shows that technology shocks may affect nominal wages and unemployment if demand is assumed to be determined from an IS-LM model rather than from a constant velocity specification. When demand is determined from an IS-LM system, the price level can be expressed as $\hat{P}_t = \hat{M}_t - \mathrm{k}\hat{Y}_t + \mathrm{y}\hat{E}_t$, where *E* represents real expenditure, and k and y are constants determined by the underlying IS and LM equations. It is demonstrated in Campbell (2007) that positive technology shocks raise both nominal wages and employment if k<1 and reduce these variables if k>1. Even if k>1, however, positive technology shocks may increase wages and employment through their effect on *E*, since positive technology shocks may raise individuals' expectations of their real permanent income and may raise the marginal product of capital, thereby increasing real consumption and investment.

When it is assumed that the economy experiences stochastic shocks to the growth rate of demand, the predicted coefficients in the wage-wage, wage-price, and price-price Phillips curve are given by equations (12), (16), and (17), respectively. If it is instead assumed that the economy experiences a combination of demand shocks and technology shocks, the coefficients in the wage-price and price-price Phillips curve will be different from the values predicted by (16) and (17), since these predicted coefficients are derived from modeling a series of demand shocks. On the other hand, the coefficients in a wage-wage Phillips curve will be the same as the values predicted by (12), since (12) is a reduced-form relationship.

IX. The Cyclical Behavior of Real Wages

We now analyze the model's predictions concerning the cyclical behavior of real wages. When the economy experiences demand shocks, the behavior of real wages is ambiguous. Appendix B shows that, in response to demand shocks, real wages and unemployment are given by the equations

$$\hat{W}_t - \hat{P}_t = \frac{\phi(\omega - e^{-1}e_u s_L)\rho^2 + \phi(1-\omega) - \rho^2}{\sqrt{1-\rho^2}} \sum_{k=1}^{t} \varepsilon_k \rho^{t-k} \sin[\psi(t-k+1)],$$

and

$$du_t = -\frac{s_L \rho^2}{\sqrt{1-\rho^2}} \sum_{k=1}^{t} \varepsilon_k \rho^{t-k} \sin[\psi(t-k+1)].$$

The coefficient on $\sum \varepsilon_k \rho^{t-k} \sin[\psi(t-k+1)]$ is strictly negative in the equation for unemployment. However, the coefficient on this term in the real wage equation is theoretically ambiguous. Thus, real wages can be either procyclical, acyclical, or countercyclical in response to aggregate demand shocks. In response to technology shocks, real wages are unambiguously procyclical, since (as previously discussed) positive (negative) technology shocks decrease (increase) prices, but do not affect nominal wages.

Since real wages can be either procyclical or countercyclical in response to demand shocks, the overall cyclical behavior of real wages is theoretically ambiguous. The fact that real wages can be either procyclical or countercyclical can explain why the cyclical behavior of real wages has appeared to change over time. For example, Huang, Liu, and Phaneuf (2004) discuss evidence from previous studies that find that real wages were countercyclical in the interwar period but have been procyclical since the end of World War II.

X. A Model with Efficiency Depending on the Real Wage

So far it has been assumed that workers' efficiency depends on the ratio between their wages and their expectations of average wages. It could also be assumed that their efficiency depends on the ratio between their wages and their expectations of the price level. There are two reasons why their efficiency may depend on their expectations of the price level. First, in the fair wage model of Akerlof and Yellen (1990), workers may view the fair wage as a function of the real wage. Second, even if workers are concerned about their relative wages, they may use information about price inflation to predict how much wages are rising at other firms, since wage inflation and price inflation are correlated and since price inflation data are more highly publicized than wage inflation data.

If efficiency depends on the ratio between a worker's wage and his or her expectations of the price level ($\bar{P}_t^e$), then (8) and (10) can be expressed as

$$(19) \qquad \hat{W}_t = \frac{e_{WW}}{e_{WW} - s_L(e_u - e_{Wu})}\hat{\bar{P}}_t^e - \frac{s_L(e_u - e_{Wu})}{e_{WW} - s_L(e_u - e_{Wu})}\hat{M}_t, \qquad \text{and}$$

$$(20) \qquad \hat{P}_t = [1 - \mathrm{f} + \mathrm{f}e^{-1}e_u s_L]\hat{M}_t - \mathrm{f}\hat{A}_t - \mathrm{f}e^{-1}e_u s_L \hat{W}_t + \mathrm{f}\hat{\bar{P}}_t^e.$$

Substituting (20) into (19) yields the following equation for the price level:

$$(21) \qquad \hat{P}_t = [1 - \mathrm{f} + \mathrm{f}e^{-1}e_u s_L z]\hat{M}_t - \mathrm{f}\hat{A}_t + \mathrm{f}[1 - \mathrm{f}e^{-1}e_u s_L z]\hat{\bar{P}}_t^e,$$

$$\text{where} \quad z = \frac{e_{WW}}{e_{WW} - s_L(e_u - e_{Wu})} > 0.$$

In addition, (9) can be expressed as

$$(22) \qquad du_t = -s_L[\hat{M}_t - z\hat{\bar{P}}_t^e - (1 - z)\hat{M}_t] = s_L z(\hat{\bar{P}}_t^e - \hat{M}_t),$$

which can be solved for $\hat{M}_t$ to yield

$$(23) \qquad \hat{M}_t = \hat{\bar{P}}_t^e - \frac{1}{s_L z}du_t.$$

If (23) is substituted into (21), the following equation is obtained:

$$(24) \qquad \hat{P}_t = \hat{\bar{P}}_t^e - \frac{[1 - \mathrm{f} + \mathrm{f}e^{-1}e_u s_L z]}{s_L z}du_t - \mathrm{f}\hat{A}_t.$$

Finally, if it is assumed that price expectations are given by

$$\hat{\hat{P}}_t^e = \omega \hat{P}_t + 2(1-\omega)\hat{P}_{t-1} - (1-\omega)\hat{P}_{t-2},$$

the Phillips curve can be expressed as

$$(25) \qquad \hat{P}_t - \hat{P}_{t-1} = \hat{P}_{t-1} - \hat{P}_{t-2} - \frac{[1-\phi+\phi e^{-1} e_u s_L z]}{(1-\omega)s_L z} du_t - \frac{\phi}{1-\omega}\hat{A}_t .$$

Thus, if workers' efficiency depends on their wage relative to their expectations of the price level, a reduced-form equation for the price-price Phillips curve can be derived. In this equation, the coefficient on lagged inflation equals 1, the coefficient on unemployment is negative, and the rate of price inflation depends both on the unemployment rate and on technology shocks.

XI. Conclusion

This study develops a model of wage setting in which firms pay efficiency wages and workers have imperfect information about average wages. Given these assumptions, it is demonstrated that the profit-maximizing behavior of firms yields a downward-sloping Phillips curve and an upward-sloping Dynamic Labor Demand (DLD) curve.

A reduced-form equation for the wage-wage Phillips curve is derived directly from the profit-maximization problem of firms. The wage-price and price-price Phillips curves are obtained by modeling a series of stochastic shocks to demand, calculating expressions for wages, prices, and unemployment, and treating these expressions as data in a regression of wage or price inflation on unemployment and lagged price inflation. In such a regression, the coefficient on lagged inflation asymptotically approaches 1, and it is very close to 1 even when the sample size is small. (In addition, a reduced-form equation for the price-price

Phillips curve can be derived if it is assumed that workers' efficiency depends on their expectations of the price level, rather than of average wages.) Thus, this study can explain why researchers find empirical evidence for the expectations-augmented Phillips curve.

The DLD curve is derived from the same profit maximization problem as the Phillips curve, and the variables that shift the DLD curve are the growth rate of demand and lagged unemployment. The intersection of the Phillips curve and DLD curve determines the economy's unemployment and wage inflation rates. In Section VII the Phillips curve – DLD framework is used to analyze the paths of unemployment and inflation in the transition between their initial equilibrium and their final equilibrium in response to a disinflationary aggregate demand shock.

The model also makes predictions about the cyclical behavior of real wages. In response to productivity shocks, real wages are strictly procyclical. However, when economic fluctuations result from shocks to aggregate demand, real wages can be either procyclical or countercyclical. Thus, the overall cyclical behavior of real wages is theoretically ambiguous. This ambiguity in the cyclical behavior of real wages can explain why real wages appear to behave differently in different time periods.

Appendix A

An expression for $\hat{L}_t$ can be obtained by totally differentiating equation (2) and dividing by the original equation. This yields

$$\hat{L}_t = \frac{\gamma\hat{W}_t - \hat{Y}_t - (\gamma-1)\hat{A}_t - \phi(\gamma-1)e^{-1}[e_W \frac{W_t}{\bar{W}_t^e}\hat{W}_t - e_W \frac{W_t}{\bar{W}_t^e}\hat{\bar{W}}_t^e + e_u du_t] - \gamma\hat{\bar{P}}_t}{\phi(\gamma-1)-\gamma}.$$

By making the substitutions $du_t = -s_L\hat{L}_t$, $W_t/\hat{\bar{W}}_t^e = ee_w^{-1}$, and $\hat{\bar{P}}_t = \hat{M}_t - \hat{Y}_t$, the solution for $\hat{L}_t$ becomes

$$\text{(A1)} \qquad \hat{L}_t = \frac{[\phi(\gamma-1)-\gamma]\hat{W}_t - \phi(\gamma-1)\hat{\bar{W}}_t^e - (\gamma-1)\hat{Y}_t + \phi(\gamma-1)\hat{A}_t + \gamma\hat{M}_t}{\eta},$$

where

$$\eta = \phi + \gamma - \phi\gamma + \phi(\gamma-1)e^{-1}s_L e_u > 0.$$

An equation for $\hat{Y}_t$ can be obtained by setting Q_t=Y_t in the production function (from assumption 3) and totally differentiating. This yields

$$\text{(A2)} \qquad \hat{Y}_t = \phi\hat{A}_t + \phi\hat{L}_t + \phi e^{-1}[e_W\hat{W}_t - e_W\hat{\bar{W}}_t^e + e_u du_t]$$

$$= \phi\hat{A}_t + \phi\hat{L}_t + \phi e^{-1}[e_W\hat{W}_t - e_W\hat{\bar{W}}_t^e - e_u s_L\hat{L}_t]$$

Substituting (A2) into (A1) yields

(A3) $\hat{L}_t = \hat{M}_t - \hat{W}_t$.

Thus, the unemployment rate can be expressed as

(A4) $du_t = -s_L(\hat{M}_t - \hat{W}_t)$.

By substituting (A4) into (6), the following equation is obtained:

$$\left[1 - s_L \frac{e_u - e_{Wu}}{e_{WW}}\right]\hat{W}_t = \hat{\overline{W}}_t^e - s_L \frac{e_u - e_{Wu}}{e_{WW}} \hat{M}_t,$$

which can be rewritten as

(A5) $$\hat{W}_t = \frac{e_{WW}}{e_{WW} - s_L(e_u - e_{Wu})} \hat{\overline{W}}_t^e - \frac{s_L(e_u - e_{Wu})}{e_{WW} - s_L(e_u - e_{Wu})} \hat{M}_t.$$

The price level is given by the equation $\hat{P}_t = \hat{M}_t - \hat{Y}_t$. Substituting (A3) into (A2) yields the following expression for $\hat{Y}_t$:

$$\hat{Y}_t = f\hat{A}_t + f(1 - e^{-1}e_u s_L)\hat{M}_t + e^{-1}e_u s_L \hat{W}_t - f\hat{\overline{W}}_t^e.$$

Thus, the price level is given by the equation:

$$\hat{P}_t = [1 - f + fe^{-1}e_u s_L]\hat{M}_t - f\hat{A}_t - e^{-1}e_u s_L \hat{W}_t + f\hat{\overline{W}}_t^e.$$

Appendix B

From equation (12), wages are determined from the second-order difference equation:

$$\text{(B1)} \quad \hat{W}_t - 2a\hat{W}_{t-1} + a\hat{W}_{t-2} = (1-a)\hat{M}_t ,$$

where 0<a<1. Equation (B1) has two imaginary roots:

$$\mathrm{m}_1 = a + i\sqrt{a - a^2}$$

$$\mathrm{m}_2 = a - i\sqrt{a - a^2} \ .$$

Then the solution to the difference equation is

$$\text{(B2)} \quad \hat{W}_t = (1-\mathrm{a})\frac{1}{\mathrm{m}_1 - \mathrm{m}_2}\sum_{j=0}^{\infty}(\mathrm{m}_1^{j+1} - \mathrm{m}_2^{j+1})\hat{M}_{t-j} .$$

We now solve for $\hat{W}_t$ when the growth rate of demand follows a stochastic process. It is assumed that

$$\hat{M}_t = 0 \qquad \text{for} \quad t \le 0$$

$$\hat{M}_t - \hat{M}_{t-1} = \hat{M}_{t-1} - \hat{M}_{t-2} + \mathrm{e}_t \qquad \text{for} \ t > 0 .$$

Thus, $\hat{M}_t$ can be expressed as

$$\hat{M}_t = 2\hat{M}_{t-1} - \hat{M}_{t-2} + \mathrm{e}_t ,$$

which means that

$$\hat{M}_t = \sum_{k=1}^{t} k\varepsilon_{t-k+1} \ .$$

This is equivalent to the equation,

$$\hat{M}_t = \sum_{k=1}^{t} (t-k+1)\varepsilon_k \ .$$

Let $b = \sqrt{a - a^2}$.

Then $\hat{W}_t$ can be expressed as

$$\hat{W}_t = (1-a)\frac{1}{\mu_1 - \mu_2}\sum_{j=0}^{t-1}\left[[(a+bi)^{j+1} - (a-bi)^{j+1}]\sum_{k=1}^{t-j}(t-k+1-j)\varepsilon_k \right]$$

$$= (1-a)\frac{1}{2bi}\sum_{k=1}^{t}\varepsilon_k \sum_{j=0}^{t-k}(t+1-k-j)\left[(a+bi)^{j+1} - (a-bi)^{j+1}\right]$$

$$= (1-a)\frac{1}{2bi}\sum_{k=1}^{t}\varepsilon_k \left[\sum_{j=0}^{t-k}(t+1-k)\left[(a+bi)^{j+1} - (a-bi)^{j+1}\right] - \sum_{j=0}^{t-k}\left[j(a+bi)^{j+1} - j(a-bi)^{j+1}\right]\right]$$

The relationships,

$$\sum_{q=0}^{n} x^q = \frac{1-x^{n+1}}{1-x} \quad \text{and} \quad \sum_{q=0}^{n} qx^{q-1} = \frac{1-(n+1)x^n + nx^{n+1}}{(1-x)^2} ,$$

allow $\hat{W}_t$ to be rewritten as

$$\hat{W}_t = (1-a)\frac{1}{2bi}\sum_{k=1}^{t}\varepsilon_k\Bigg[(t-k+1)(a+bi)\frac{1-(a+bi)^{t-k+1}}{1-a-bi}$$
$$-(t-k+1)(a-bi)\frac{1-(a-bi)^{t-k+1}}{1-a+bi}$$
$$-(a+bi)^2\frac{1-(t-k+1)(a+bi)^{t-k}+(t-k)(a+bi)^{t-k+1}}{(1-a-bi)^2}$$
$$+(a-bi)^2\frac{1-(t-k+1)(a-bi)^{t-k}+(t-k)(a-bi)^{t-k+1}}{(1-a+bi)^2}\Bigg]$$

$$= (1-a)\frac{1}{2bi}\sum_{k=1}^{t}\varepsilon_k\Bigg[(t-k+1)(a+bi)\frac{[1-(a+bi)^{t-k+1}][1-a+bi]}{[1-a-bi][1-a+bi]}$$
$$-(t-k+1)(a-bi)\frac{[1-(a-bi)^{t-k+1}][1-a-bi]}{[1-a+bi][1-a-bi]}$$
$$-(a+bi)^2\frac{[1-(t-k+1)(a+bi)^{t-k}+(t-k)(a+bi)^{t-k+1}][1-a+bi]^2}{[1-a-bi]^2[1-a+bi]^2}$$
$$+(a-bi)^2\frac{[1-(t-k+1)(a-bi)^{t-k}+(t-k)(a-bi)^{t-k+1}][1-a-bi]^2}{[1-a+bi]^2[1-a-bi]^2}\Bigg]$$

$$= (1-a)\frac{1}{2bi}\sum_{k=1}^{t}\varepsilon_k\Bigg[(t-k+1)(a+bi)\frac{[1-(a+bi)^{t-k+1}][1-a+bi][(1-a)^2+b^2]}{[(1-a)^2+b^2]^2}$$
$$-(t-k+1)(a-bi)\frac{[1-(a-bi)^{t-k+1}][1-a-bi][(1-a)^2+b^2]}{[(1-a)^2+b^2]^2}$$
$$-(a+bi)^2\frac{[1-(t-k+1)(a+bi)^{t-k}+(t-k)(a+bi)^{t-k+1}][1-a+bi]^2}{[(1-a)^2+b^2]^2}$$
$$+(a-bi)^2\frac{[1-(t-k+1)(a-bi)^{t-k}+(t-k)(a-bi)^{t-k+1}][1-a-bi]^2}{[(1-a)^2+b^2]^2}\Bigg]$$

Let $\rho = \sqrt{a}$.

Then, $b = \rho\sqrt{1-\rho^2}$.

Then

$$\hat{W}_t = \sum_{k=1}^{t} \varepsilon_k \left[t-k+1-\frac{\rho(a+bi)^{t-k+1}-\rho(a-bi)^{t-k+1}}{2\sqrt{1-\rho^2}\,i} \right]$$

$$\hat{W}_t = \sum_{k=1}^{t} \varepsilon_k \left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}} \right]$$

This last equation is obtained from the fact that

$$(a+bi)^{t-k+1}-(a-bi)^{t-k+1} = 2i\rho^{t-k+1}\sin[\psi(t-k+1)],$$

where $\rho = \sqrt{a^2+b^2}$ and $\psi = \arccos(a/\rho)$.

Given the above expression for wages, the change in wages between periods *t*-1 and *t* is

$$\hat{W}_t - \hat{W}_{t-1} = \sum_{k=1}^{t} \varepsilon_k \left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}} \right] - \sum_{k=1}^{t-1} \varepsilon_k \left[t-k-\frac{\rho^{t-k+1}\sin[\psi(t-k)]}{\sqrt{1-\rho^2}} \right]$$

$$= \sum_{k=1}^{t} \varepsilon_k \left[1-\rho^{t-k+1}\cos[\psi(t-k+1)] \right]$$

The unemployment rate can also be expressed in terms of the underlying demand shocks. From equation (9),

$$du_t = -s_L(\hat{M}_t - \hat{W}_t)$$

$$= -\frac{s_L}{\sqrt{1-\rho^2}}\sum_{k=1}^{t}\varepsilon_k\,\rho^{t-k+2}\sin[\psi(t-k+1)]\,.$$

An expression for the price level can be derived by substituting (11) into (10), yielding

$$\hat{P}_t = [1-\phi+\phi e^{-1}e_u s_L]\hat{M}_t - \phi\hat{A}_t + \phi(\omega - e^{-1}e_u s_L)\hat{W}_t + 2\phi(1-\omega)\hat{W}_{t-1} - \phi(1-\omega)\hat{W}_{t-2}$$

$$\begin{aligned}
&= [1-\phi+\phi e^{-1}e_u s_L]\sum_{k=1}^{t}\varepsilon_k(t-k+1)\\
&\quad + \phi(\omega - e^{-1}e_u s_L)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right]\\
&\quad + 2\phi(1-\omega)\sum_{k=1}^{t-1}\varepsilon_k\left[t-k-\frac{\rho^{t-k+1}\sin[\psi(t-k)]}{\sqrt{1-\rho^2}}\right]\\
&\quad - \phi(1-\omega)\sum_{k=1}^{t-2}\varepsilon_k\left[t-k-1-\frac{\rho^{t-k}\sin[\psi(t-k-1)]}{\sqrt{1-\rho^2}}\right]
\end{aligned}$$

$$\begin{aligned}
&= [1-\phi+\phi e^{-1}e_u s_L]\sum_{k=1}^{t}\varepsilon_k(t-k+1)\\
&\quad + \phi(\omega - e^{-1}e_u s_L)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right]\\
&\quad + 2\phi(1-\omega)\sum_{k=1}^{t-1}\varepsilon_k\left[t-k-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)] - \rho^{t-k+1}\sqrt{1-\rho^2}\cos[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right]\\
&\quad - \phi(1-\omega)\sum_{k=1}^{t-2}\varepsilon_k\left[t-k-1-\frac{\rho^{t-k}(2\rho^2-1)\sin[\psi(t-k+1)] - 2\sqrt{1-\rho^2}\,\rho^{t-k+1}\cos[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right]
\end{aligned}$$

$$
\begin{aligned}
&= [1-\varphi+\varphi e^{-1}e_u s_L]\sum_{k=1}^{t}\varepsilon_k(t-k+1) \\
&\quad +\varphi(\omega-e^{-1}e_u s_L)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right] \\
&\quad +2\varphi(1-\omega)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right]+2\varphi(1-\omega)\rho^2\varepsilon_t \\
&\quad -2\varphi(1-\omega)\sum_{k=1}^{t}\varepsilon_k \\
&\quad -\varphi(1-\omega)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k}(2\rho^2-1)\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right]+2\varphi(1-\omega)\sum_{k=1}^{t}\varepsilon_k \\
&\quad -\varphi(1-\omega)\varepsilon_t+2\varphi(1-\omega)\rho^2(2\rho^2-1)\varepsilon_{t-1}-2\varphi(1-\omega)\rho^2(2\rho^2-1)\varepsilon_{t-1} \\
&\quad -\varphi(1-\omega)(2\rho^2-1)\varepsilon_t
\end{aligned}
$$

$$
\begin{aligned}
&= [1-\varphi+\varphi e^{-1}e_u s_L]\sum_{k=1}^{t}\varepsilon_k(t-k+1) \\
&\quad +\varphi(\omega-e^{-1}e_u s_L)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right] \\
&\quad +2\varphi(1-\omega)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k+2}\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right] \\
&\quad -\varphi(1-\omega)\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\rho^{t-k}(2\rho^2-1)\sin[\psi(t-k+1)]}{\sqrt{1-\rho^2}}\right]
\end{aligned}
$$

$$
=\sum_{k=1}^{t}\varepsilon_k\left[t-k+1-\frac{\varphi(\omega-e^{-1}e_u s_L)\rho^2+\varphi(1-\omega)}{\sqrt{1-\rho^2}}\,\rho^{t-k}\sin[\psi(t-k+1)]\right]
$$

The rate of price inflation is

$$\hat{P}_t - \hat{P}_{t-1} = \sum_{k=1}^{t} \mathrm{e}_k \left[1 - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{\,t-k-1} \cos[\mathrm{y}\,(t-k+1)]\right]$$

Also,

$$\hat{P}_{t-1} - \hat{P}_{t-2} = \sum_{k=1}^{t-1} \mathrm{e}_k \left[1 - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{\,t-k-2} \cos[\mathrm{y}\,(t-k)]\right]$$

$$\hat{P}_{t-1} - \hat{P}_{t-2} = \sum_{k=1}^{t-1} \mathrm{e}_k \Big[1 - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{\,t-k-1} \cos[\mathrm{y}\,(t-k+1)]$$
$$- \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{\,t-k-2} \sqrt{1-\mathrm{r}^2}\, \sin[\mathrm{y}\,(t-k+1)]\Big]$$

Subtracting $\hat{P}_t$ from $\hat{W}_t$ yields an expression for real wages. Accordingly,

$$\hat{W}_t - \hat{P}_t = \sum_{k=1}^{t} \mathrm{e}_k \left[t - k + 1 - \frac{\mathrm{r}^{\,t-k+2} \sin[\mathrm{y}\,(t-k+1)]}{\sqrt{1-\mathrm{r}^2}} \right]$$
$$- \sum_{k=1}^{t} \mathrm{e}_k \left[t - k + 1 - \frac{\mathrm{f}\,(\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + \mathrm{f}\,(1-\mathrm{w})}{\sqrt{1-\mathrm{r}^2}}\, \mathrm{r}^{\,t-k} \sin[\mathrm{y}\,(t-k+1)] \right]$$

$$= \frac{\mathrm{f}\,(\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + \mathrm{f}\,(1-\mathrm{w}) - \mathrm{r}^2}{\sqrt{1-\mathrm{r}^2}} \sum_{k=1}^{t} \mathrm{e}_k\, \mathrm{r}^{\,t-k} \sin[\mathrm{y}\,(t-k+1)]$$

Appendix C

Demand shocks

Regression : $(\hat{W}_t - \hat{W}_{t-1}) = \beta_1 du_t + \beta_2(\hat{P}_{t-1} - \hat{P}_{t-2}) + e_t$

$$\mathbf{X'X} = \begin{bmatrix} \sum_{t=1}^{T} du_t^2 & \sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2}) \\ \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})du_t & \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2 \end{bmatrix}$$

$$(\mathbf{X'X})^{-1} = \frac{\begin{bmatrix} \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2 & -\sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2}) \\ -\sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2}) & \sum_{t=1}^{T} du_t^2 \end{bmatrix}}{\sum_{t=1}^{T} du_t^2 \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2 - \left(\sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2})\right)^2}$$

$$\mathbf{X'y} = \begin{bmatrix} \sum_{t=1}^{T} du_t(\hat{W}_t - \hat{W}_{t-1}) \\ \sum_{t=1}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2}) \end{bmatrix}$$

$$\hat{\boldsymbol{\beta}} = \frac{\begin{bmatrix} \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2 \sum_{t=1}^{T} du_t(\hat{W}_t - \hat{W}_{t-1}) - \sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2}) \sum_{t=1}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2}) \\ -\sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2}) \sum_{t=1}^{T} du_t(\hat{W}_t - \hat{W}_{t-1}) + \sum_{t=1}^{T} du_t^2 \sum_{t=1}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2}) \end{bmatrix}}{\sum_{t=1}^{T} du_t^2 \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2 - \left(\sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2})\right)^2}$$

Define

$$D = \sum_{t=1}^{T} du_t^2 \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2 - \left(\sum_{t=1}^{T} du_t (\hat{P}_{t-1} - \hat{P}_{t-2}) \right)^2$$

$$N_1 = \sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2 \sum_{t=1}^{T} du_t (\hat{W}_t - \hat{W}_{t-1}) - \sum_{t=1}^{T} du_t (\hat{P}_{t-1} - \hat{P}_{t-2}) \sum_{t=1}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2})$$

$$N_2 = -\sum_{t=1}^{T} du_t (\hat{P}_{t-1} - \hat{P}_{t-2}) \sum_{t=1}^{T} du_t (\hat{W}_t - \hat{W}_{t-1}) + \sum_{t=1}^{T} du_t^2 \sum_{t=1}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2})$$

Then $\beta_1 = \frac{N_1}{D}$

$$\beta_2 = \frac{N_2}{D}.$$

$$\lim_{T\to\infty} \beta_1 = \frac{\lim_{T\to\infty} (N_1 / T^3)}{\lim_{T\to\infty} (D / T^3)}$$

$$\lim_{T\to\infty} \beta_2 = \frac{\lim_{T\to\infty} (N_2 / T^3)}{\lim_{T\to\infty} (D / T^3)}$$

Calculations of sums are derived in Appendix D.

$$\sum_{t=t_0}^{T} du_t^2 = \sum_{t=t_0}^{T} \left[-\frac{s_L \rho^2}{\sqrt{1-\rho^2}} \sum_{k=1}^{t} \varepsilon_k \rho^{t-k} \sin[\psi (t-k+1)] \right]^2$$

$$= \frac{\sigma^2 s_L^2 \rho^4}{1-\rho^2} \sum_{t=t_0}^{T} \sum_{k=1}^{t} \rho^{2(t-k)} \sin^2[\psi (t-k+1)]$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} du_t^2 = \frac{\sigma^2 s_L^2 \rho^4}{1-\rho^2} \frac{1-\rho^4}{(1-\rho^2)(1+2\rho^2-3\rho^4)}$$

$$\sum_{t=t_0}^{T}(\hat{P}_{t-1}-\hat{P}_{t-2})^2$$

$$=\sum_{t=t_0}^{T}\left\{\sum_{k=1}^{t-1}\mathrm{e}_k\left[1-\mathrm{f}\left((\mathrm{w}-e^{-1}e_u s_L)\,\mathrm{r}^{\,2}+1-\mathrm{w}\right)\mathrm{r}^{\,t-k-2}\cos[\mathrm{y}\,(t-k)]\right]\right\}^2$$

$$=\mathrm{s}^{\,2}\sum_{t=t_0}^{T}\left\{\sum_{k=1}^{t-1}\left[1-2\mathrm{f}\left((\mathrm{w}-e^{-1}e_u s_L)\,\mathrm{r}^{\,2}+1-\mathrm{w}\right)\mathrm{r}^{\,t-k-2}\cos[\mathrm{y}\,(t-k)]\right.\right.$$
$$\left.\left.+\mathrm{f}^{\,2}\left((\mathrm{w}-e^{-1}e_u s_L)\,\mathrm{r}^{\,2}+1-\mathrm{w}\right)^2\mathrm{r}^{\,2t-2k-4}\cos^2[\mathrm{y}\,(t-k)]\right]\right\}$$

$$=\mathrm{s}^{\,2}\,\frac{(T+t_0-2)(T-t_0+1)}{2}$$
$$+\mathrm{s}^{\,2}\sum_{t=t_0}^{T}\left\{\sum_{k=1}^{t-1}\left[-2\mathrm{f}\left((\mathrm{w}-e^{-1}e_u s_L)\,\mathrm{r}^{\,2}+1-\mathrm{w}\right)\mathrm{r}^{\,t-k-2}\cos[\mathrm{y}\,(t-k)]\right.\right.$$
$$\left.\left.+\mathrm{f}^{\,2}\left((\mathrm{w}-e^{-1}e_u s_L)\,\mathrm{r}^{\,2}+1-\mathrm{w}\right)^2\mathrm{r}^{\,2t-2k-4}\cos^2[\mathrm{y}\,(t-k)]\right]\right\}$$

$$\lim_{T\to\infty}\frac{1}{T^2}\sum_{t=t_0}^{T}(\hat{P}_{t-1}-\hat{P}_{t-2})^2=\frac{\mathrm{s}^{\,2}}{2}$$

$$\sum_{t=t_0}^{T}du_t(\hat{P}_{t-1}-\hat{P}_{t-2})$$

$$=\sum_{t=t_0}^{T}\left\{-\frac{s_L\,\mathrm{r}^{\,2}}{\sqrt{1-\mathrm{r}^{\,2}}}\sum_{k=1}^{t}\mathrm{e}_k\,\mathrm{r}^{\,t-k}\sin[\mathrm{y}\,(t-k+1)]\right.$$
$$\left.\times\sum_{k=1}^{t-1}\mathrm{e}_k\left[1-\mathrm{f}\left((\mathrm{w}-e^{-1}e_u s_L)\,\mathrm{r}^{\,2}+1-\mathrm{w}\right)\mathrm{r}^{\,t-k-1}\cos[\mathrm{y}\,(t-k)]\right]\right\}$$

$$=\sum_{t=t_0}^{T}\left\{-\frac{\mathrm{s}^{\,2}s_L\,\mathrm{r}^{\,2}}{\sqrt{1-\mathrm{r}^{\,2}}}\sum_{k=1}^{t-1}\left[\mathrm{r}^{\,t-k}\sin[\mathrm{y}\,(t-k+1)]\right.\right.$$
$$\left.\left.-\mathrm{f}\left((\mathrm{w}-e^{-1}e_u s_L)\,\mathrm{r}^{\,2}+1-\mathrm{w}\right)\mathrm{r}^{\,2t-2k-1}\sin[\mathrm{y}\,(t-k+1)]\cos[\mathrm{y}\,(t-k)]\right]\right\}$$

$$= \sum_{t=t_0}^{T} \left\{ -\frac{s_L \mathrm{r}^2}{\sqrt{1-\mathrm{r}^2}} \sum_{k=1}^{t} \mathrm{e}_k \mathrm{r}^{t-k} \sin[\mathrm{y}(t-k+1)] \right.$$

$$\times \sum_{k=1}^{t-1} \mathrm{e}_k \left[1 - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L) \mathrm{r}^2 + 1 - \mathrm{w} \right) \mathrm{r}^{t-k-1} \cos[\mathrm{y}(t-k+1)] \right.$$

$$\left. \left. - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L) \mathrm{r}^2 + 1 - \mathrm{w} \right) \mathrm{r}^{t-k-2} \sqrt{1-\mathrm{r}^2} \sin[\mathrm{y}(t-k+1)] \right] \right\}$$

$$= -\frac{\mathrm{s}^2 s_L \mathrm{r}^2}{\sqrt{1-\mathrm{r}^2}} \sum_{t=t_0}^{T} \left\{ \sum_{k=1}^{t-1} \mathrm{r}^{t-k} \sin[\mathrm{y}(t-k+1)] \right.$$

$$- \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L) \mathrm{r}^2 + 1 - \mathrm{w} \right) \left[\sum_{k=1}^{t-1} \mathrm{r}^{2t-2k-1} \sin[\mathrm{y}(t-k+1)] \cos[\mathrm{y}(t-k+1) \right.$$

$$\left. \left. + \sqrt{1-\mathrm{r}^2} \sum_{k=1}^{t-1} \mathrm{r}^{2t-2k-2} \sin^2[\mathrm{y}(t-k+1)] \right] \right\}$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} du_t (\hat{P}_{t-1} - \hat{P}_{t-2})$$

$$= -\frac{\mathrm{s}^2 s_L \mathrm{r}^2}{\sqrt{1-\mathrm{r}^2}} \left[\frac{\mathrm{r}^2}{\sqrt{1-\mathrm{r}^2}} - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L) \mathrm{r}^2 + 1 - \mathrm{w} \right) \frac{3(1-\mathrm{r}^2)\mathrm{r}^2 \sqrt{1-\mathrm{r}^2}}{(1-\mathrm{r}^2)(1+2\mathrm{r}^2-3\mathrm{r}^4)} \right]$$

$$= -\mathrm{s}^2 s_L \mathrm{r}^2 \left[\frac{\mathrm{r}^2(1+2\mathrm{r}^2-3\mathrm{r}^4) - 3\mathrm{r}^2(1-\mathrm{r}^2)\mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L) \mathrm{r}^2 + 1 - \mathrm{w} \right)}{(1-\mathrm{r}^2)(1+2\mathrm{r}^2-3\mathrm{r}^4)} \right]$$

$$\sum_{t=t_0}^{T} du_t (\hat{W}_t - \hat{W}_{t-1})$$

$$= \sum_{t=t_0}^{T} \left\{ -\frac{s_L \mathrm{r}^2}{\sqrt{1-\mathrm{r}^2}} \sum_{k=1}^{t} \mathrm{e}_k \mathrm{r}^{t-k} \sin[\mathrm{y}(t-k+1)] \right.$$

$$\left. \times \sum_{k=1}^{t} \mathrm{e}_k \left[1 - \mathrm{r}^{t-k+1} \cos[\mathrm{y}(t-k+1)] \right] \right\}$$

$$= -\frac{\sigma^2 s_L \rho^2}{\sqrt{1-\rho^2}} \sum_{t=t_0}^{T} \left\{ \sum_{k=1}^{t} \rho^{t-k} \sin[\psi(t-k+1)] \right.$$

$$\left. - \sum_{k=1}^{t} \rho^{2t-2k+1} \sin[\psi(t-k+1)] \cos[\psi(t-k+1)] \right\}$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} du_t (\hat{W}_t - \hat{W}_{t-1})$$

$$= -\frac{\sigma^2 s_L \rho^2}{\sqrt{1-\rho^2}} \left[\frac{1}{\sqrt{1-\rho^2}} - \frac{\rho^2 \sqrt{1-\rho^2}}{1+2\rho^2-3\rho^4} \right]$$

$$= -\sigma^2 s_L \rho^2 \left[\frac{1+\rho^2-2\rho^4}{(1-\rho^2)(1+2\rho^2-3\rho^4)} \right]$$

$$\sum_{t=t_0}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2})$$

$$= \sum_{t=t_0}^{T} \left\{ \sum_{k=1}^{t} \varepsilon_k \left[1 - \rho^{t-k+1} \cos[\psi(t-k+1)] \right] \right.$$

$$\left. \times \sum_{k=1}^{t-1} \varepsilon_k \left[1 - \phi\left((\omega - e^{-1} e_u s_L) \rho^2 + 1 - \omega \right) \rho^{t-k-2} \cos[\psi(t-k)] \right] \right\}$$

$$= \sigma^2 \sum_{t=t_0}^{T} \left\{ \sum_{k=1}^{t-1} \left[1 - \phi\left((\omega - e^{-1} e_u s_L) \rho^2 + 1 - \omega \right) \rho^{t-k-2} \cos[\psi(t-k)] \right. \right.$$

$$- \rho^{t-k+1} \cos[\psi(t-k+1)$$

$$\left. \left. + \phi\left((\omega - e^{-1} e_u s_L) \rho^2 + 1 - \omega \right) \rho^{2t-2k-1} \cos[\psi(t-k+1)] \cos[\psi(t-k)] \right] \right\}$$

$$= \sigma^2 \frac{(T+t_0-2)(T-t_0+1)}{2}$$

$$+\sigma^2 \sum_{t=t_0}^{T} \left\{ \sum_{k=1}^{t-1} \left[-\phi \left((\omega - e^{-1} e_u s_L)\rho^2 + 1 - \omega \right) \rho^{t-k-2} \cos[\psi(t-k)] \right. \right.$$

$$- \rho^{t-k+1} \cos[\psi(t-k+1)$$

$$\left. \left. + \phi \left((\omega - e^{-1} e_u s_L)\rho^2 + 1 - \omega \right) \rho^{2t-2k-1} \cos[\psi(t-k+1)] \cos[\psi(t-k)] \right] \right\}$$

$$\lim_{T\to\infty} \frac{1}{T^2} \sum_{t=t_0}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2}) = \frac{\sigma^2}{2}$$

$$\lim_{T\to\infty} \frac{D}{T^3} = \lim_{T\to\infty} \left[\frac{\sum_{t=1}^{T} du_t^2}{T} \frac{\sum_{j=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2}{T^2} - \frac{\left(\sum_{t=1}^{T} du_t (\hat{P}_{t-1} - \hat{P}_{t-2}) \right)^2}{T^3} \right]$$

$$= \frac{\sigma^4 s_L^2 \rho^4 (1-\rho^4)}{2(1-\rho^2)^2 (1+2\rho^2 - 3\rho^4)}$$

$$\lim_{T\to\infty} \frac{N_1}{T^3}$$

$$= \lim_{T\to\infty} \left[\frac{\sum_{t=1}^{T} (\hat{P}_{t-1} - \hat{P}_{t-2})^2}{T^2} \frac{\sum_{t=1}^{T} du_t (\hat{W}_t - \hat{W}_{t-1})}{T} \right.$$

$$\left. - \frac{\sum_{t=1}^{T} du_t (\hat{P}_{t-1} - \hat{P}_{t-2})}{T} \frac{\sum_{t=1}^{T} (\hat{W}_t - \hat{W}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2})}{T^2} \right]$$

$$= -\frac{\sigma^4 s_L \rho^2 (1+\rho^2-2\rho^4)}{2(1-\rho^2)(1+2\rho^2-3\rho^4)} + \sigma^4 s_L \rho^2 \left[\frac{\rho^2(1+2\rho^2-3\rho^4)}{2(1-\rho^2)(1+2\rho^2-3\rho^4)} \right.$$

$$\left. -\phi\left((\omega - e^{-1} e_u s_L)\rho^2 + 1 - \omega\right) \frac{3\rho^2(1-\rho^2)}{2(1-\rho^2)(1+2\rho^2-3\rho^4)} \right]$$

$$= \frac{\sigma^4 s_L \rho^2}{2(1-\rho^2)(1+2\rho^2-3\rho^4)} \left[-1+4\rho^4-3\rho^6-3\rho^2(1-\rho^2)\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)\right]$$

$$\lim_{T\to\infty} \frac{N_2}{T^3}$$

$$= \lim_{T\to\infty} \left[-\frac{\sum_{t=1}^{T} du_t(\hat{P}_{t-1}-\hat{P}_{t-2})}{T} \frac{\sum_{t=1}^{T} du_t(\hat{W}_t-\hat{W}_{t-1})}{T^2} + \frac{\sum_{t=1}^{T} du_t^2}{T} \frac{\sum_{t=1}^{T}(\hat{W}_t-\hat{W}_{t-1})(\hat{P}_{t-1}-\hat{P}_{t-2})}{T^2} \right]$$

$$= \frac{\sigma^2 s_L^2 \rho^4 (1-\rho^4)}{2(1-\rho^2)^2(1+2\rho^2-3\rho^4)}$$

$$\lim_{T\to\infty} \hat{\beta}_1 = \lim_{T\to\infty} \frac{(N_1/T^3)}{(D/T^3)}$$

$$= \frac{(1-\rho^2)\left[-1+4\rho^4-3\rho^6-3\rho^2(1-\rho^2)\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)\right]}{s_L \rho^2 (1-\rho^4)}$$

$$= \frac{(1-\rho^2)\left[-1+\rho^4+3\rho^4-3\rho^6-3\rho^2(1-\rho^2)\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)\right]}{s_L \rho^2 (1-\rho^4)}$$

$$= -\frac{1-\rho^2}{s_L \rho^2} + 3\frac{(1-\rho^2)\left[\rho^4(1-\rho^2)-\rho^2(1-\rho^2)\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)\right]}{s_L \rho^2(1+\rho^2)(1-\rho^2)}$$

$$= -\frac{1-a}{s_L a} + 3(1-\rho^2)\frac{\rho^2-\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)}{s_L(1+\rho^2)}$$

$$= \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}} + 3(1-a)\frac{a-\phi\left((\omega-e^{-1}e_u s_L)a+1-\omega\right)}{s_L(1+a)}$$

$$\lim_{T\to\infty} \hat{\beta}_2 = \lim_{T\to\infty} \frac{(N_2/T^3)}{(D/T^3)} = 1$$

$$\underline{\text{Regression : } (\hat{P}_t - \hat{P}_{t-1}) = \beta_1 du_t + \beta_2(\hat{P}_{t-1} - \hat{P}_{t-2}) + e_t}$$

$$\hat{\boldsymbol{\beta}} = \frac{\left[\begin{array}{c} \sum_{t=1}^{T}(\hat{P}_{t-1} - \hat{P}_{t-2})^2 \sum_{t=1}^{T} du_t(\hat{P}_t - \hat{P}_{t-1}) - \sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2}) \sum_{t=1}^{T}(\hat{P}_t - \hat{P}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2}) \\ - \sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2}) \sum_{t=1}^{T} du_t(\hat{P}_t - \hat{P}_{t-1}) + \sum_{t=1}^{T} du_t^2 \sum_{t=1}^{T}(\hat{P}_t - \hat{P}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2}) \end{array}\right]}{\sum_{t=1}^{T} du_t^2 \sum_{t=1}^{T}(\hat{P}_{t-1} - \hat{P}_{t-2})^2 - \left(\sum_{t=1}^{T} du_t(\hat{P}_{t-1} - \hat{P}_{t-2})\right)^2}$$

$$\sum_{t=t_0}^{T} du_t(\hat{P}_t - \hat{P}_{t-1})$$

$$= \sum_{t=t_0}^{T} \left\{ -\frac{s_L \rho^2}{\sqrt{1-\rho^2}} \sum_{k=1}^{t} \varepsilon_k \rho^{t-k} \sin[\psi(t-k+1)] \right.$$
$$\left. \times \sum_{k=1}^{t} \varepsilon_k \left[1 - \phi\left((\omega - e^{-1}e_u s_L)\rho^2 + 1 - \omega\right)\rho^{t-k-1} \cos[\psi(t-k+1)]\right] \right\}$$

$$= -\frac{\sigma^2 s_L \rho^2}{\sqrt{1-\rho^2}} \sum_{t=t_0}^{T} \left\{ \sum_{k=1}^{t} \rho^{t-k} \sin[\psi(t-k+1)] \right.$$
$$\left. - \phi\left((\omega - e^{-1}e_u s_L)\rho^2 + 1 - \omega\right)\rho^{-2} \sum_{k=1}^{t} \rho^{2t-2k+1} \cos[\psi(t-k+1)] \sin[\psi(t-k+1)] \right\}$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} du_t(\hat{P}_t - \hat{P}_{t-1})$$

$$= -\frac{\mathrm{s}^2 s_L \mathrm{r}^2}{\sqrt{1-\mathrm{r}^2}}\left[\frac{1}{\sqrt{1-\mathrm{r}^2}} - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\frac{\sqrt{1-\mathrm{r}^2}}{1+2\mathrm{r}^2-3\mathrm{r}^4}\right]$$

$$= -\mathrm{s}^2 s_L \mathrm{r}^2\left[\frac{1+2\mathrm{r}^2-3\mathrm{r}^4}{(1-\mathrm{r}^2)(1+2\mathrm{r}^2-3\mathrm{r}^4)} - \frac{\mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)(1-\mathrm{r}^2)}{(1-\mathrm{r}^2)(1+2\mathrm{r}^2-3\mathrm{r}^4)}\right]$$

$$\sum_{t=t_0}^{T}(\hat{P}_t - \hat{P}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2})$$

$$= \sum_{t=t_0}^{T}\left\{\sum_{k=1}^{t}\mathrm{e}_k\left[1 - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{t-k-1}\cos[\mathrm{y}\,(t-k+1)]\right]\right.$$
$$\left.\times\sum_{k=1}^{t-1}\mathrm{e}_k\left[1 - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{t-k-2}\cos[\mathrm{y}\,(t-k)]\right]\right\}$$

$$= \mathrm{s}^2\sum_{t=t_0}^{T}\left\{\sum_{k=1}^{t-1}\left[1 - \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{t-k-2}\cos[\mathrm{y}\,(t-k)]\right.\right.$$
$$- \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{t-k-1}\cos[\mathrm{y}\,(t-k+1)]$$
$$\left.\left.+ \mathrm{f}^2\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)^2\mathrm{r}^{2t-2k-3}\cos[\mathrm{y}\,(t-k)]\cos[\mathrm{y}\,(t-k+1)]\right]\right\}$$

$$= \mathrm{s}^2\,\frac{(T+t_0-2)(T-t_0+1)}{2}$$
$$+ \mathrm{s}^2\sum_{t=t_0}^{T}\left\{\sum_{k=1}^{t-1}\left[-\mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{t-k-2}\cos[\mathrm{y}\,(t-k)]\right.\right.$$
$$- \mathrm{f}\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)\mathrm{r}^{t-k-1}\cos[\mathrm{y}\,(t-k+1)]$$
$$\left.\left.+ \mathrm{f}^2\left((\mathrm{w} - e^{-1} e_u s_L)\,\mathrm{r}^2 + 1 - \mathrm{w}\right)^2\mathrm{r}^{2t-2k-3}\cos[\mathrm{y}\,(t-k)]\cos[\mathrm{y}\,(t-k+1)]\right]\right\}$$

$$\lim_{t\to\infty}\frac{1}{T^2}\sum_{t=t_0}^{T}(\hat{P}_t - \hat{P}_{t-1})(\hat{P}_{t-1} - \hat{P}_{t-2}) = \frac{\mathrm{s}^2}{2}$$

In this case, D is the same as before.

$$\lim_{T\to\infty}\frac{N_1}{T^3}$$

$$=\lim_{T\to\infty}\left[\frac{\sum_{t=1}^{T}(\hat{P}_{t-1}-\hat{P}_{t-2})^2}{T^2}\frac{\sum_{t=1}^{T}du_t(\hat{P}_t-\hat{P}_{t-1})}{T}-\frac{\sum_{t=1}^{T}du_t(\hat{P}_{t-1}-\hat{P}_{t-2})}{T}\frac{\sum_{t=1}^{T}(\hat{P}_t-\hat{P}_{t-1})(\hat{P}_{t-1}-\hat{P}_{t-2})}{T^2}\right]$$

$$=-\sigma^4 s_L\rho^2\left[\frac{1+2\rho^2-3\rho^4}{2(1-\rho^2)(1+2\rho^2-3\rho^4)}-\frac{\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)(1-\rho^2)}{2(1-\rho^2)(1+2\rho^2-3\rho^4)}\right]$$
$$+\sigma^4 s_L\rho^2\left[\frac{\rho^2(1+2\rho^2-3\rho^4)-3\rho^2(1-\rho^2)\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)}{2(1-\rho^2)(1+2\rho^2-3\rho^4)}\right]$$

$$=-\frac{\sigma^4 s_L\rho^2}{2(1-\rho^2)(1+2\rho^2-3\rho^4)}\Big[1+\rho^2-5\rho^4+3\rho^6$$
$$-(1-3\rho^2)(1-\rho^2)\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)\Big]$$

$$\lim_{T\to\infty}\frac{N_2}{T^3}$$

$$=\lim_{T\to\infty}\left[-\frac{\sum_{t=1}^{T}du_t(\hat{P}_{t-1}-\hat{P}_{t-2})}{T}\frac{\sum_{t=1}^{T}du_t(\hat{P}_t-\hat{P}_{t-1})}{T^2}+\frac{\sum_{t=1}^{T}du_t^2}{T}\frac{\sum_{t=1}^{T}(\hat{P}_t-\hat{P}_{t-1})(\hat{P}_{t-1}-\hat{P}_{t-2})}{T^2}\right]$$

$$=\frac{\sigma^2 s_L^2\rho^4(1-\rho^4)}{2(1-\rho^2)^2(1+2\rho^2-3\rho^4)}$$

$$\lim_{T\to\infty}\hat{\beta}_1=\lim_{T\to\infty}\frac{(N_1/T^3)}{(D/T^3)}$$

$$=\frac{-(1-\rho^2)\Big[1+\rho^2-5\rho^4+3\rho^6-(1-3\rho^2)(1-\rho^2)\phi\left((\omega-e^{-1}e_u s_L)\rho^2+1-\omega\right)\Big]}{s_L(1-\rho^4)}$$

$$= -\frac{(1-\rho^2)(1-\rho^4)}{s_L\rho^2(1-\rho^4)} - \frac{(1-\rho^2)\left[\rho^2 - 4\rho^4 + 3\rho^6 - (1-3\rho^2)(1-\rho^2)\varphi\left((\omega - e^{-1}e_u s_L)\rho^2 + 1 - \omega\right)\right]}{s_L\rho^2(1-\rho^4)}$$

$$= -\frac{(1-\rho^2)}{s_L\rho^2} - \frac{(1-\rho^2)\left[\rho^2(1-3\rho^2)(1-\rho^2) - (1-3\rho^2)(1-\rho^2)\varphi\left((\omega - e^{-1}e_u s_L)\rho^2 + 1 - \omega\right)\right]}{s_L\rho^2(1-\rho^4)}$$

$$= -\frac{1-a}{s_L a} - \frac{(1-\rho^2)(1-3\rho^2)(1-\rho^2)\left[\rho^2 - \varphi\left((\omega - e^{-1}e_u s_L)\rho^2 + 1 - \omega\right)\right]}{s_L\rho^2(1-\rho^2)(1+\rho^2)}$$

$$= \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}} - \frac{(1-3\rho^2)(1-\rho^2)\left[\rho^2 - \varphi\left((\omega - e^{-1}e_u s_L)\rho^2 + 1 - \omega\right)\right]}{s_L\rho^2(1+\rho^2)}$$

$$= \frac{e_u - e_{Wu}}{(1-\omega)e_{WW}} - \frac{(1-3a)(1-a)\left[a - \varphi\left((\omega - e^{-1}e_u s_L)a + 1 - \omega\right)\right]}{s_L a(1+a)}$$

$$\lim_{T\to\infty}\hat{\beta}_2 = \lim_{T\to\infty}\frac{(N_2/T^3)}{(D/T^3)} = 1$$

Appendix D
Calculations of Sums

$$\sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{\,t-k}\sin[y\,(t-k+1)]$$

$$= \sum_{t=t_0}^{T}\sum_{x=0}^{t-1} r^{\,x}\sin[y\,(x+1)]$$

$$= \sum_{t=t_0}^{T}\left\{\sum_{x=0}^{\infty} r^{\,x}\sin[y\,(x+1)] - \sum_{x=t}^{\infty} r^{\,x}\sin[y\,(x+1)]\right\}$$

$$= \sum_{t=t_0}^{T}\left\{\frac{1}{\sqrt{1-r^{2}}} - r^{\,t}\sum_{x=0}^{\infty} r^{\,x}\big[\sin[y t]\cos[y\,(x+1)] + \cos[y t]\sin[y\,(x+1)]\big]\right\}$$

$$= \sum_{t=t_0}^{T}\left\{\frac{1}{\sqrt{1-r^{2}}} - \frac{r^{\,t}\cos[y t]}{\sqrt{1-r^{2}}}\right\}$$

$$\lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{\,t-k}\sin[y\,(t-k+1)]}{T} = \frac{1}{\sqrt{1-r^{2}}}$$

$$\lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t-1} r^{\,t-k}\sin[y\,(t-k+1)]}{T}$$

$$= \lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{\,t-k}\sin[y\,(t-k+1)]}{T} - \lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sin[y\ ]}{T}$$

$$= \frac{1}{\sqrt{1-r^{2}}} - \sqrt{1-r^{2}} = \frac{r^{2}}{\sqrt{1-r^{2}}}$$

$$\sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{\,2(t-k)}\sin^{2}[y\,(t-k+1)]$$

$$= \sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{\,2(t-k)}\,\frac{1}{2}\big[1-\cos[2y\,(t-k+1)]\big]$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\sum_{x=0}^{t-1} \mathrm{r}^{2x}\left[1-\cos[2\mathrm{y}(x+1)]\right]$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\left\{\frac{1-\mathrm{r}^{2t}}{1-\mathrm{r}^{2}} - \sum_{x=0}^{\infty}\mathrm{r}^{2x}\cos[2\mathrm{y}(x+1)] + \sum_{x=t}^{\infty}\mathrm{r}^{2x}\cos[2\mathrm{y}(x+1)]\right\}$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\left\{\frac{1-\mathrm{r}^{2t}}{1-\mathrm{r}^{2}} - \frac{\mathrm{r}^{2}-1}{1+2\mathrm{r}^{2}-3\mathrm{r}^{4}} + \mathrm{r}^{2t}\cos(2\mathrm{y}t)\sum_{x=0}^{\infty}\mathrm{r}^{2x}\cos[2\mathrm{y}(x+1)] \right.$$
$$\left. - \mathrm{r}^{2t}\sin(2\mathrm{y}t)\sum_{x=0}^{\infty}\mathrm{r}^{2x}\sin[2\mathrm{y}(x+1)]\right\}$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\left\{\frac{1-\mathrm{r}^{2t}}{1-\mathrm{r}^{2}} - \frac{\mathrm{r}^{2}-1}{1+2\mathrm{r}^{2}-3\mathrm{r}^{4}} + \mathrm{r}^{2t}\cos(2\mathrm{y}t)\frac{\mathrm{r}^{2}-1}{1+2\mathrm{r}^{2}-3\mathrm{r}^{4}} \right.$$
$$\left. - 2\mathrm{r}^{2t}\sin(2\mathrm{y}t)\frac{\mathrm{r}\sqrt{1-\mathrm{r}^{2}}}{1+2\mathrm{r}^{2}-3\mathrm{r}^{4}}\right\}$$

$$\lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t}\mathrm{r}^{2(t-k)}\sin^2[\mathrm{y}(t-k+1)]}{T} = \frac{1}{2}\left[\frac{1}{1-\mathrm{r}^{2}} - \frac{\mathrm{r}^{2}-1}{1+2\mathrm{r}^{2}-3\mathrm{r}^{4}}\right]$$

$$= \frac{1-\mathrm{r}^{4}}{(1-\mathrm{r}^{2})(1+2\mathrm{r}^{2}-3\mathrm{r}^{4})}$$

$$\lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t-1}\mathrm{r}^{2t-2k-2}\sin^2[\mathrm{y}(t-k+1)]}{T}$$

$$= \lim_{T\to\infty}\mathrm{r}^{-2}\left[\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t}\mathrm{r}^{2(t-k)}\sin^2[\mathrm{y}(t-k+1)]}{T} - \frac{\sum_{t=t_0}^{T}\sin^2[\mathrm{y}]}{T}\right]$$

$$= \mathrm{r}^{-2}\left[\frac{1-\mathrm{r}^{4}}{(1-\mathrm{r}^{2})(1+2\mathrm{r}^{2}-3\mathrm{r}^{4})} - (1-\mathrm{r}^{2})\right]$$

$$= r^2 \frac{5-8r^2+3r^4}{(1-r^2)(1+2r^2-3r^4)}$$

$$\sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{2t-2k+1}\sin[y(t-k+1)]\cos[y(t-k+1)]$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{2t-2k+1}\sin[2y(t-k+1)]$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\sum_{x=0}^{t-1} r^{2x+1}\sin[2y(x+1)]$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\left\{\sum_{x=0}^{\infty} r^{2x+1}\sin[2y(x+1)] - \sum_{x=t}^{\infty} r^{2x+1}\sin[2y(x+1)]\right\}$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\left\{\frac{2r^2\sqrt{1-r^2}}{1+2r^2-3r^4} - r^{2t+1}\sum_{x=0}^{\infty} r^{2x}\left[\sin[2yt]\cos[2y(x+1)]+\cos[2yt]\sin[2y(x+1)]\right]\right\}$$

$$= \frac{1}{2}\sum_{t=t_0}^{T}\left\{\frac{2r^2\sqrt{1-r^2}}{1+2r^2-3r^4} - r^{2t+1}\left[\sin[2yt]\sum_{x=0}^{\infty} r^{2x}\cos[2y(x+1)]+\cos[2yt]\sum_{x=0}^{\infty} r^{2x}\sin[2y(x+1)]\right]\right\}$$

$$\lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t} r^{2t-2k+1}\sin[y(t-k+1)]\cos[y(t-k+1)]}{T} = \frac{r^2\sqrt{1-r^2}}{1+2r^2-3r^4}$$

$$\lim_{T\to\infty}\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t-1} r^{2t-2k-1}\sin[y(t-k+1)]\cos[y(t-k+1)]}{T}$$

$$= \lim_{T\to\infty} \mathrm{r}^{-2}\left[\frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t} \mathrm{r}^{2t-2k+1}\sin[\mathrm{y}(t-k+1)]\cos[\mathrm{y}(t-k+1)]}{T} - \frac{\sum_{t=t_0}^{T} \mathrm{r}\sin\mathrm{y}\cos\mathrm{y}}{T}\right]$$

$$= \frac{\sqrt{1-\mathrm{r}^2}}{1+2\mathrm{r}^2-3\mathrm{r}^4} - \sqrt{1-\mathrm{r}^2}$$

$$= \mathrm{r}^2\sqrt{1-\mathrm{r}^2}\,\frac{3\mathrm{r}^2-2}{1+2\mathrm{r}^2-3\mathrm{r}^4}$$

$$\lim_{T\to\infty} \frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t-1} \mathrm{r}^{2t-2k-2}\sin[\mathrm{y}(t-k+1)]\cos[\mathrm{y}(t-k)]}{T}$$

$$= \lim_{T\to\infty} \frac{\sum_{t=t_0}^{T}\sum_{k=1}^{t-1} \mathrm{r}^{2t-2k-2}\sin[\mathrm{y}(t-k+1)]\left(\mathrm{r}\cos[\mathrm{y}(t-k+1)] + \sqrt{1-\mathrm{r}^2}\sin[\mathrm{y}(t-k+1)]\right)}{T}$$

$$= \lim_{T\to\infty} \frac{\left[\sum_{t=t_0}^{T}\sum_{k=1}^{t-1} \mathrm{r}^{2t-2k-1}\sin[\mathrm{y}(t-k+1)]\cos[\mathrm{y}(t-k+1)] + \sqrt{1-\mathrm{r}^2}\,\mathrm{r}^{2t-2k-2}\left(\sin^2[\mathrm{y}(t-k+1)]\right)\right]}{T}$$

$$= \mathrm{r}^2\sqrt{1-\mathrm{r}^2}\,\frac{3\mathrm{r}^2-2}{1+2\mathrm{r}^2-3\mathrm{r}^4} + \sqrt{1-\mathrm{r}^2}\,\mathrm{r}^2\,\frac{5-8\mathrm{r}^2+3\mathrm{r}^4}{(1-\mathrm{r}^2)(1+2\mathrm{r}^2-3\mathrm{r}^4)}$$

$$= \mathrm{r}^2\sqrt{1-\mathrm{r}^2}\,\frac{3(1-\mathrm{r}^2)}{(1-\mathrm{r}^2)(1+2\mathrm{r}^2-3\mathrm{r}^4)}$$

References

Akerlof, George A., “Labor Contracts as Partial Gift Exchange,” *Quarterly Journal of Economics*, November 1982, 97, 543-69.

________, “Gift Exchange and Efficiency Wage Theory: Four Views,” *American Economic Review*, May 1984, 74, 79-83.

Akerlof, George A. and Yellen, Janet L., “The Fair Wage-Effort Hypothesis and Unemployment,” *Quarterly Journal of Economics*, May 1990, 105, 255-83.

Ball, Laurence, “Credible Disinflation with Staggered Price-Setting,” *American Economic Review*, March 1994, 84, 282-289.

Bewley, Truman F., *Why Wages Don’t Fall During a Recession*, Cambridge, MA: Harvard University Press, 1999.

Calvo, Guillermo A., “Staggered Contracts in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, September 1983, 12, 383-398.

Campbell, Carl M., “A Model of the Determinants of Effort,” *Economic Modelling*, March 2006, 23, 215-37.

________, “An Efficiency Wage – Imperfect Information Model of the Aggregate Supply Curve,” Working Paper, Northern Illinois University, October 2007.

________, (2008a) “An Efficiency Wage Approach to Reconciling the Wage Curve and the Phillips Curve,” *Labour Economics*, 2008, doi:10.1016/j.labeco.2008.01.002.

________, (2008b) “The Formation of Wage Expectations in the Effort and Quit Decisions of Workers” Working Paper, Northern Illinois University, January 2008.

Campbell, Carl M., and Kunal S. Kamlani, “The Reasons for Wage Rigidity: Evidence from a Survey of Firms,” *Quarterly Journal of Economics*, August 1997, 112, 759-789.

Friedman, Milton, “The Role of Monetary Policy,” *American Economic Review*, March 1968, 58, 1-17.

Fuhrer, Jeffrey C., “The Phillips Curve is Alive and Well,” *New England Economic Review*, March/April 1995, 41-56.

Fuhrer, Jeff, and George Moore, “Inflation Persistence,” *Quarterly Journal of Economics*, February 1995, 110, 127-159.

Galí, Jordi, and Mark Gertler, “Inflation Dynamics: A Structural Econometric Analysis,” *Journal of Monetary Economics*, October 1999, 44, 195-222.

Huang, Kevin X. D., Zheng Liu, and Louis Phaneuf, "Why Does the Cyclical Behavior of Real Wages Change Over Time?" *American Economic Review*, September 2004, 94, 836-856.

King, Robert G., and Mark W. Watson, "The Post-War U.S. Phillips Curve: A Revisionist Econometric History," *Carnegie-Rochester Conference Series on Public Policy*, 1994, 41, 157-219.

Liu, Zheng and Louis Phaneuf, "Technology Shocks and Labor Market Dynamics: Some Evidence and Theory," *Journal of Monetary Economics*, November 2007, 54, 2534-2553.

Mankiw, N. Gregory and Reis, Ricardo, "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics*, November 2002, 117, 1295-1328.

McCallum, Bennett, "Comment," in *NBER Macroeconomics Annual: 1997*, Cambridge, MA: MIT Press, 1997.

Phelps, Edmund S., "Money-Wage Dynamics and Labor-Market Equilibrium," *Journal of Political Economy*, July-August 1968, 76, 678-711.

Phillips, A.W., "The Relationship between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957," *Economica*, November 1958, 25, 283-99.

Roberts, John M., "New Keynesian Economics and the Phillips Curve," *Journal of Money, Credit, and Banking*, November 1995, 27, 975-984.

________, "Inflation Expectations and the Transmission of Monetary Policy," mimeo, Federal Reserve Board of Governors, March 1998.

Rotemberg, Julio J., "Sticky Prices in the United States," *Journal of Political Economy*, November 1982, 60, 1187-1211.

Salop, Steven C., "A Model of the Natural Rate of Unemployment," *American Economic Review*, March 1979, 69, 117-25.

Schlicht, Ekkehart, "Labor Turnover, Wage Structure and Natural Unemployment," *Zeitschrift für die Gesamte Staatswissewnschaft*, June 1978, 134, 337-46.

Shapiro, Carl and Stiglitz, Joseph E., "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review*, June 1984, 74, 433-44.

Samuelson, Paul A. and Solow, Robert M., "Analytical Aspects of Anti-Inflation Policy," *American Economic Review*, May 1960, 50(2), 177-194.

Solow, Robert M., "Another Possible Source of Wage Stickiness," *Journal of Macroeconomics*, Winter 1979, 1, 79-82.

Stiglitz, Joseph E., "Alternative Theories of Wage Determination and Unemployment in L.D.C.'s: The Labor Turnover Model," *Quarterly Journal of Economics*, May 1974, 88, 194-227.

Taylor, John B., "Staggered Contracts in a Macro Model," *American Economic Review*, May 1979, 69, 108-113.

________, "Output and Price Stability: An International Comparison," *Journal of Economic Dynamics and Control*, February 1980, 2, 109-132.

Wadhwani, Sushil B., and Wall, Martin, "A Direct Test of the Efficiency Wage Model Using UK Micro-Data," *Oxford Economic Papers*, October 1991, 43, 529-548.

Weiss, Andrew, "Job Queues and Layoffs in Labor Markets with Flexible Wages," *Journal of Political Economy*, June 1980, 88, 526-38.

Weisskopf, Thomas E., "The Effect of Unemployment on Labour Productivity: An International Comparative Analysis," *International Review of Applied Economics*, June 1987, 1, 127-51.

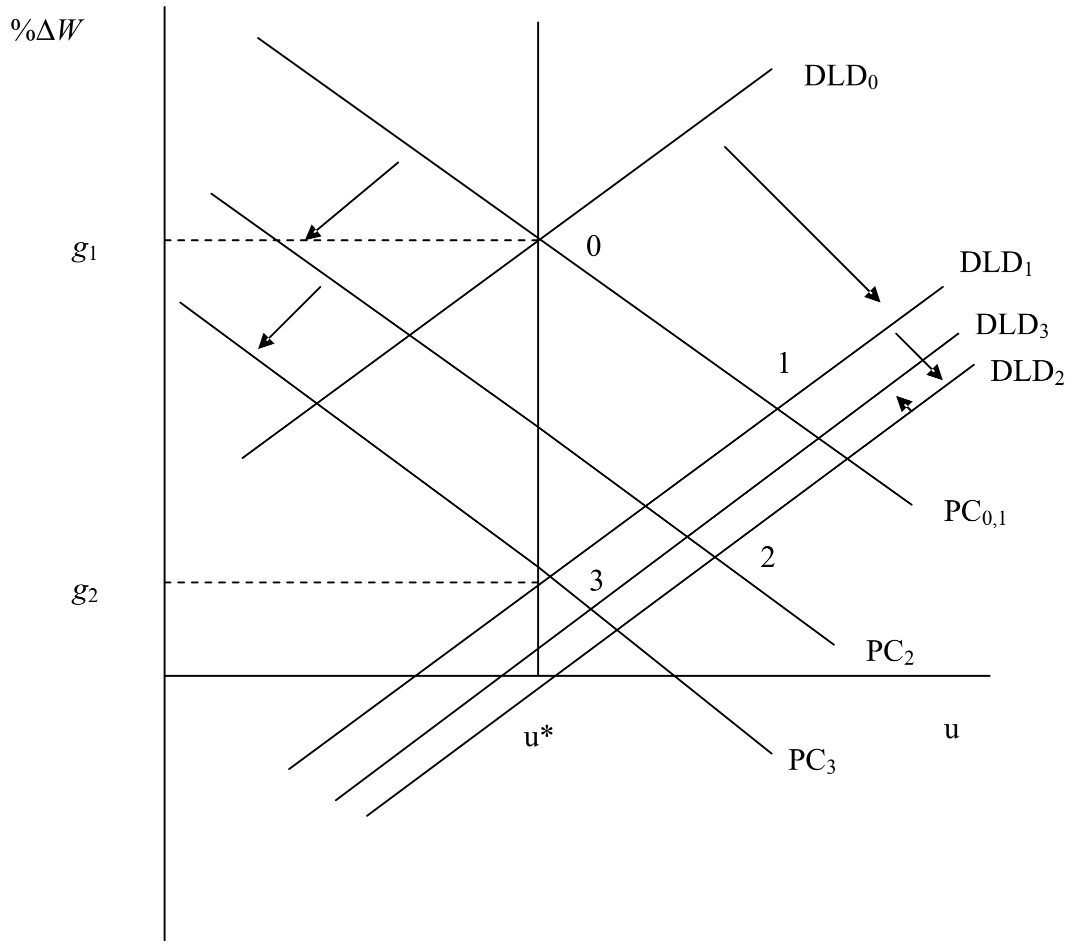

Figure 1

Response of wage inflation and unemployment to a decrease in nominal demand from g_1 to g_2

[1] See King and Watson (1994) and Fuhrer (1995) for empirical evidence for the Phillips curve.

[2] For example, Fuhrer (1995, p. 43) writes, "Perhaps the greatest weakness of the Phillips curve is its lack of theoretical underpinnings: No one has derived a Phillips curve from first principles, beginning with the fundamental concerns and constraints of consumers and firms."

[3] According to McCallum (1997), the Calvo-Rotemberg model of the Phillips curve, has become "the closest thing there is to a standard specification."

[4] The rationale for the assumption that $e_{Wu} < 0$ is discussed in Campbell (2008a).

[5] While the present study treats $e[W_t / \overline{W}_t^e, u_t]$ as a general functional form, Campbell (2006) develops a model of workers' effort, based on the shirking and fair wage versions of efficiency wage theory. It is demonstrated that the utility-maximization problem of workers yields an equation for workers' efficiency in which efficiency depends positively on the worker's relative wage and depends negatively on the unemployment rate. Thus, the efficiency function can be viewed as being determined from utility maximizing behavior.

[6] For example, these errors may be due to firms' lack of perfect information about the level of product demand or about the parameters in their profit functions.

[7] As discussed in Campbell (2008a), assuming a positive relationship between efficiency and wages does not guarantee that there will be excess supply of labor. Whether a firm operates on its labor supply curve or to the left of its labor supply curve (i.e., pays an efficiency wage) depends on the elasticity of output with respect to the wage, calculated at the market-clearing wage.

[8] Since $\hat{W}_t$, $\hat{W}_{t-1}$, and $\hat{W}_{t-2}$ are the percentage differences in wages from their initial values, the difference between $\hat{W}_t$ and $\hat{W}_{t-1}$ is the percentage change in wages between period *t*-1 and period *t*, and the difference between $\hat{W}_{t-1}$ and $\hat{W}_{t-2}$ is the percentage change in wages between period *t*-2 and period *t*-1.

[9] In Figure 1, the unemployment rate is lower in period 2 than in period 1. However, depending on the model's parameters, it is possible that the DLD curve shifts far enough right in period 2 so that the unemployment rate is higher in period 2 than in period 1.