Unsportsmanlike behaviour: An investigation into the commodity-bundling practices of Sky Television

by

Ross Kendall

June 2012

Acknowledgements

I would like to thank my supervisor Dr Seamus Hogan for all his time and effort spent reviewing my work and providing guidance. I also thank those who read my drafts and provided valuable feedback.

Abstract: It is puzzling that Sky TV, effectively a monopolist in the pay television market in New Zealand, does not offer its Sky Sport channels as a stand-alone package. This is contrary to what the bulk of literature suggests would be optimal. We develop a simple one-period model with reasonable assumptions that demonstrates this inconsistency. We expand the model to allow for learning by introducing a second period and uncertainty. Our initial two-period model fails to definitively explain the behaviour, but when adjustments are made, several possibilities for why Sky's behaviour may be profit maximising are revealed.

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1. Introduction

Commodity bundling (the sale of two or more goods as a package) is a common practice employed by firms. An often-used example of commodity bundling is the case of pay television. The cable television market in the USA has been the subject of a lot of study and debate. This paper seeks to examine the case of Sky TV (Sky) in New Zealand. Sky effectively has a monopoly on the pay television market in New Zealand, and offers several different packages to consumers. However, Sky does not allow purchase of extra channels (for example Sky Sport) without the purchase of a basic package of other channels. This practice appears to conflict with theoretical models of profit-maximising behaviour, and is what this paper seeks to investigate.

A large volume of literature has been generated by the desire to understand how and why firms choose to bundle their products. Adams and Yellen (1976) made a particularly significant contribution to this literature, showing that even with independent utility of consumption for two goods, bundling could be profitable due to its ability to sort consumers into different groups, based on the characteristics of their reservation prices. This improves the firm's ability to extract consumer surplus.

Their paper is highly relevant to the situation of Sky, as they also show that mixed bundling (offering the bundled goods separately as well as packaged) is at least as profitable as pure bundling (only offering the goods in a package). This is a fairly trivial result as mixed bundling degenerates to pure bundling when the prices of the individual goods are such that no-one purchases them. A more interesting result is that pure bundling can only be optimal when no-one consumes a good whose marginal cost exceeds his or her reservation price for it.

For the purposes of this paper, we will consider only two of the groups of channels that Sky offers. Sky sells a main bundle of channels ("Other") on its own, and the main bundle and their sports channels ("Sky Sport"), as a package ("the bundle"). This paper investigates possible reasons why Sky Sport is not offered on its own. This situation is a candidate for one where pure bundling could be optimal, due to the negligible marginal costs involved in offering an additional channel to a customer, but even then it is unusual that Other would be offered alone, while Sky Sport is only offered in a package with Other.

The behaviour of Sky is similar to the practices of pay television companies in other countries, for example the United States, where it is common to require consumers to purchase a “basic” package before they can choose to purchase additional channels. However, in many cases outside of New Zealand, there is more than one firm in the pay television market, in which case the models in this paper will not apply.

The paper proceeds as follows: Section 2 reviews some of the relevant literature on commodity bundling, especially surrounding the pay television market. Section 3 develops a basic one-period model with some simple assumptions, showing that it is optimal to offer Sky Sport alone in addition to packaged. Then, we examine the assumptions in turn to see if the behaviour of Sky can be explained if some are relaxed. Section 4 extends the model to a two-period framework with uncertainty, to examine whether the ability to learn new information about the value of the bundle can potentially explain the lack of a separate Sky Sport option. In an extension, we examine the model in the case where the value of Sky Sport is also uncertain. Section 5 concludes, summarising the findings of the various models, and looking at possible extensions.

2. Literature Review

Much of the previous commodity-bundling literature has focused on the welfare implications of bundling, given interest in the United States around regulating the bundling behaviour of pay television firms. Kobayashi (2005) provides a survey of this literature and concludes that, while there is the possibility for potentially harmful welfare effects due to bundling, the strict assumptions and lack of empirical and robustness testing of theoretical models means that there is not sufficient reason to justify the regulation of bundling behaviour, given the uncertainty around the costs and benefits of doing so.

Some research has examined whether mixed or pure bundling is more profitable, usually as a means to another goal. Schmalensee (1994) investigates the case of reservation prices following a bivariate-normal distribution and finds that mixed bundling is in general strictly more profitable than either no bundling or pure bundling. Chae (1992) shows that in a subscription television market, mixed bundling is the most profitable strategy for a monopolist, assuming reservation prices follow a uniform distribution. These findings, in combination with those of Adams and Yellen (1976) discussed earlier, provide evidence that the models developed in this paper where mixed bundling is found to be optimal are not isolated cases. Our framework is different from those used in the papers discussed, yet yields many similar results.

3. One-Period Model

Adams and Yellen (1976) gave simple examples of a situation where pure bundling was optimal with a small discrete number of consumers. This model attempts to examine a situation with a continuous distribution of consumers over the range of possible reservation prices.

3.1 Model Set-up

Sky can offer at most two packages: Sky Sport alone (price, p_s), and Sky Sport bundled with Other (price, p_b). Both packages are provided with zero marginal cost.

There is a continuum of consumers of measure 1, each of whom will buy either one unit of Sky Sport, one unit of the bundle, or no Sky. Let the total demands for no Sky, Sky Sport alone, and the bundle be, respectively,

$$q_0(p_s, p_b), \quad q_s(p_s, p_b), \quad q_b(p_s, p_b)$$

so that

$$q_0(p_s, p_b) + q_s(p_s, p_b) + q_b(p_s, p_b) = 1$$

There is a joint density of reservation values for the two packages that determines the quantities purchased of each package. The reservation value for Sky Sport is V_s, the reservation value for the bundle is V_b, and the joint density is given by $f(V_s, V_b)$. Consumers purchase the package with the maximum difference between their reservation price and the purchase price, or no package if both of their reservation values are lower than the prices.

3.2 Assumptions

To examine the optimal behaviour of Sky, we must first make some base assumptions to represent the situation.

> A1: *Offering a Sky Sport alone package incurs no additional fixed costs over provision of the bundle package alone.*

This assumption is empirically reasonable, given that Sky already offers multiple packages and will have systems in place to easily allow new packages to be created.

> A2: *The minimum of the support of the distribution of additional willingness to pay for the bundle over Sky Sport ($V_b - V_s$) is zero, which implies that if*
>
> $$q_b(p_s, p_b) > 0,$$
>
> *then*
>
> $$q_s(p_s, p_b) > 0 \quad \forall p_s < p_b,$$
>
> *so there is no neighbourhood of prices below the price of the bundle with zero demand for Sky Sport.*

Assumption A2 is simply saying that some consumers exist who only care about Sky Sport, which is a reasonable assumption to make, especially given that Assumption A3 ensures that there is no mass point of such consumers.

> A3: *The joint distribution of reservation values has positive density at all points in the convex hull of the support with no mass points.*

This assumption means that the reservation values of consumers are distributed continuously, with no irregular spikes or shifts. This is an approximation to the real situation, where there is a discrete number of consumers, but given the large number of consumers in the market, is very reasonable.

3.3 Outcomes

Given Assumptions A1-3 we can examine the behaviour of a profit-maximising firm. The firm's objective can be expressed as

$$\max_{p_s, p_b} \pi ,$$

where

$$\pi = p_s q_s(p_s, p_b) + p_b q_b(p_s, p_b) .$$

This gives first order conditions:

$$\frac{\partial \pi}{\partial p_b} = -p_b \frac{\partial q_0}{\partial p_b} + q_b + (p_s - p_b)\frac{\partial q_s}{\partial p_b} = 0 ,$$

$$\frac{\partial \pi}{\partial p_s} = -p_s \frac{\partial q_0}{\partial p_s} + q_s + (p_b - p_s)\frac{\partial q_b}{\partial p_s} = 0 . \qquad (1.1)$$

Sky Sport will be offered as a separate package if $p_s < p_b$ at the profit maximum.

Theorem 1: $p_s = p_b$ cannot be a profit maximum.

Proof: To see whether $p_s = p_b$ can be a profit maximum, consider whether Equation (1.1) can hold at $p_s = p_b$.

$$\left.\frac{\partial \pi}{\partial p_s}\right|_{p_s = p_b} = -p_s \frac{\partial q_0}{\partial p_s} \qquad (1.2)$$

From Equation (1.2) we see that for $p_s = p_b$ to be an optimum, $-p_s \, \partial q_0 / \partial p_s$ must be non-negative. If it is negative, there is an incentive for Sky to lower the price of Sky Sport, and hence offer Sky Sport alone. However, Assumptions A2 and A3 ensure that $\partial q_0 / \partial p_s > 0$ for the entire relevant range, and hence it is optimal for Sky to offer a separate Sky Sport package, at some price $p_s < p_b$.

Q.E.D.

Relaxing some of the assumptions can lead to situations where it may be optimal to set $p_s = p_b$ and hence only offer the bundle.

It is trivial that if A1 does not hold, for a large enough additional fixed cost it will not be optimal for Sky to offer Sky Sport alone. However, this does not seem plausible, as with systems already set up for billing customers and controlling access to channels, any fixed costs associated with adding another package should be minimal.

If A2 does not hold, then for some price $p_s < p_b$ there will be zero demand for Sky Sport. This implies that over this range $\partial q_0 / \partial p_s = 0$ and hence $p_s = p_b$ is a local maximum, and in some cases a global maximum. However, A2 holding seems more defensible than its not holding, as it would be expected that there would be at least some consumers who have no additional willingness to pay for the bundle, and a continuous distribution over their reservation price for Sky Sport such that at any price, quantity demanded would increase if the price fell.

If A3 does not hold there could be, for example, a mass point of consumers with a reservation price of zero for the incremental value of the bundle, and a reservation price for Sky Sport just below the price of the bundle. Mathematically,

$$\lim_{p_s \to p_b^-} q_s(p_s, p_b) = \hat{q}_s > 0$$

By lowering the price of Sky Sport, Sky can gain a mass of customers at negligible cost, trivialising the result that Sky Sport should be offered alone. It seems more reasonable to assume that A3 holds, as we would expect something close to a smooth, continuous distribution of consumers in a believable situation, rather than large discrete changes in the density of reservation values.

It is worth noting that in this framework, and in the extensions presented throughout this paper, there exists the possibility that although it is optimal for Sky to offer Sky Sport separately, the price differential is so low that the difference in profit is negligible. This is not an appealing solution however, as it is still suboptimal behaviour, even if it is only slightly worse than the optimum.

The simple model in this section, although narrow, provides useful insight into the situation of Sky. By showing that Sky's behaviour cannot be easily explained in a framework with the key elements of multi-channel bundling, we have demonstrated that the situation is worth investigating further. The next section looks at a richer model that allows for learning, to see if any new insights into the problem can be gleaned.

4. Two-Period Model

This section considers the behaviour of a monopoly attempting to maximise the sum of its profits over two periods. We wish to consider the possibility that consumers can learn about the products over time, and so we introduce a two-period model with a degree of uncertainty. We assume that consumers know their reservation price for Sky Sport with certainty, but are uncertain as to their reservation price for the bundle. Consumers know the distribution of possible values for the bundle they each face, and their realisation from that distribution in period one is revealed to them if they purchase the bundle. This informs theirs decision about what channels to purchase in the second period.

The addition of uncertainty adds a quasi-option value of information to the bundle, as each consumer has a chance of learning that he has a high valuation and can then continue to purchase the bundle in the second period[1]. If he learns that he has a low valuation of the bundle, he can simply choose a different package in the second period.

There is the potential that this set-up could explain the behaviour of Sky. Consumers know that they will gain information from the bundle, but as Sky is a monopoly, the price of the bundle is set above its marginal cost, meaning that consumers will not capture the full value of information. Hence there may be potential for Sky to gain by restricting the choices available to consumers.

We then extend the model to examine the case when the values of both Sky Sport and the bundle are not known with certainty, as this may influence the model outcomes.

[1] The concept of a "quasi-option value" is due to Arrow and Fisher (1974) and Henry (1974).

4.1 Model Set-up

The model set-up is largely the same as the one-period model, but adapted to the new two-period setting.

Sky can offer at most two packages: Sky Sport alone (price, p_s), and Sky Sport bundled with Other (price, p_b). Both packages are provided with zero marginal cost. Sky is committed to maintaining the same prices for both packages in both periods.

There is a continuum of consumers of measure 1, each of whom will buy either one unit of Sky Sport, one unit of the bundle, or no Sky in each period. Let the total demands for each combination be given by Table 1:

Table 1: Notation for quantity demanded of combinations of purchases

Period-one purchase	Period-two purchase	Quantity demanded
No Sky	No Sky	q_0
Sky Sport	Sky Sport	q_1
Bundle	No Sky	q_2
Bundle	Sky Sport	q_3
Bundle	Bundle	q_4

so that

$$\sum_{i=0}^{4} q_i = 1 \qquad (2.1)$$

Any other combinations of purchases will have zero quantity, as with no new information gained, consumers will not alter their decision in the second period.

There is a joint density of reservation values for the two packages that determines the quantities purchased of each package. The reservation value for Sky Sport is V_s, the realised reservation value for the bundle is V_b, the reservation value for the bundle for a consumer who has yet to purchase it (equal to the expected value of V_b plus any information value it possesses) is $\bar{V}_b$ and the joint density is given by $f(V_s, V_b, \bar{V}_b)$.

Consumers purchase the package that maximises the expected sum of their utilities over the two periods, where utility in each period is the realised reservation value minus price paid for the package selected:

$$U_2 = \max_{i_2}\left(E\left[V_{i_2}\right] - p_{i_2}\right)$$

$$U_1 = \max_{i_1}\left(E\left[V_{i_1}\right] - p_{i_1} + E\left[U_2 \middle| i_1\right]\right)$$

where

$$i_1 = 0, s, b$$
$$i_2 = 0, s, b$$
$$V_0 = p_0 = 0$$

4.2 Assumptions

Assumptions A1 and A3 from the one-period model are retained, but Assumption A2 is replaced with Assumptions B1 and B2 to reflect the new set-up, still remaining analogous to the previous situation.

B1: *The minimum of the support of the distribution of realisations of additional willingness to pay for the bundle over Sky Sport ($V_b - V_s$), is zero, which implies that if*

$$q_4(p_s, p_b) > 0,$$

then

$$q_3(p_s, p_b) > 0 \quad \forall p_s < p_b.$$

Hence there is no neighbourhood of prices below the price of the bundle with zero demand for Sky Sport, once the realisations for additional willingness to pay have been revealed.

B2: *The minimum of the support of the distribution of additional willingness to pay for the bundle over Sky Sport, for consumers who have not yet purchased the bundle ($\bar{V}_b - V_s$), is zero, which implies that if*

$$q_3(p_s, p_b) + q_4(p_s, p_b) > 0,$$

then

$$q_1(p_s, p_b) > 0 \quad \forall p_s < p_b .$$

This means that there is a positive density of consumers (but not a mass as A3 still holds) who receive zero information value from the bundle. This implies that there is variation in the degree of uncertainty facing consumers, from no uncertainty upwards. This in turn implies that there is no neighbourhood of prices below the price of the bundle with zero demand for Sky Sport in period one.

4.3 Outcomes

Sky's objective can now be expressed as

$$\max_{p_s, p_b} \pi ,$$

where

$$\pi = p_b \left(q_2 + q_3 + 2q_4\right) + p_s \left(2q_1 + q_3\right).$$

The first-order condition with respect to the price of Sky Sport is then

$$\frac{\partial \pi}{\partial p_s} = p_b \left(\frac{\partial q_2}{\partial p_s} + \frac{\partial q_3}{\partial p_s} + 2\frac{\partial q_4}{\partial p_s} \right) + 2q_1 + q_3 + p_s \left(2\frac{\partial q_1}{\partial p_s} + \frac{\partial q_3}{\partial p_s} \right) = 0 . \qquad (2.2)$$

Once again, to see whether it is optimal to offer Sky Sport separately we must examine the derivative in Equation (2.2) when $p_s = p_b$:

$$\left.\frac{\partial \pi}{\partial p_s}\right|_{p_s = p_b} = p_b \left[2\left(\frac{\partial q_1}{\partial p_s} + \frac{\partial q_3}{\partial p_s} + \frac{\partial q_4}{\partial p_s} \right) + \frac{\partial q_2}{\partial p_s} \right]. \qquad (2.3)$$

From Equation (2.1),

$$\sum_{i=0}^{4} \frac{\partial q_i}{\partial p_s} = 0 ,$$

$$\frac{\partial q_0}{\partial p_s} = -\sum_{i=1}^{4} \frac{\partial q_i}{\partial p_s} . \qquad (2.4)$$

Applying (2.4) to (2.3),

$$\left.\frac{\partial \pi}{\partial p_s}\right|_{p_s = p_b} = p_b \left[-\frac{\partial q_2}{\partial p_s} - 2\frac{\partial q_0}{\partial p_s} \right]. \qquad (2.5)$$

From Equation (2.5), it must be the case that $-\partial q_2/\partial p_s - 2\,\partial q_0/\partial p_s$ is non-negative for it to be optimal for Sky to not offer Sky Sport on its own. However, it is clear that $\partial q_0/\partial p_s$, the rate at which the number of consumers who purchase no Sky in either period changes as the price of Sky Sport increases, is positive. Mathematically,

$$q_0 = \int_0^{p_s}\int_0^{p_b}\int_0^{\infty} f(V_s, V_b, \bar{V}_b)dV_b d\bar{V}_b dV_s$$

$$\frac{\partial q_0}{\partial p_s} = \int_0^{p_b}\int_0^{\infty} f(p_s, V_b, \bar{V}_b)dV_b d\bar{V}_b > 0,$$

which is positive for all relevant prices due to Assumption A3.

With some careful reasoning, it can also be seen that $\partial q_2/\partial p_s$, the rate at which the number of consumers who purchase the bundle in the first period and no Sky in the second period changes as the price of Sky Sport increases, is non-negative. Mathematically,

$$q_2 = \int_0^{p_s}\int_0^{p_b}\int_{p_b}^{\infty} f(V_s, V_b, \bar{V}_b)d\bar{V}_b dV_b dV_s$$

$$\frac{\partial q_2}{\partial p_s} = \int_0^{p_b}\int_{p_b}^{\infty} f(p_s, V_b, \bar{V}_b)\,d\bar{V}_b\,dV_b > 0.$$

Once again, Assumption A3 ensures that this expression is greater than zero.

We can also reason through non-mathematically why $\partial q_2/\partial p_s$ cannot be negative. Break down the consumers who purchase the bundle in the first period into two types: those who are willing to buy Sky Sport but prefer the bundle, and those who are not willing to buy Sky Sport.

For the consumers who are not willing to buy Sky Sport, an increase in p_s is irrelevant, so we can ignore these consumers.

For consumers who are willing to purchase Sky Sport, when p_s increases some consumers who were on the margin of choosing between the bundle and Sky Sport will now choose the bundle, as the implicit cost of Other has fallen. So the number of consumers who purchase the bundle in the first period has increased. All of the consumers that would have consumed the bundle in period one and then no Sky in period two before the increase in p_s still choose to do so. But now there are additional consumers purchasing the bundle in period one and some of these will switch to no Sky. Therefore the number of consumers switching from the bundle to nothing is higher than before the increase in p_s and so $\partial q_2 / \partial p_s$ must be positive. Therefore Equation (2.5) is negative, and it must be optimal for Sky to offer a package of Sky Sport alone at some price $p_s < p_b$.

Figures 1 and 2 show graphically that q_2 cannot fall when p_s rises.

Figure 1: Reservation values and package choice

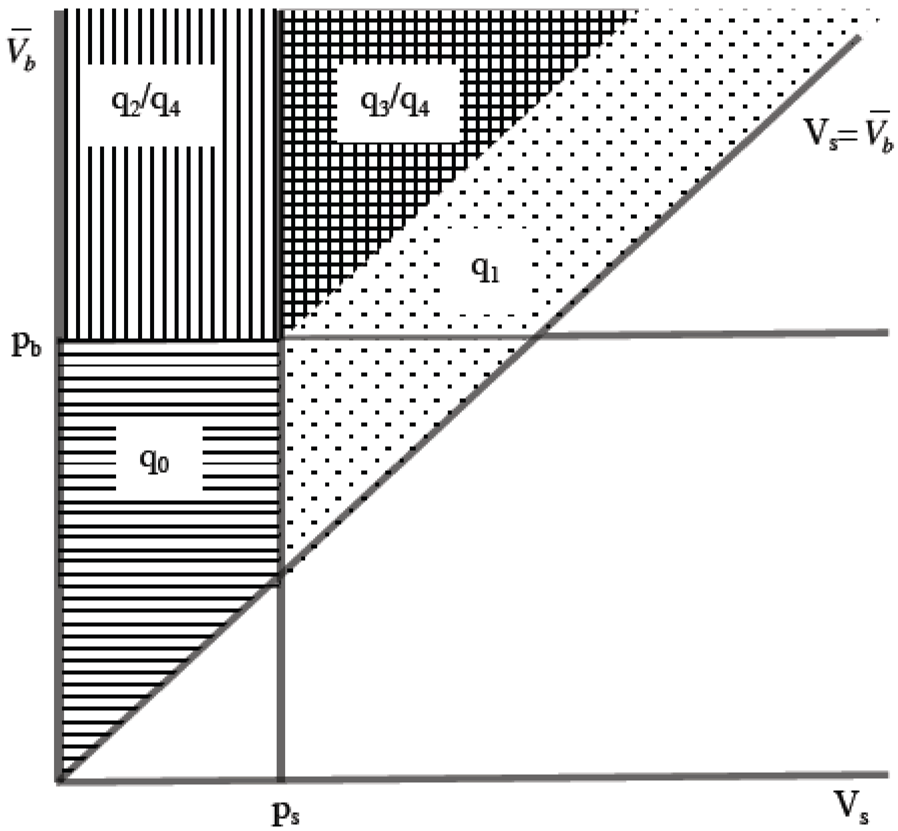
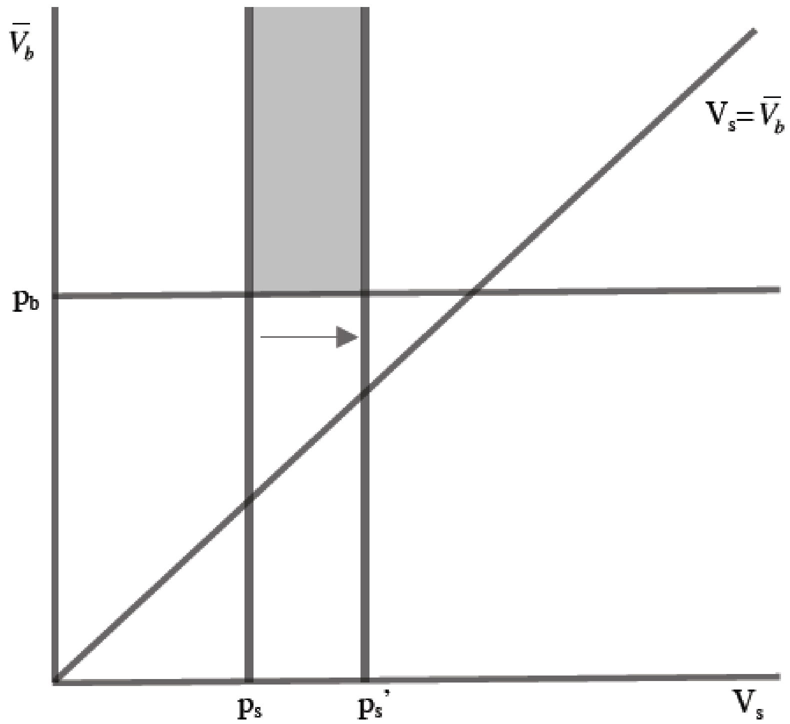


Figure 1 shows what package consumers will purchase given the information available to them.

Figure 2: Change in potential q_2 purchases when p_s rises



From Figure 1, we know that the shaded area in Figure 2 is the increase in consumers who could purchase q_2 when p_s increases. The change in p_s does not affect the probability of consumers in the original area being categorised as q_2 (ie purchasing no Sky in period two), as none of these customers were willing to buy Sky Sport anyway. So now, all of the consumers who originally chose q_2 still do, while there is a new area of consumers who may also choose q_2, hence q_2 increases.

This is the same result as the one-period model, and demonstrates that this result is robust to the introduction of uncertainty in the way modelled here.

It is interesting to examine the case where Assumption B2 is relaxed. It could be the case for example, that all consumers value the information of the bundle more than some positive amount. This is not unreasonable, as it seems likely that all consumers face at least some degree of uncertainty as to how much they will enjoy the additional channels, and hence be willing to pay something for resolving that uncertainty.

In this case, although it is optimal to offer Sky Sport alone at a price $p_s < p_b$, the presence of uncertainty means that for p_s close enough to p_b, no consumer will purchase Sky Sport in period one. This is the case as the bundle has positive information value for all consumers. Therefore it may be interesting to examine the case where Sky sets prices such that no consumers would purchase Sky Sport in period one. Let the lowest price that fulfils this criterion be given by $p_s = p_b - I$, where I is the lowest value of information amongst all consumers. Examining the derivative in Equation (2.2) at this point gives

$$\left.\frac{\partial \pi}{\partial p_s}\right|_{p_s = p_b - I} = p_b\left(\frac{\partial q_2}{\partial p_s} + \frac{\partial q_3}{\partial p_s} + 2\frac{\partial q_4}{\partial p_s}\right) + q_3 + (p_b - I)\left(2\frac{\partial q_1}{\partial p_s} + \frac{\partial q_3}{\partial p_s}\right).$$

Simplifying and applying Equation (2.3) gives

$$\left.\frac{\partial \pi}{\partial p_s}\right|_{p_s = p_b - I} = p_b\left[-\frac{\partial q_2}{\partial p_s} - 2\frac{\partial q_0}{\partial p_s}\right] + q_3 - I\left[2\frac{\partial q_1}{\partial p_s} + \frac{\partial q_3}{\partial p_s}\right]. \qquad (2.6)$$

The first term in Equation (2.6) is the same as Equation (2.5), which is negative. However, q_3 is positive and the sign of the third term is likely positive too. The sign of the derivative at these prices is therefore ambiguous, so a pricing scheme satisfying $p_b - I \le p_s < p_b$ could be an optimum.

This would explain the behaviour of Sky in the real world if they offered Sky Sport alone at a price a little below that of the bundle, such that anyone seeking to buy Sky Sport would purchase the bundle first just to see if they like it. However, this is not the case. But given the simplicity of the model, we would not expect it to exactly model the actual behaviour of Sky,

This model does show the important point that information is valuable. It is conceivable that in reality, uncertainty is not resolved after one period, and new uncertainty about the value of channels appears in each period of a many-period game. In this case, it could be that while it is theoretically optimal to offer Sky Sport at a price below that of the bundle in a finite-period game, Sky does not because it believes no-one would purchase it, given the constant flow of valuable new information for those who purchase the bundle in a world with no obvious final period (which would in this case be the only period anyone would buy Sky Sport).

4.4 Extension

In this section, we retain the Assumptions from the previous model, but alter the set-up slightly to allow for uncertainty in the value of Sky Sport in addition to uncertainty in the additional value of the bundle.

4.4.1 Model set-up

Once again, there is a joint density of reservation values for the two packages that determines the quantities purchased of each package. The realised reservation value for the Sky Sport is V_s, the reservation value for Sky Sport for a consumer who has yet to purchase it (equal to the expected value of V_s plus any information value it possesses) is $\bar{V}_s$, the realised reservation value for the bundle is V_b, the reservation value for the bundle for a consumer who has yet to purchase it (equal to the expected value of V_b plus any information value it possesses) is $\bar{V}_b$ and the joint density is given by $g(V_s, \bar{V}_s, V_b, \bar{V}_b)$.

The previous notation for the quantity demanded remains, but we must introduce two new quantities: let $q_5(p_s, p_b)$ be the number of consumers who purchase Sky Sport in the first period, and no Sky in the second period and $q_6(p_s, p_b)$ be the number of consumers who purchase Sky Sport in the first period and the bundle in the second period. Similarly to the base two-period model, the following property holds:

$$\sum_{i=0}^{6} q_i = 1$$

Hence,

$$\frac{\partial q_0}{\partial p_s} = -\sum_{i=1}^{6} \frac{\partial q_i}{\partial p_s}. \qquad (2.7)$$

4.4.2 Outcomes

With this set-up, Sky's objective can be expressed as

$$\max_{p_s, p_b} \pi ,$$

where

$$\pi = p_b \left(q_2 + q_3 + 2q_4 + q_6\right) + p_s \left(2q_1 + q_3 + q_5 + q_6\right).$$

The first-order condition with respect to the price of Sky Sport is:

$$\begin{aligned} \frac{\partial \pi}{\partial p_s} &= p_b \left(\frac{\partial q_2}{\partial p_s} + \frac{\partial q_3}{\partial p_s} + 2\frac{\partial q_4}{\partial p_s} + \frac{\partial q_6}{\partial p_s} \right) + 2q_1 + q_3 + q_5 + q_6 \\ &+ p_s \left(2\frac{\partial q_1}{\partial p_s} + \frac{\partial q_3}{\partial p_s} + \frac{\partial q_5}{\partial p_s} + \frac{\partial q_6}{\partial p_s} \right) = 0 \end{aligned} \qquad (2.8)$$

Once again, to see whether it is optimal to offer Sky Sport separately we must examine the derivative in Equation (2.8) when $p_s = p_b$:

$$\left.\frac{\partial \pi}{\partial p_s}\right|_{p_s = p_b} = p_b \left[2\left(\frac{\partial q_1}{\partial p_s} + \frac{\partial q_3}{\partial p_s} + \frac{\partial q_4}{\partial p_s} + \frac{\partial q_6}{\partial p_s} \right) + \frac{\partial q_2}{\partial p_s} + \frac{\partial q_5}{\partial p_s} \right]. \qquad (2.9)$$

Applying (2.7) to (2.9),

$$\left.\frac{\partial \pi}{\partial p_s}\right|_{p_s = p_b} = p_b \left[-\frac{\partial q_2}{\partial p_s} - 2\frac{\partial q_0}{\partial p_s} - \frac{\partial q_5}{\partial p_s} \right]. \qquad (2.10)$$

So $-\partial q_2/\partial p_s - 2\,\partial q_0/\partial p_s - \partial q_5/\partial p_s$ must be non-negative to explain Sky's decision not to offer Sky Sport alone. With a slight adjustment, the same reasoning used in the previous section can be used to show that $\partial q_0/\partial p_s$ and $\partial q_2/\partial p_s$ are positive. If $\bar{V}_s$ is substituted for V_s, and V_s is integrated from zero to infinity, then the same results are arrived at.

The next step is therefore to examine $\partial q_5/\partial p_s$ to see if it can be signed. Let $A = \max\left\{p_s, \bar{V}_b + p_s - p_b\right\}$, then

$$q_5 = \int_0^{p_s} \int_0^{\infty} \int_A^{\infty} \int_0^{\infty} g(V_s, \bar{V}_b, \bar{V}_s, V_b)\, dV_b\, d\bar{V}_s\, d\bar{V}_b\, dV_s$$

$$\frac{\partial q_5}{\partial p_s} = \int_0^{\infty} \int_A^{\infty} \int_0^{\infty} g(p_s, \bar{V}_b, \bar{V}_s, V_b)\, dV_b\, d\bar{V}_s\, d\bar{V}_b - \int_0^{p_s} \int_0^{\infty} \int_0^{\infty} g(V_s, \bar{V}_b, A, V_b)\, dV_b d\bar{V}_b dV_s \quad (2.11)$$

From (2.11) we can see that the sign of $\partial q_5/\partial p_s$ is ambiguous, as by Assumption A3, the first term is positive and the second is negative. But by considering the effect of an increase in p_s on q_5, we can get an idea about $\partial q_5/\partial p_s$.

Intuitively, $\partial q_5/\partial p_s$ cannot be signed because although the number of consumers who purchase Sky Sport in period one falls when the price increases, the proportion of customers who still purchase Sky Sport in period one, but switch to no Sky in period two, could change. If the first effect dominates, or the second effect works in the same direction, then $\partial q_5/\partial p_s$ will be negative. This possibility means that we can no longer sign $\partial\pi/\partial p_s\big|_{p_s=p_b}$, leaving open the possibility that it is optimal for Sky to not offer Sky Sport alone. This will be the case if $\partial q_5/\partial p_s$ is negative and dominates the effect of the other two terms.

If offering only the bundle is in fact optimal, then contrasting this model with the previous model, where only the value of the bundle was uncertain, should reveal the reason why. This contrast seems to suggest that it is beneficial for Sky to force consumers to buy the bundle when they would prefer to buy Sky Sport, because some of these consumers who would switch to no Sky in the second period will get a high realisation for their value of the bundle, allowing Sky to both retain some of these consumers, and receive a higher price.

The results of this model suggest that for some distributions of reservation values and uncertainty, Sky's behaviour is optimal. Unfortunately, this model does not give any insight into the likelihood that reservation prices actually follow such distributions, and so it is difficult to fully evaluate the credibility of this explanation.

5. Conclusion

This paper has examined the question of why Sky, essentially a monopolist in the pay television market in New Zealand, does not offer a package to consumers containing only Sky Sport channels - instead offering it only as part of a larger bundle. This behaviour appears inconsistent with literature that suggests mixed bundling is superior to pure bundling under believable conditions.

We have shown that in a simple one-period model of Sky's behaviour, under reasonable assumptions, Sky's behaviour is not optimal. Given this, it is either the case that one of the assumptions is flawed, which seems unlikely, or that this simple model does not reveal the full story about the situation.

A more developed model with two periods and uncertainty from consumers about their valuation of non-sport channels, allowing for learning, supports the main finding of the single-period model. However, it does leave open the possibility that Sky Sport is not offered separately because no-one would purchase it at the price it would be set at if the model were extended to the real world. When this model is extended to include uncertainty in the value of Sky Sport also, we find that Sky's behaviour is optimal for some distributions of reservation values and uncertainty. It is not known how likely it is that such distributions would be representative of reality, but nonetheless this result is the most appealing explanation for Sky's behaviour explored in this paper.

The model set-up serves to simplify the problem, as the presence of the Other-only package that is actually offered would add considerable complexity to the model. It is true that it could have a substantive effect on the outcomes and hence this set-up is limiting. The suggestion that marginal costs are zero is very close to the truth. For customers that have the hardware installed, there is very close to zero costs for Sky in allowing them access to their package. Even if there are small marginal costs, if they are the same for both packages and are low, then it is easy to show that the one-period result that Sky Sport should be offered alone still holds.

Although Sky's behaviour has not been definitively explained, investigation has revealed several candidate explanations with differing degrees of believability. Some of these explanations are simple but unsatisfying, such as fixed costs, discontinuities in distributions, and negligible differences. The two-period model and its extension do however present some more appealing explanations. The base two-period model, with slight relaxation of its assumptions, could lead to a situation where Sky Sport is not offered alone if the model were to be applied to a real world with many periods. The extended two-period model leaves open the possibility that Sky's behaviour is indeed optimal, if reservation values and uncertainty follow some subset of possible distributions.

Further investigation could involve examining the effects of deviations from traditional economic modelling, such as systematically low expectations for the value of the other channels (people might not be willing to pay for channels they know very little about, even if they would enjoy them if they actually could watch), or public resentment for instance. Additional modelling with more standard assumptions could involve adding more periods or adding to the model the fact that Sky offers the other channels on their own. An examination of the welfare implications of requiring Sky Sport to be offered alone could be undertaken, allowing analysis of potential policies that seek to regulate the pay television market.

6. References

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