# Inequality and Growth: What are the Tradeoffs?

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Address to NZAE, Palmerston North, June 2012

# Background and Overview

- Growth and inequality relationship dates back to Kuznets (AER, 1955)

  --Inverted-U between inequality and development
  --Controversial and inconclusive
  --Kuznets originally associated it with "dual economy dynamics"
  --transition from agricultural to industrial economy

- Key feature of recent empirical research has focused on the tradeoffs between income inequality and growth

- Addressed by running regressions of growth rates on measures of inequality (and other variables)

- Anand and Kanbur (JDE, 1993), Alesina and Rodrik (QJE, 1994), Persson and Tabellini (AER, 1994), Perotti (JEG, 1996), and others obtain **negative** relationship. The various explanations for this include:

  -- Political economy consequences of inequality
  -- Negative impact of inequality on education
  -- Capital market imperfections and credit constraints, etc

- Other studies find **positive**, or more ambiguous, relationship; see e.g. Li and Zou (RDE, 1998), Forbes (AER, 2000), and Barro (JEG, 2000). Explanations include

  -- Relative savings propensities of rich vs. poor
  -- Investment indivisibilities
  -- Incentives.

- Lundberg and Squire (EJ, 2003) introduce a number of factors that may potentially influence both inequality and growth; test for joint significance.

- They identify two common variables [openness and civil liberties] that impact both variables in the same direction, thereby implying a positive relationship between income inequality and growth.

- From theoretical perspective, empirical controversy not surprising.
- Growth rate and income distribution are both endogenous outcomes.
- Income inequality-growth relationship will reflect the underlying set of forces to which both are simultaneously reacting.
- This can be understood only within the context of a consistently specified general equilibrium growth model.
- “Association” rather than “causality” is more appropriate characterization.

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- Key element is heterogeneity
- Many sources (tastes, endowments, technology….)
- In most general context to solve for growth and inequality simultaneously is intractable (see Sorger, ET, 2000)
- But if assume that underlying utility function is homogeneous, then aggregation is possible (Gorman, Econom.,1953)
- Problem becomes tractable
- Assumption of homogeneity is routine throughout modern growth theory
- Leads to “representative agent theory of distribution” (Caselli and Ventura, AER 2000)
- Macroeconomic equilibrium has a simple recursive structure:

  --Aggregate equilibrium is independent of distribution,
  distribution depends upon aggregate.

# My Research

- My work in this area has followed this approach
- As the source of heterogeneity I have taken the initial endowments of capital (assets) and in some cases ability
- Quite an extensive literature taking this approach
- Alternative approach is to assume that people start out identical and that heterogeneity is endogenized through idiosyncratic random shocks (Krusell and Smith, JPE 1998, and others).

- Build models of increasing realism (complexity)

- Fall into three categories, structurally

1. Economies always on balanced growth path (One-sector endogenous growth model model)

   --No dynamics either in aggregates or distributions.

2. Models with transitional dynamics (e.g. neoclassical growth models; two-state variable endogenous growth model)

   --Have transitional dynamics of aggregates and distribution, but no mobility

   (inequality may expand or contract but no changing in rank)

   --Introduces **intertemporal** as well as **intratemporal** tradeoffs.

3. Two sources of heterogeneity in endowments (e.g. capital and skills)

   --Permits both wealth and income inequality dynamics and mobility. These need not move together.

# Applications

- Effects of structural changes (e.g. productivity increases)
- Tax and expenditure policies (e.g. government investment vs. government consumption and financing)
- Extensions to international economy (e.g. distributional consequences of international transfers)
- Path dependence of structural changes and their distributional consequences (particularly relevant for issues relating to foreign aid)
- Human capital vs. physical capital
- Role of externalities
- Heterogeneity vs. demographic distribution

# Remainder of Talk

- A simple canonical model
- A dynamic model
- Role of path dependence and consequences for distribution
- Skills and capital endowments

# A Canonical Model

## Firms

- Firms face identical production conditions; make identical decisions.

$$Y_j = A(L_j K)^{\alpha} K_j^{1-\alpha} \qquad 0 < \alpha < 1 \qquad (1a)$$

- Economy-wide capital stock yields externality such that in equilibrium the aggregate (average) production function is linear in aggregate capital stock

$$Y = AL^{\alpha} K \equiv \Omega(L)K \qquad (1b)$$

- Wage rate and the return to capital, $r$, are determined by marginal products

$$\left(\frac{\partial F}{\partial L_j}\right)_{K_j=K, L_j=L} = \alpha\Omega L^{-1} K = \alpha A L^{\alpha-1} K \equiv wK \qquad (2a)$$

$$\left(\frac{\partial F}{\partial K_j}\right)_{K=K, L=L} = (1-\alpha)\Omega = (1-\alpha) A L^{\alpha} \equiv r \qquad (2b)$$

# Consumers

- Consumers are identical except for initial endowments of capital, $K_{i0}$. Agent $i$'s share of total capital stock is $k_i \equiv K_i / K$. Relative capital has initial distribution function $G_0(k_i)$, mean $\sum_i k_i = 1$, and coefficient of variation $\sigma_k$.

- Agents endowed with a unit of time that can be allocated either to leisure, $l_i$ or to work, $1 - l_i \equiv L_i$.

- The agent's decision problem is

$$\max \int_0^\infty \frac{1}{\gamma}\left(C_i(t) l_i^\eta\right)^\gamma e^{-\beta t} dt, \quad \text{with} \quad -\infty < \gamma < 1, \eta > 0, \gamma\eta < 1 \tag{3}$$

subject to the capital accumulation constraint

$$\dot{K}_i = rK_i + (1 - l_i)wK - C_i \tag{4}$$

Optimality conditions are

$$C_i^{\gamma-1} l_i^{\eta\gamma} = \lambda_i \tag{5a}$$

$$\eta C_i^{\gamma} l_i^{\eta\gamma-1} = wK\lambda_i \tag{5b}$$

$$r = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{5c}$$

together with the transversality (intertemporal solvency) condition

## Macroeconomic equilibrium

- Economy is in fact always on its balanced growth path.
- All agents face same real wage and same rate of return on capital and thus choose same growth rate for consumption and leisure (although levels differ).

- So that summing over all agents we get

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C}; \ \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l} \Rightarrow l_i = \nu_i l \quad \text{for } l_i(t) \text{all } i \qquad (6)$$

and $\nu_i$ is constant for each *i*, and yet to be determined.

- Macroeconomic equilibrium reduces to differential equation in *l*, of form:

$$\frac{dl}{dt} = G(l) \quad \text{where} \quad G'(l) > 0 \qquad (7)$$

- Only solution that is intertemporally viable is that *l* be at its steady-state value at all points of time.

- The next step: show that the change in the relative stock of capital of agent *i* is described by the following equation

$$\dot{k}_i(t) = w(l)\left[\left(1-\nu_i l\left(1+\frac{1}{\eta}\right)\right)-\left(1-l\left(1+\frac{1}{\eta}\right)\right)k_i(t)\right] \tag{8}$$

- Transversality condition (intertemporal solvency) imposes the restriction

$$r > \psi$$

which can be further written as

$$l > \frac{\eta}{1+\eta}. \tag{9}$$

- Transversality condition implies only viable solution is $\dot{k}_i = 0$ for all time.

- Since $k_i$ reflects capital stocks that evolve gradually over time, this is accomplished by agents selecting their respective leisure, $l_i$, in accordance with the "relative labor supply" function

$$l_i - l = \left( l - \frac{\eta}{1+\eta} \right)\left(k_i - 1\right) \tag{10}$$

- This condition implies a positive relationship between relative wealth and leisure. Setting $l_i$ in accordance with this equation implies that the relative wealth position of agents, $k_i$, is unchanging over time.

- This is key mechanism generating the endogenous distribution of income.

**Intuition:** Wealthier agents have lower marginal utility of wealth. They work less, enjoy more leisure, and given their relative capital endowments, this translates to an endogenously determined distribution of income.

- Supported by empirical evidence from a range of sources (e.g. data from stock market boom, effect of inheritance)

# Macroeconomic equilibrium

- Economy is always on its balanced growth path.
- Equilibrium as in Romer (1986) model, but have aggregated over agents having different endowments and supplying different labor and having different consumption levels:

**Equilibrium growth rate:**

$$\psi = \frac{r(l) - \beta}{1 - \gamma} \quad (11a)$$

**Aggregate consumption-capital ratio:**

$$\frac{C}{K} = \frac{w(l)}{\eta} l \quad (11b)$$

**Goods market equilibrium:**

$$\psi = \Omega(l) - \frac{C}{K} \quad (11c)$$

- Summarize by (12a,b): jointly determine growth rate $\psi$ and leisure time, $l$:

**RR** $$\psi = \frac{(1-\alpha)\Omega(l) - \beta}{1-\gamma}, \tag{12a}$$

**PP** $$\psi = \Omega(l)\left[1 - \frac{\alpha}{\eta}\frac{l}{1-l}\right]. \tag{12b}$$

- **RR** ensures equality between return to capital and return to consumption.
- **PP** describes combinations of $\psi$ and $l$ for product market equilibrium.
- Focus attention on equilibria that are (i) viable, [i.e. satisfy transversality condition], (ii) also generate positive growth.

$$\frac{\eta}{\alpha+\eta} > l > \frac{\eta}{1+\eta} \tag{13a}$$

Both schedules are concave; sufficient conditions for a unique equilibrium:

$$\alpha - \gamma + \frac{\beta}{A} > 0; \quad 1 + \frac{1-\alpha l}{\eta(1-l)} > \frac{1-\alpha}{1-\gamma} \tag{13b}$$

# Distribution of Wealth, Income and Welfare

- Wealth distribution remains unchanged at its original level
- Income of individual *i:* $Y_i = rK_i + wK(1-l_i)$,
- Average economy-wide income is: $Y = rK + wK(1-l)$.
- Relative income of individual *i*, $y_i \equiv Y_i / Y$

$$y_i(l,k_i) = 1 + \rho(l)(k_i - 1), \quad \text{where} \quad \rho(l) \equiv 1 - \frac{\alpha}{(1+\eta)(1-l)}, \tag{14}$$

- Relative income depends upon *two* factors,

  (i) the initial (unchanging) relative holding of capital, and
  (ii) the equilibrium allocation of time between labor and leisure

- As long as the equilibrium is one of positive growth,

$$0 < \rho(l) < 1 \qquad (15)$$

- Relative income is increasing in $k_i$; richer individuals supply less labor, but this effect is insufficiently strong to offset impact of their higher capital income.

- The DD locus illustrates the relationship between the standard deviation of relative income, $\sigma_y$, [our measure of income inequality], and the standard deviation of capital endowments, $\sigma_k$,

**DD** $$\sigma_y = \rho(l)\sigma_k \qquad (12c)$$

- Given $\sigma_k$, the standard deviation of income is a decreasing and concave function of aggregate leisure time.

- Having determined equilibrium allocation of labor from the upper panel in Fig. 1, (12c) determines corresponding variability of income across agents.

# Welfare Measures

- Individual welfare equals the value of the intertemporal utility function (3) evaluated along the equilibrium growth path. Starting from $K_{i,0}$, this is

$$X(K_{i0}) = \frac{1}{\gamma}\frac{\left((C_i / K_i)l_i^{\eta}\right)^{\gamma}}{\beta - \gamma\psi}K_{i,0}^{\gamma} \qquad (16)$$

- Welfare of individual *i* relative to that of the individual with average wealth

$$x(k_i) = \frac{\left(C_i/K_i\right)^{\gamma}}{\left(C/K\right)^{\gamma}}\frac{l_i^{\eta\gamma}}{l^{\eta\gamma}}k_i^{\gamma} = \left(\frac{l_i}{l}\right)^{(1+\eta)\gamma} = \left[1+\left(1-\frac{\eta}{1+\eta}\frac{1}{l}\right)\left(k_i - 1\right)\right]^{\gamma(1+\eta)}$$

- The better endowed agent will have the higher absolute level of welfare.

- We can now compute the following natural measure of welfare inequality.

$$x(k_i)^{1/\gamma(1+\eta)} = u(k_i) = 1 + \varphi(l)(k_i - 1) \qquad \text{where} \qquad \varphi(l) \equiv 1 - \frac{\eta}{1+\eta}\frac{1}{l} \qquad (17a)$$

Welfare inequality, expressed in terms of equivalent units of capital, can be measured by the standard deviation of relative utility

**UU** $$\sigma_u = \varphi(l)\sigma_k \qquad (17b)$$

In the absence of taxes, and assuming positive growth

$$\sigma_k > \sigma_y > \sigma_u \qquad (17c)$$

- Fig 1 provides simple framework for analyzing effects of structural changes in growth-inequality relationship.

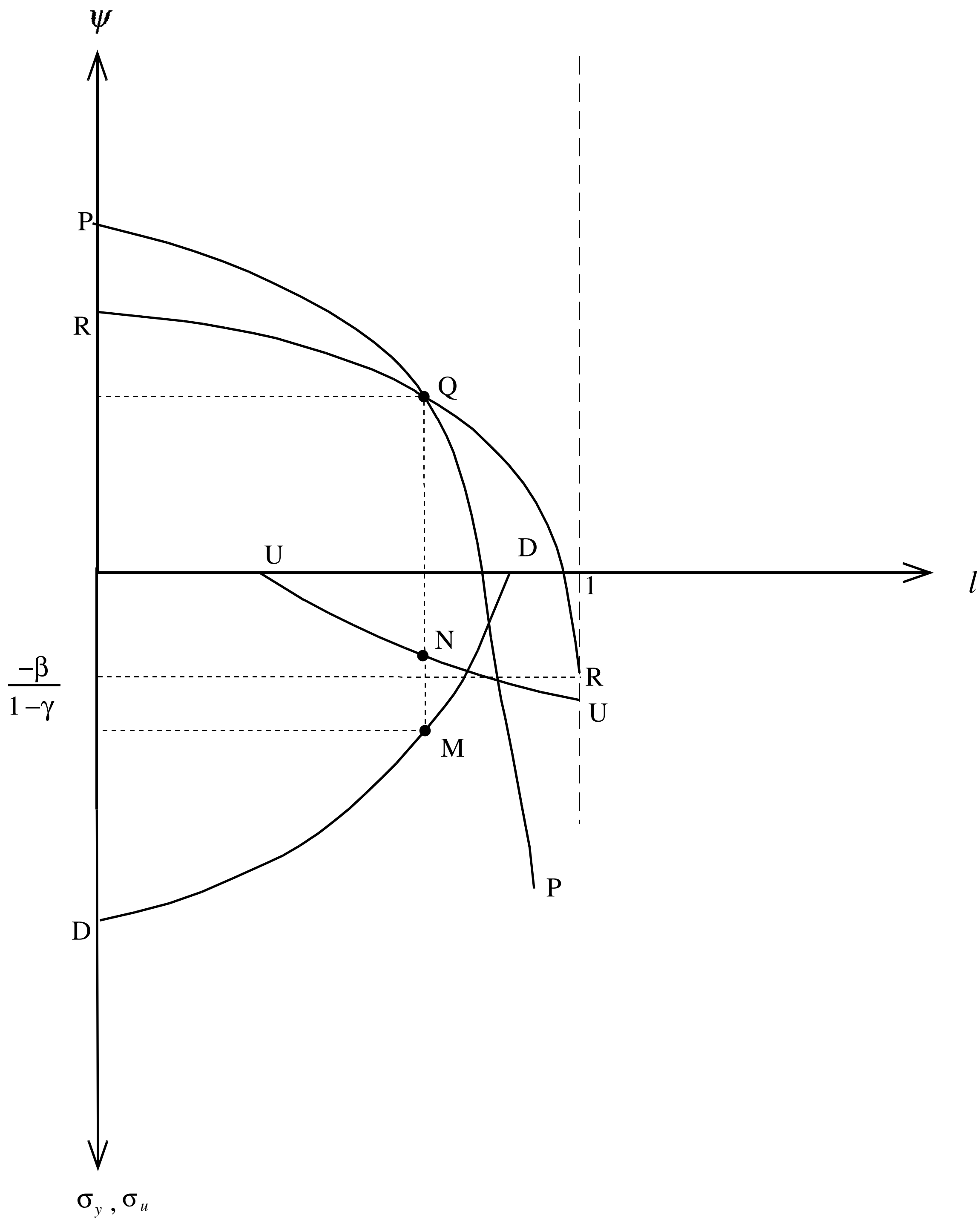



Fig 1: Equilibrium Growth, Employment, and Income Distribution

## Application to Taxes: (Garcia-Penalosa-Turnovsky, JMCB, 2007)

- Need to distinguish **before-tax** and **after-tax** income inequality
- The general effects of fiscal policy on these distributional measures can be summarized by the following proposition:

  Given the initial distribution of capital across agents:

  **(i)** Fiscal policy influences the before-tax distribution of income and the distribution of welfare through effect on labor supply.

  **(ii)** Any fiscal policy that increases (decreases) the supply of labor increases (decreases) before-tax income inequality and decreases (increases) welfare inequality.

  **(iii)** A labor income tax and a capital income tax both influence after-tax distribution in two ways;

  --Direct redistributive impact,

  --Indirect effect through changes in the labor supply.

  **(iv)** A consumption tax or investment subsidy has only latter effect.

- Proposition highlights how, when labor supply is endogenous, income inequality is a poor measure of welfare inequality. In fact, changes in pre-tax inequality are inversely related with those of welfare inequality.
- Post-tax income inequality not necessarily better measure of welfare inequality.
- Fiscal policy has two effects on the distribution of post-tax income,
- Net income inequality need not move together with welfare inequality.

- Specific issue they address relates to financing an investment subsidy
- Compare financing using
  (i) capital income tax, (ii) labor income tax, (iii) consumption tax

(i) Investment subsidy financed by a capital income tax:
**increases** the growth rate, labor supply, and before-tax income inequality. It **reduces** welfare inequality and has an **ambiguous effect** on after-tax income inequality.

(ii) An investment subsidy financed by a labor income tax:
**increases** the growth rate. Although it has an **ambiguous effect** on labor supply, and therefore on before-tax income inequality and welfare inequality, it **increases** after-tax income inequality.

(iii) An investment subsidy financed by a consumption tax:
**increases** the growth rate, labor supply, and both before-tax and after-tax income inequality, while it **reduces** welfare inequality.

- Highlights the different growth-inequality tradeoffs generated by this policy.

1. Changes in fiscal policy that increase the growth rate will also increase pre-tax income inequality, in line with some recent empirical evidence.

**Reason:** Faster growth $\Rightarrow$ greater supply of labor, $\Rightarrow$ reducing the wage rate, $\Rightarrow$ raising the return to capital, $\Rightarrow$ the distribution of income more unequal.

This apparent conflict between efficiency and equity disappears when inequality is measured in terms of utility.

2. Pre-tax and post-tax inequality need not move together.

**Indirect effect** of subsidy is always the same (increasing the growth rate, labor supply, and hence pre-tax inequality),

**Direct effect** depends upon the tax used to finance it. Consumption tax has no direct redistributive impact, labor income tax redistributes toward those with higher incomes, capital income tax redistributes in the reverse direction. These direct effects can be large enough to dominate the indirect effect.

- Can carry out similar analysis for model of human capital
- If human capital is only form of capital then economy is always on balanced growth path (Turnovsky, JHC 2011)

# A Dynamic Model

- Two differences:

(i) Population is growing at constant rate $n$

(ii) Neoclassical production function

$$Y = F\left(K, L\right) \tag{18}$$

$$w = \frac{\partial Y_j}{\partial L_j} = \frac{\partial Y}{\partial L} = F_L(K, L) \equiv w(K, L) \tag{19a}$$

$$r = \frac{\partial Y_j}{\partial K_j} = \frac{\partial Y}{\partial K} = F_K(K, L) \equiv r(K, L) \tag{19b}$$

where $w_K = F_{KL} > 0;\ w_L = F_{LL} < 0;\ r_K = F_{KK} < 0;\ r_L = F_{KL} > 0$.

- Otherwise, framework remains as before

# Aggregate Equilibrium Dynamics

- Summing over individuals, aggregate macroeconomic equilibrium is described by the following pair of equations in $K(t)$ and $l(t)$:

$$\dot{K} = F(K,L) - \frac{F_L(K,L)l}{\eta} - nK \tag{20a}$$

$$\dot{l} = \frac{F_K(K,L) - \beta - n - (1-\gamma)\dfrac{F_{KL}(K,L)}{F_L(K,L)}\left[F(K,L) - \dfrac{F_L(K,L)l}{\eta} - nK\right]}{G(l)} \tag{20b}$$

where

$$G(l) \equiv \frac{1-\gamma(1+\eta)}{l} - \frac{(1-\gamma)F_{LL}}{F_L} \tag{20c}$$

- This is conventional Ramsey model with endogenous labor supply
- Involves dynamic adjustment
- Aggregate behavior is independent of distribution

**Steady State**

$$F_K(\tilde{K}, \tilde{L}) = \beta + n \qquad (21a)$$

$$F(\tilde{K}, \tilde{L}) - n\tilde{K} = \frac{F_L(\tilde{K}, \tilde{L})\tilde{l}}{\eta} \qquad (21b)$$

$$\tilde{L} + \tilde{l} = 1 \qquad (21c)$$

These equations imply the key relationship:

$$\tilde{l} > \frac{\eta}{1+\eta} \qquad (22)$$

**Local Aggregate Dynamics**

$$\begin{pmatrix} \dot{K} \\ \dot{l} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} K - \tilde{K} \\ l - \tilde{l} \end{pmatrix} \tag{23}$$

where $a_{11}a_{22} - a_{12}a_{21} < 0$.

- The stable path for $K$ and $l$ is

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t} \tag{24a}$$

$$l(t) = \tilde{l} + \frac{a_{21}}{\mu - a_{22}}\left(K(t) - \tilde{K}\right) = \frac{\mu - a_{11}}{a_{12}}\left(K(t) - \tilde{K}\right) \tag{24b}$$

- In general $l(t)$ varies inversely with $K(t)$
- The evolution of aggregate leisure over time is an essential determinant of the time path of wealth inequality, and thus of income inequality.
- Suppose economy is subject to an expansionary structural shock ($K_0 < \tilde{K}$). This will lead to a jump in average leisure, such that $l(0) < \tilde{l}$; thereafter, leisure will increase monotonically during the transition.

# The distribution of income and wealth

- The dynamics of relative capital stock

$$\dot{k}_i(t) = \frac{w(K,l)}{K}\left[1-\nu_i l\left(1+\frac{1}{\eta}\right)-\left(1-l\left(1+\frac{1}{\eta}\right)\right)k_i\right] \qquad (25)$$

where $K, l$ evolve in accordance with (24a, b).

- To solve for time path of $k_i(t)$, note that agent *i's* steady-state share of capital is

$$1-\nu_i\tilde{l}\left(1+\frac{1}{\eta}\right)-\left(1-\tilde{l}\left(1+\frac{1}{\eta}\right)\right)\tilde{k}_i = 0 \qquad \text{for each } i \qquad (26)$$

or, equivalently [cf equation (10)]

$$\tilde{l}_i - \tilde{l} = \left(\tilde{l} - \frac{\eta}{1+\eta}\right)(\tilde{k}_i - 1) \qquad \text{for each } i \qquad (27)$$

- Next, linearize (25) around the steady-state $\tilde{K}, \tilde{l}, \tilde{k}_i$,

$$\dot{k}_i(t) = \frac{w(\tilde{K}, \tilde{l})}{\tilde{K}} \left[ \left(1 + \frac{1}{\eta}\right)(\tilde{k}_i - \nu_i)(l(t) - \tilde{l}) + \left[ \left( \tilde{l}\left(1 + \frac{1}{\eta}\right) - 1 \right) \left( k_i(t) - \tilde{k}_i \right) \right] \right] \tag{28}$$

The bounded solution to this equation is

$$k_i(t) - 1 = \theta(t)(\tilde{k}_i - 1) \tag{19}$$

where

$$\theta(t) \equiv 1 + \left(\frac{1}{\beta - \mu}\right) \frac{F_L(\tilde{K}, \tilde{L})}{\tilde{K}} \left(1 - \frac{l(t)}{\tilde{l}}\right), \tag{30}$$

$$k_{i,0} - 1 = \theta(0)(\tilde{k}_i - 1) = \left(1 + \left(\frac{1}{\beta - \mu}\right) \frac{F_L(\tilde{K}, \tilde{L})}{\tilde{K}} \left(1 - \frac{l(0)}{\tilde{l}}\right)\right)(\tilde{k}_i - 1) \tag{31}$$

and $k_{i,0}$ is given from the initial distribution of capital endowments.

- The evolution of agent *i*'s relative capital stock is determined as follows.

(i) Given the time path of aggregate economy, and distribution of initial capital endowments, (31) determines the steady-state relative capital, $(\tilde{k}_i - 1)$,

(ii) Together with (29) this yields entire time path for the distribution of capital.

(iii) We can express the time path for $k_i(t)$ in the form

$$k_i(t) - \tilde{k}_i = \left(\frac{\theta(t)-1}{\theta(0)-1}\right)(k_{i,0} - \tilde{k}_i) = \left(\frac{l(t)-1}{l(0)-1}\right)(k_{i,0} - \tilde{k}_i) = e^{\mu t}(k_{i,0} - \tilde{k}_i) \qquad (32)$$

- Because of linearity, we can transform these equations into corresponding results for the coefficient of variation of the distribution of capital:

$$\sigma_k(t) = \theta(t)\tilde{\sigma}_k \quad (29')$$

$$\sigma_{k,0} = \theta(0)\tilde{\sigma}_k \quad (31')$$

$$\sigma_k(t) - \tilde{\sigma}_k = \left(\frac{\theta(t)-1}{\theta(0)-1}\right)(\sigma_{k,0} - \tilde{\sigma}_k) = \left(\frac{l(t)-1}{l(0)-1}\right)(\sigma_{k,0} - \tilde{\sigma}_k) = e^{\mu t}(\sigma_{k,0} - \tilde{\sigma}_k) \quad (32')$$

- Two critical factors:

  (i) how agent *i*'s relative wealth evolves over time and

  (ii) the consequences of this for the distribution of income.

- A critical determinant of this is the magnitude of $\theta(t)$. From (30) this is seen to depend upon $l(t)/\tilde{l}$, which in turn depends upon how the underlying shock affects the evolution of the aggregate capital stock, $(K(t) - \tilde{K})$.

**Proposition (Wealth dynamics):**

(i) The long-run distribution of wealth converges to a non-degenerate steady-state distribution, which is proportional to initial distribution.

(ii) If economy experiences an expansion (contraction) in its aggregate capital stock, i.e. $K_0 < \tilde{K}$ ($K_0 > \tilde{K}$), then wealth inequality will decrease (increase) during transition, and the long-run distribution of wealth will be less (more) unequal than is the initial distribution.

## Income Distribution

- Income of individual $i$ and average economy-wide income at time $t$ are

$$Y_i(t) = r(t)K_i(t) + w(t)(1 - l_i(t)),$$
$$Y(t) = r(t)K(t) + w(t)(1 - l(t)),$$

- Relative income

$$y_i(t) \equiv Y_i(t)/Y(t).$$

- $s(t) \equiv F_K K/Y$ denotes the share of output going to capital,

$$y_i(t) - 1 = s(t)(k_i(t) - 1) + (1 - s(t))\frac{l(t)}{1 - l(t)}(1 - \nu_i) \tag{33}$$

The relative income of agent $i$ has two components,

(i) relative capital income,
(ii) relative labor income.

We can express (33) as

$$y_i(t)-1=\varphi(t)(k_i(t)-1), \tag{34}$$

where $$\varphi(t)\equiv 1-(1-s(t))\left[1+\frac{l(t)}{1-l(t)}\left(1-\frac{1}{\tilde{l}}\frac{\eta}{1+\eta}\right)\frac{1}{\theta(t)}\right]. \tag{35}$$

Because of linearity of (28') in $(k_i(t)-1)$ we can express this relationship between relative income and relative capital in terms of standard deviations

$$\sigma_y(t)=\varphi(t)\sigma_k(t) \qquad \varphi(t)<1 \tag{28'}$$

implying that income is more equally distributed than is capital.

In steady state:

$$\tilde{\sigma}_y = \tilde{\varphi}\tilde{\sigma}_k \qquad (28")$$

$$\tilde{\varphi} = \lim_{t\to\infty}\varphi(t) = 1 - \frac{1}{1+\eta}\left(\frac{1-\tilde{s}}{1-\tilde{l}}\right) = 1 - \frac{1}{1+\eta}\frac{F_L(\tilde{K},\tilde{L})}{F(\tilde{K},\tilde{L})}$$

$$\frac{\tilde{\sigma}_y}{\tilde{\sigma}_{y,0}} = \frac{\tilde{\varphi}}{\tilde{\varphi}_0}\frac{\tilde{\sigma}_k}{\sigma_{k,0}} = \left(\frac{1-\left(1/(1+\eta)\right)\left(F_L(\tilde{K}/\tilde{L})/F(\tilde{K}/\tilde{L})\right)}{1-\left(1/(1+\eta)\right)\left(F_L(\tilde{K}_0/\tilde{L}_0)/F(\tilde{K}_0/\tilde{L}_0)\right)}\right)\frac{\tilde{\sigma}_k}{\sigma_{k,0}} \qquad (36)$$

- Whether long-run distribution, following a structural change, is more or less unequal than initial distribution depends on long-run change in the distribution of capital, as reflected in $\tilde{\sigma}_k/\sigma_{k,0}$, and factor returns, as reflected in $\tilde{\varphi}/\tilde{\varphi}_0$.

- Dynamics of relative income

$$\frac{dy_i(t)}{dt} = s(t)\frac{dk_i(t)}{dt} + (1-s(t))\frac{1-\nu_i}{\left(1-l(t)\right)^2}\frac{dl(t)}{dt} + \left( k_i(t) - 1 + \left(\nu_i - 1\right)\frac{l(t)}{1-l(t)} \right)\frac{ds(t)}{dt}$$

- The evolution of the relative income of agent *i* depends upon two factors,

  (i) Evolution of relative capital income (first term),

  (ii) Evolution of relative labor income. [This can be expressed as a function of the evolution of aggregate leisure, and of the relative rewards to capital and labor, as reflected by the capital share, $s(t)$].

- For Cobb-Douglas production function, $s(t)$ remains constant.

- Whether income inequality increases or decreases depends on whether the economy converges to the steady state from below or from above.

- If economy that starts below the steady state, so that $K_0 < \tilde{K}$. Then $l(0) < \tilde{l}$ and leisure is rising, while wealth inequality is decreasing.

- Consider an agent with above average wealth, $(k_i - 1) > 0$, then $dk_i / dt < 0$ and $(\nu_i - 1) > 0$, implying that the first two terms are negative and that the relative income of the agent is decreasing during the transition.

- The evolution of factor shares may reinforce or offset these effects.

**Proposition (Income dynamics)**: The evolution of income inequality for an economy that converges to its steady state from below, i.e. $K_0 < \tilde{K}$, (respectively, from above, i.e. $K_0 > \tilde{K}$) is driven by three factors:

(i) Decreasing (increasing) wealth inequality, which tends to reduce (raise) income inequality;

(ii) Increasing (decreasing) leisure, which tends to increase (decrease) the relative labor income of the capital-poor and hence reduce(raise) income inequality;

(iii) Change in the share of capital in income, which depends both on whether the economy is converging from below or above, and on the elasticity of substitution in production.

If the share of capital is constant, income inequality will decrease (increase) during the transition to the steady state from below (above).

- Elastic labor supply is important
- With inelastic labor supply, there is no change in leisure time.
- Hence, the evolution of the income distribution is driven by two forces, the change in wealth inequality and that of labor share.
- With a high elasticity of substitution in production, these two forces have opposite signs, and it is possible that the second dominates, making the distribution of income more dispersed.
- When the labor supply can respond, the changes in average leisure will change the distribution of work-time and tend to reinforce the wealth-distribution effect.
- The presence of this effect implies that income inequality may move in opposite directions depending on the elasticity of leisure.

# Numerical Simulations

| | |
|---|---|
| Production function: | $Y = A(\alpha K^{-\rho} + (1-\alpha)L^{-\rho})^{-1/\rho}$ |
| Utility function: | $U = (1/\gamma)(Cl^{\eta})^{\gamma}$ |
| Basic parameters: | $A = 1, \alpha = 0.4$ |
| | $\rho = 1/3,\ 0,\ -0.2$ (elast of sub $\varepsilon = 0.75,\ 1,\ 1.25$) |
| | $\beta = 0.04,\ \gamma = -1.5, n = 0.015$ |
| Endogenous labor | $\eta = 1.75$ |
| Exogenous labor | $\bar{L} = 0.316,\ 0.278,\ 0.193$ corresponding to $\varepsilon = 0.75,\ 1,\ 1.25$ |

- Economy is initially in a steady state in which aggregate fraction of time devoted to leisure is $l_0$ and the average stock of capital is $K_0$.
- For Cobb-Douglas economy with endogenous labor supply, $l_0 = 0.722$, $L_0 = 0.278$.
- To preserve comparability, in the case of inelastic labor supply we normalize the fixed labor supply to the same level; implying that all aggregate magnitudes will be the same in the two cases.

- Starting from initial steady state, we investigate time paths in response to:

  (i) An increase in the level of technology $A$ from 1 to 1.5 (Fig. 1);
  (ii) A decrease in the rate of population growth rate, $n$, from 1.5% to 0 (Fig. 2).
  (iii) A decrease in the rate of time preference from 0.04 to 0.02 (Fig 3).

- Plot the time paths for the distribution of wealth and income, relative to their respective initial values, [$\sigma_k(t)/\tilde{\sigma}_{k,0}$ and $\sigma_y(t)/\tilde{\sigma}_{y,0}$], where normalize $\tilde{\sigma}_{k,0}=1$.

- Left-hand panels plot the time paths of the wealth and income inequality when the labor supply is exogenous,

- Right-hand panels present the case with endogenous labor, for case $\eta$=1.75.

Fig 1: Increase in A from 1 to 1.5

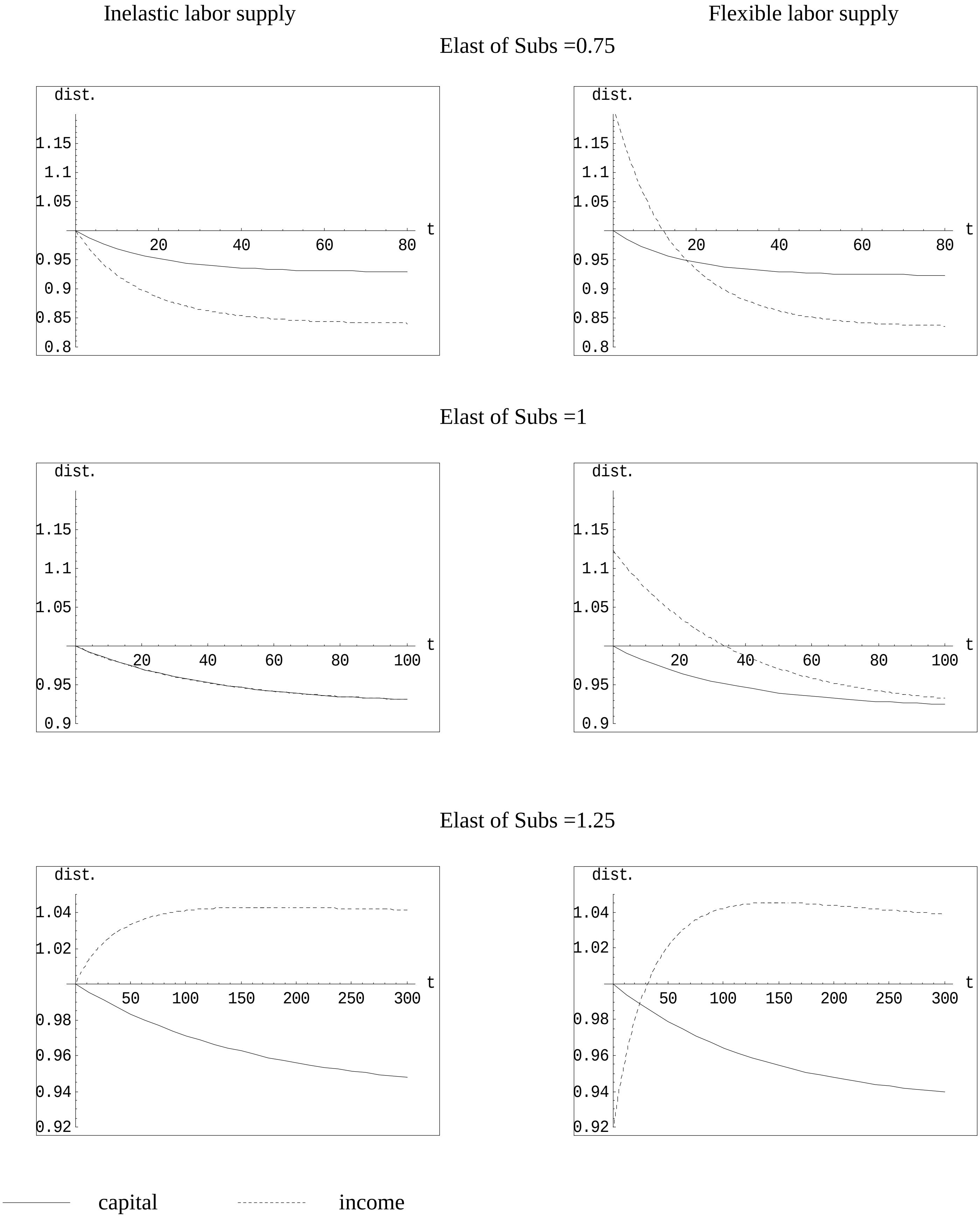
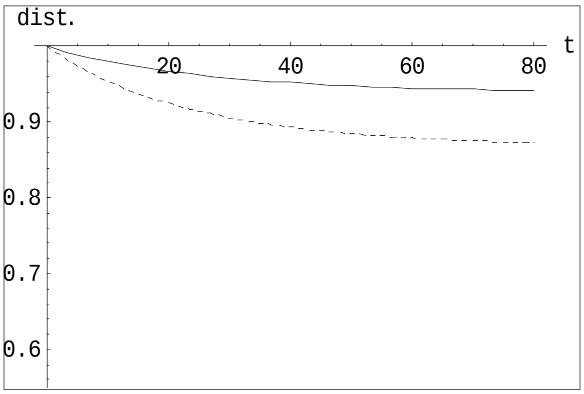
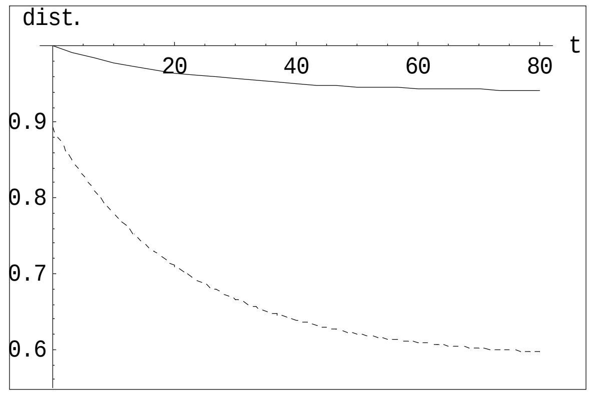


Fig 2: Decrease in n from 0.015 to 0

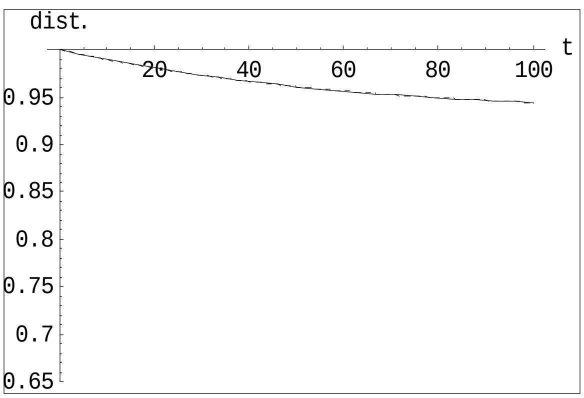
Inelastic labor supply | Flexible labor supply

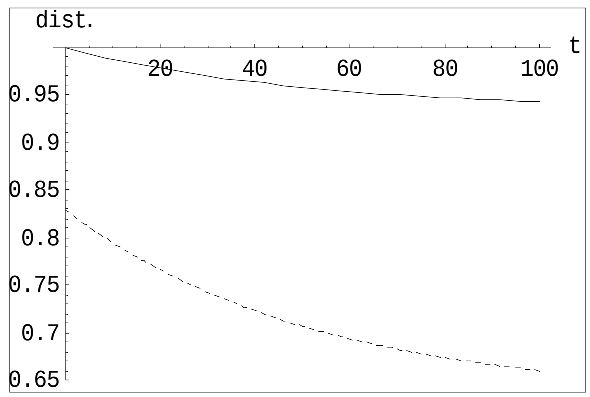
Elast of Subs =0.75



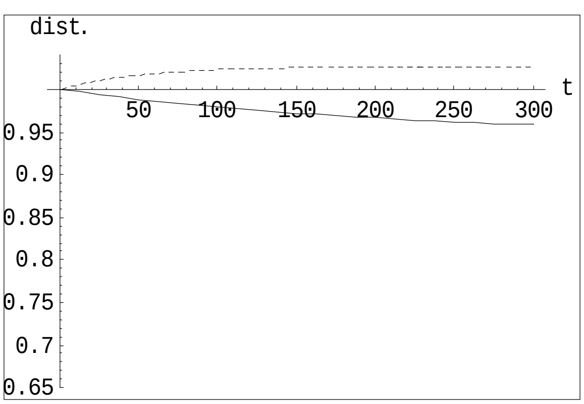



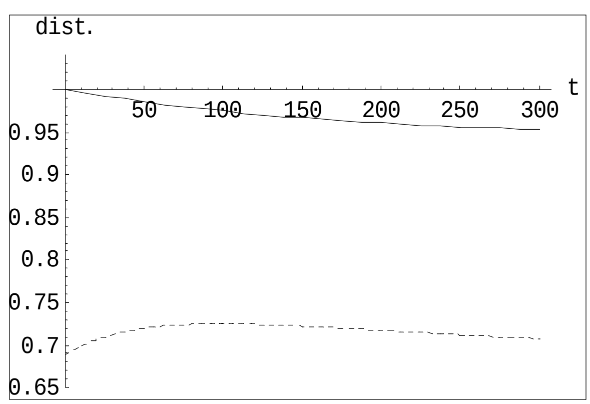

Elast of Subs =1



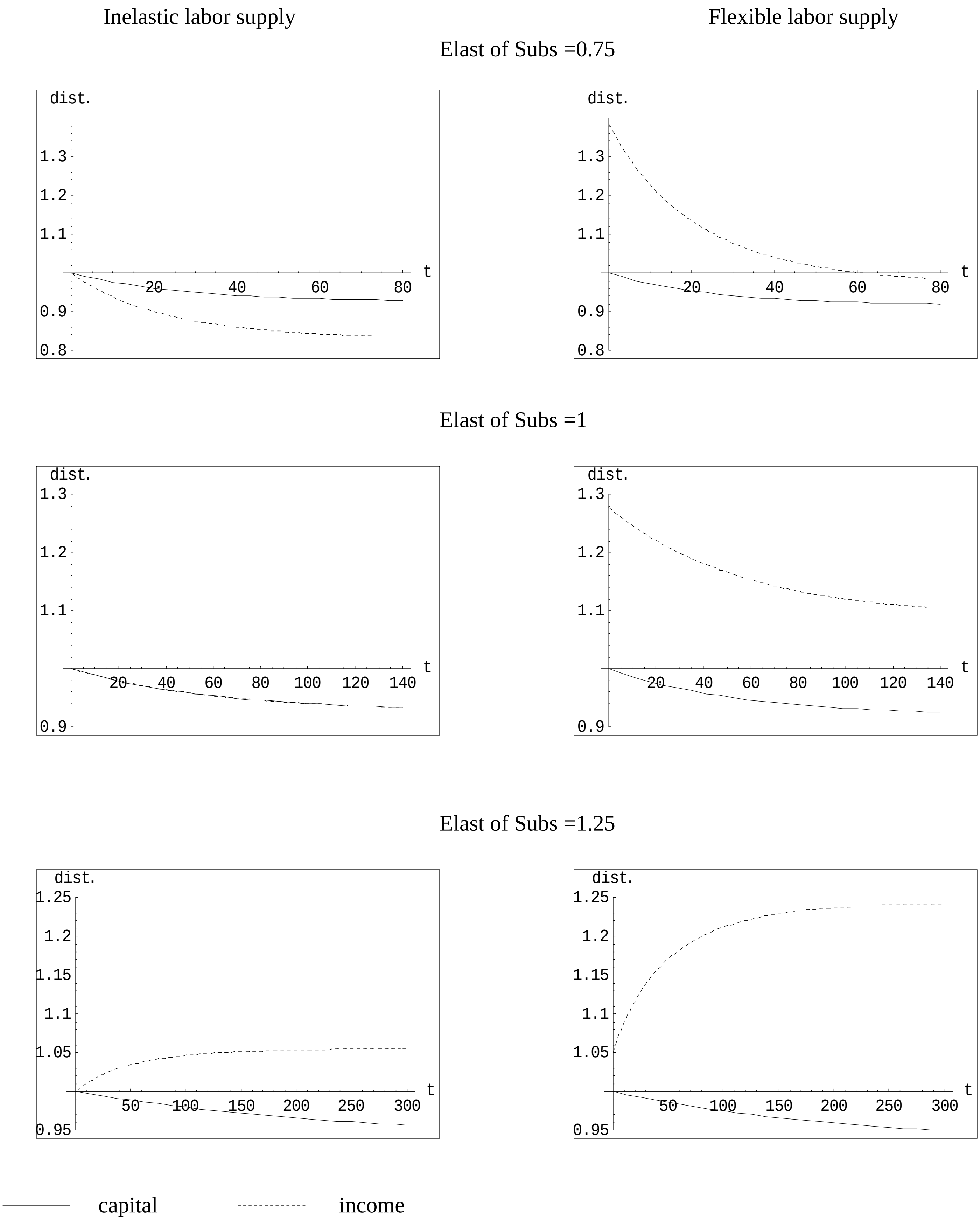

Elast of Subs =1.25




——— capital  - - - - - income

Fig 3: Decrease in β  from 0.04 to 0.02



# Path Dependence and Inequality

- Effect of structural change on dynamics of wealth and income inequality, depends critically upon initial response of leisure (labor supply) to shock.
- This initial response, in turn, depends upon time path that structural change is expected (known) to follow.
- Impacts of a productivity increase of a **given** magnitude on the **long-run distributions** of both wealth and income are crucially dependent upon the time path that the productivity increase is assumed to follow.
- Sharp contrast to the aggregate economy. In that case, the path followed by the productivity increase affects only the transitional path of the aggregate economy; it has no impact on steady-state aggregates.

- Model thus far adopts standard procedure of specifying structural changes as an unanticipated discrete permanent increase.
- For example a 50% increase in the level of productivity, which induces an instantaneous increase in labor supply and decrease in leisure.
- But if same (50%) productivity level increase is acquired *gradually* over a known time path, and is therefore anticipated after the first instant, the initial response of leisure is exactly the opposite. It increases rather than declines, leading to profoundly different distributional consequences.
- Important aspect

  --Model can admit a diverse set of distributional equilibria for countries having similar levels of aggregate development (per capita income), in accordance with empirical evidence.

  --Important in foreign aid which typically is granted over time

- Gradual implementation of productivity increase leads to dramatic differences along the transitional path, from case where its full impact is immediate.

  1. **Discrete** productivity increase leads to **steady decline in wealth inequality**.
  2. **Gradual** increase leads to **initial increase in wealth inequality**, followed by **eventual decline**.
  3. In long run, likely to lead to more rather than less wealth inequality, unless flexibility of production is very high. Even if long-run wealth inequality declines, will still be greater than if productivity increase is discrete.
  4. **Discrete** productivity increase leads to **initial increase in income inequality**, followed by a **steady decline** to below its initial level.
  5. **Gradual** introduction generates essentially the **opposite** time profile.
  6. But the most striking aspect of the transitional behavior is that a gradual productivity increase can generate a form of the Kuznets-type inverted-U relationship between inequality and per-capita income.

## Analytical framework

- Level of productivity, $A(t)$ is assumed to increase gradually from its initial level, $A_0$, to its enhanced long-run level, $\tilde{A}$, (both known to the firm).

$$A(t) = \tilde{A} + \left(A_0 - \tilde{A}\right)e^{-\kappa t} \qquad (37)$$

- $\kappa$ defines the time path followed by the increase in productivity. The conventional approach is approximated by letting $\kappa \to \infty$.
- Contrast between how $\kappa$ affects the **aggregate** behavior of the economy and its consequences for **distribution**. While it affects the transitional path of the aggregate economy, it has no effect on the aggregate steady state.
- But the choice of $\kappa$ has profound impacts on **both** the time paths and the steady-state equilibria of wealth and income distributions.

- Macrodynamic equilibrium is described by the following equations:

$$\dot{K} = A(t)F(K,L) - \frac{A(t)F_L(K,L)l}{\eta} - nK \tag{38a}$$

$$\dot{l} = \frac{1}{G(K,l)}\left\{A(t)F_K(K,L) - \beta - n - (1-\gamma)\frac{F_{KL}K,L)}{F_L}\left[A(t)F(K,L) - \frac{A(t)F_L(K,L)l}{\eta} - nK\right]\right\} \tag{38b}$$

$$\dot{A}(t) = \kappa\left(\tilde{A} - A(t)\right) \tag{38c}$$

where

$$G(K,l) \equiv \frac{1-\gamma(1+\eta)}{l} - (1-\gamma)\frac{F_{LL}}{F_L} > 0, \qquad l + L = 1$$

The steady-state equilibrium is

$$\tilde{A}F(\tilde{K},\tilde{L}) - n\tilde{K} = \frac{F_L(\tilde{K},\tilde{L})\tilde{l}}{\eta} \tag{39a}$$

$$\tilde{A}F_K(\tilde{K},\tilde{L}) = \beta + n \tag{39b}$$

$$\tilde{L} + \tilde{l} = 1 \tag{39c}$$

- Since the steady state is independent of $\kappa$, the long-run effects of an increase in productivity are independent of the time path by which it is achieved.
- However, the transitional responses may be significantly different (as illustrated later in the numerical experiments).
- More importantly, this has fundamental consequences for wealth and income inequality, both in transition and across steady states.

## 3. Distributional dynamics

- Dynamic equation for the relative capital stock is now:

$$\dot{k}_i(t) = \frac{A(t)F_L(K,l)}{K}\left[1-\nu_i l\left(1+\frac{1}{\eta}\right)-\left\{1-l\left(1+\frac{1}{\eta}\right)\right\}k_i\right] \tag{40}$$

where $K, l, A$ evolve in accordance with (38a)-(38c).

- The bounded solution is

$$k_i(t)-1=\theta(t)(\tilde{k}_i-1) \tag{41a}$$

where

$$\theta(t) \equiv \left[1+\frac{\tilde{A}F_L}{\tilde{K}}\int_t^{\infty}\left(1-\frac{l(\tau)}{\tilde{l}}\right)e^{-\beta(\tau-t)}d\tau\right] \tag{41b}$$

$$k_{i,0}-1=\theta(0)(\tilde{k}_i-1)=\left(1+\frac{\tilde{A}F_L}{\tilde{K}}\int_0^{\infty}\left(1-\frac{l(\tau)}{\tilde{l}}\right)e^{-\beta\tau}d\tau\right)(\tilde{k}_i-1) \tag{41a'}$$

- The crucial difference between this analysis from previous lies in the form of the productivity shock $A(t)$, which is reflected in the time path of $\theta(t)$.

- If the complete productivity increase occurs instantaneously,

  $l(\tau)-\tilde{l} = \left(l(0)-\tilde{l}\right)e^{\mu\tau}$ and $\theta(t)$, $\theta(0)$ simplify to

$$\theta(t) = 1+\left(\frac{1}{\beta-\mu}\right)\frac{\tilde{A}F_L(\tilde{K},\tilde{L})}{\tilde{K}}\left(1-\frac{l(t)}{\tilde{l}}\right);$$

$$\theta(0) = 1+\left(\frac{1}{\beta-\mu}\right)\frac{\tilde{A}F_L(\tilde{K},\tilde{L})}{\tilde{K}}\left(1-\frac{l(0)}{\tilde{l}}\right)$$

- Only current allocation of time to leisure relative to steady-state allocation is relevant in determining current wealth inequality relative to its long run.

- When the productivity increase occurs gradually over time, the entire time profile of $A(t)$, reflected in $l(t)$, needs to be taken into account.

- If during transition $l(\tau)<\tilde{l}$, so that leisure approaches its long-run steady state from below, then $\theta(t)<1$ and wealth inequality will decline over time.

- As simulations show, this is the case for the discrete productivity increase, when following an initial drop, leisure increases monotonically over time.

- Simulations also show that a gradual productivity increase leads to an initial increase in leisure, taking it initially above its new steady-state level.

- Since transitional path is of inverted U-shaped form eventually approaching $\tilde{l}$ from below, whether inequality rises or falls over time depends upon the extent to which $l(\tau) > \tilde{l}$ during the early phases of the adjustment.

- Closer $l(\tau)$ is to its steady state, $\tilde{l}$, the smaller the subsequent adjustment in $l(t)$, and hence smaller is the overall change in the distribution of wealth.

- This is because if economy and therefore all individuals fully adjust their respective leisure times instantaneously, they will all accumulate wealth at the same rate, causing the wealth distribution to remain unchanged.

## Numerical analysis

- Compare aggregate and distributional consequences of a 50% increase in productivity, which we allow to take effect in two alternate ways:

  (i) An immediate unanticipated jump in productivity from 1 to 1.5.

  (ii) A 50% increase in $A$ that occurs gradually over time, [$A(t)$ adjusts at the (known) rate 10 % per period (year)].

- In the latter case, the enhanced productivity level is achieved asymptotically. As a result, the instant it starts to increase, the subsequent growing levels of productivity are fully anticipated along the transition path.

**Figure 1 : Aggregate Dynamics**

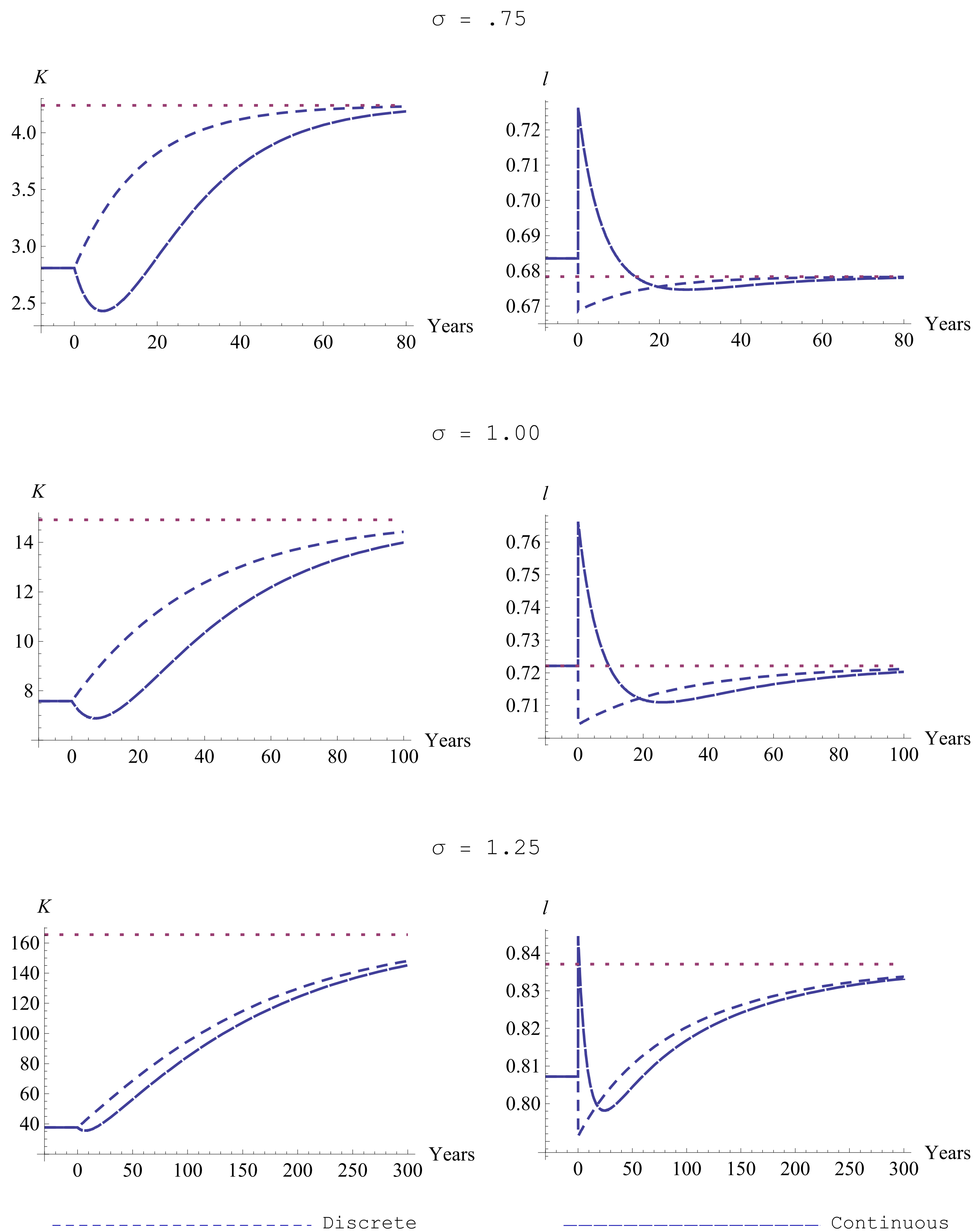


**Figure 2 : Distributional Dynamics**

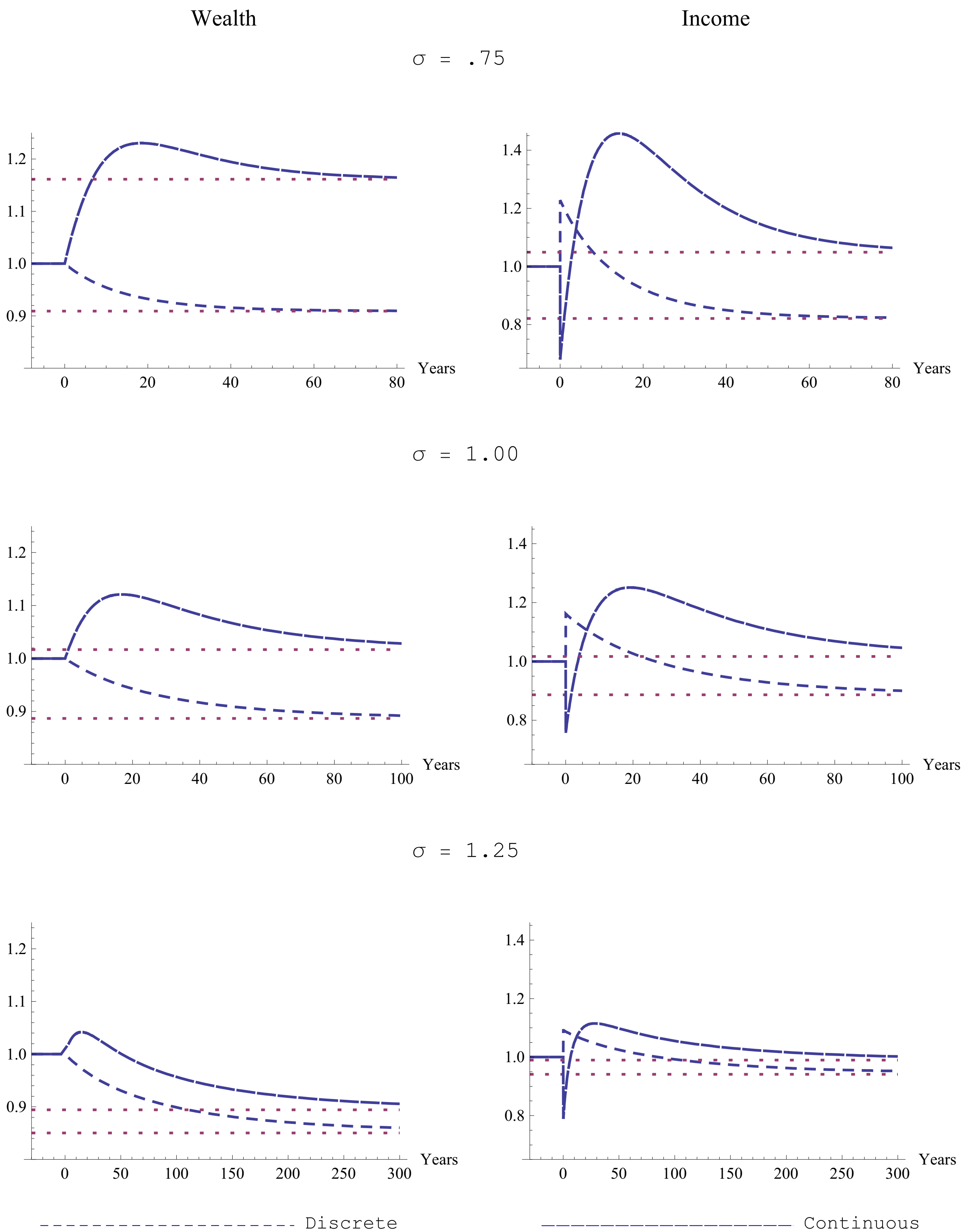


**Figure 3 : Robustness of Distributional Dynamics**

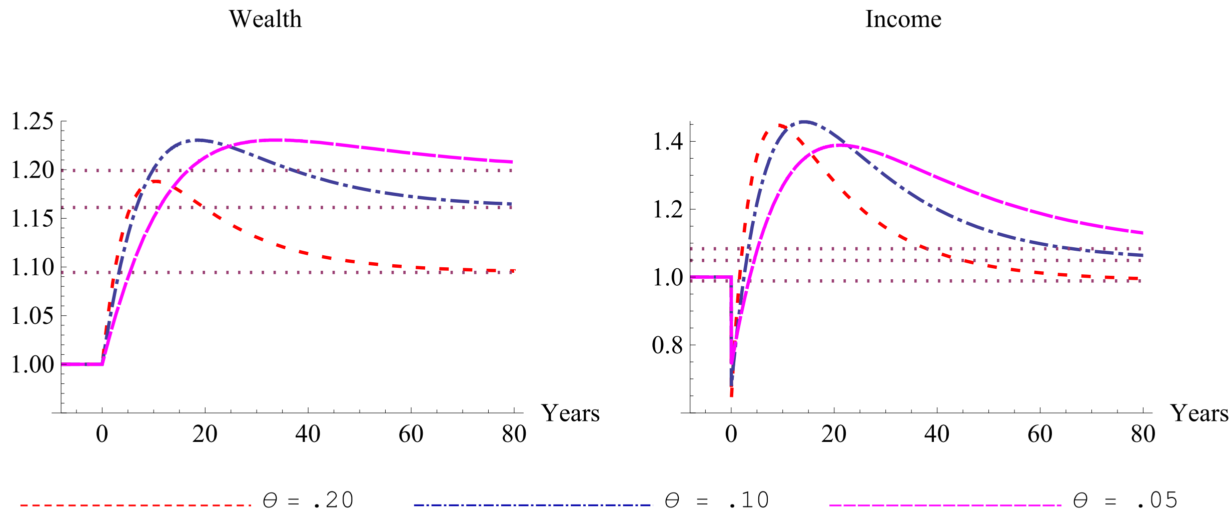
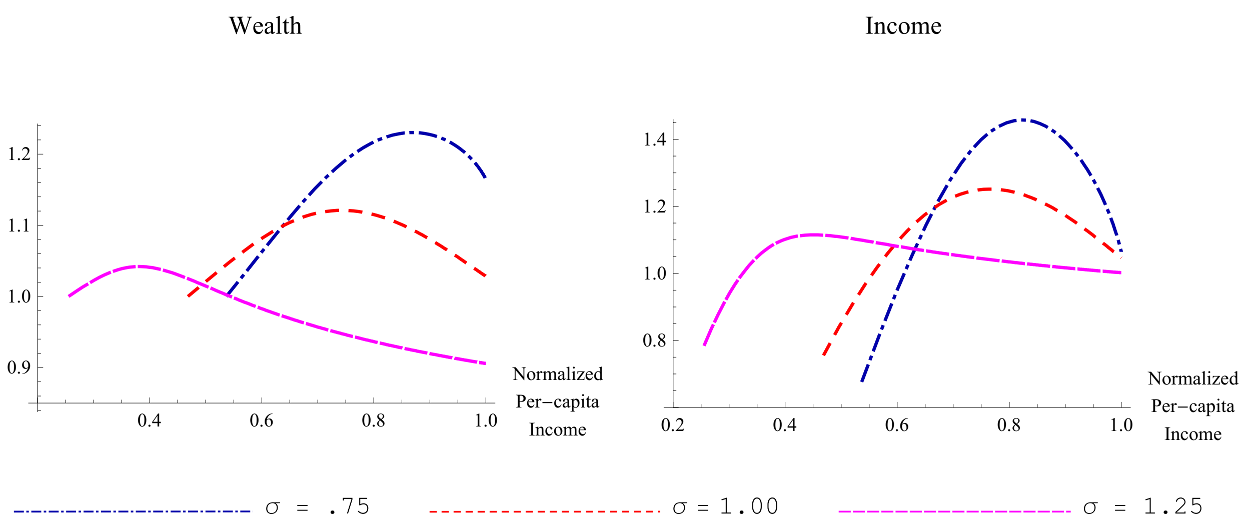



**Figure 4 : Kuznets' Curve**



## The growth-inequality relationship

- Consequences of the nature of productivity change for the Kuznets' curve?
- Such a relationship cannot be generated by a **discrete** increase in productivity. This is because after initial jumps in output and income inequality, subsequent increases in income are associated with a uniform decline in inequality.
- The more flexible specification where the level of productivity increases gradually is capable of generating such a pattern for both wealth and income inequality along the transition path, after initial adjustments.
- The inverted U-shaped relationship between inequality and per-capita income generated by this model relies on a mechanism that is very different from the traditional explanations of the Kuznets' curve.
- While the sectoral composition of capital, labor, and knowledge is the traditional mechanism behind the Kuznets' curve, we focus on the role of consumption-smoothing in response to a gradual, but anticipated, change in productivity, and its effects on investment and the choice of labor and leisure.

# Heterogeneous Skills

## Consumers

- Two sources of heterogeneity: agents' initial endowments of capital, $K_{i,0}$, and fixed relative skill levels, $a_i$.

- Relative capital (wealth) is $k_i(t) \equiv K_i(t)/K(t)$, has mean 1, and dispersion across agents, $\sigma_k(t)$, with initial (given) dispersion being $\sigma_{k,0}$.

- Relative skills across agents has mean zero and constant dispersion $\sigma_a$.
  --previous model corresponds to $a_i = 1$

- $k_{i,0}$, $a_i$ possibly correlated across agents.

# Technology and factor payments

- Effective labor employed by the firm: $L = \frac{1}{N}\sum_i a_i L_i$

- Firms pay capital and labor according to their marginal physical products,

$$r(t) \equiv r(K,L) = F_K(K,L) \qquad (22a)$$

$$w_i(t) = w_i(K,L) = F_{L_i}(K,L) = a_i F_L(K,L) \qquad (22b)$$

- Average wage rate

$$w(t) \equiv w(K,L) = F_L(K,L) \qquad (22b')$$

- Distribution of relative wage rates, $w_i(t)/w(t)$, is given, and reflects the given distribution of skill levels across agents.

# Macroeconomic equilibrium

$$\dot{K} = F(K,L) - F_L(K,L)\frac{(1-L)}{\eta} - \delta K$$

$$\dot{L} = \frac{1}{Z(L)}\left\{(1-\gamma)\frac{F_{KL}}{F_L}\left(F - F_L\frac{(1-L)}{\eta} - \delta K\right) + \left[(\beta+\delta) - F_K\right]\right\}$$

$$Z(L) \equiv \frac{1-\gamma(1+\eta)}{1-\tilde{L}} - \frac{(1-\gamma)F_{LL}}{F_L} > 0$$

- Two differences from previous model

  --no population growth, but have depreciation of capital

  -- more convenient to work in terms of *L* rather than *l*

- Assuming stability, aggregate quantities converge to steady state

$$F_K(\tilde{K},\tilde{L}) = \beta + \delta \tag{43a}$$

$$F(\tilde{K},\tilde{L}) - \delta\tilde{K} = F_L(\tilde{K},\tilde{L})\frac{(1-\tilde{L})}{\eta} \tag{43b}$$

$$\tilde{L} + \Omega\tilde{l} = 1 \tag{43c}$$

- Again get constraint

$$\frac{1}{1+\eta} > \tilde{L} > 0 \tag{44}$$

- (Locally) stable paths for $K(t)$ and $L(t)$ as before.

# The dynamics of relative wealth and income

## Relative capital stock (wealth)

- Dynamics of individual *i's* relative capital stock, $k_i(t) \equiv K_i(t)/K(t)$:

$$\dot{k}_i(t) = \frac{w(K,L)}{K}\left[\left(a_i - a_i \nu_i l \frac{1+\eta}{\eta}\right) - \left(1 - \Omega l \frac{1+\eta}{\eta}\right) k_i(t)\right] \qquad (45)$$

- (45) implies steady-state relationship:

$$\tilde{L}_i - \tilde{L} = \left(\tilde{L} - \frac{1}{1+\eta}\right)\left(\frac{\tilde{k}_i}{a_i} - 1\right) \qquad \text{for each } i \qquad (46)$$

- An analogous relationship holds at all points of time

$$L_i(t) - L(t) = \frac{l(t)}{\tilde{l}}\left(\tilde{L} - \frac{1}{1+\eta}\right)\left(\frac{\tilde{k}_i}{a_i} - 1\right) \qquad (46')$$

- Bounded solution for $k_i(t)$ can be expressed as

$$k_i(t) = \frac{1+\theta(t)}{1+\theta(0)} k_{i,0} + \frac{\theta(0)-\theta(t)}{1+\theta(0)} a_i \tag{47}$$

$$\theta(0) \equiv \frac{F_L(\tilde{K},\tilde{L}) / \tilde{K}}{\beta - \mu} \left( \frac{L(0)-\tilde{L}}{1-\tilde{L}} \right) \tag{48}$$

$$\theta(t) = \theta(0) e^{\mu t} \tag{48'}$$

- At any point of time, agent's relative capital is weighted average of $k_{i,0}$ and $a_i$.
- Weights of the two endowments change over time. As economy converges to new steady state, factor prices change, altering relative contributions of wealth and skill endowments to individual's income, and hence to savings.
- In expanding economy, $L(0) > \tilde{L}$, and $\theta(0) > \theta(t) > 0$, $\dot{\theta}(t) = \mu\theta(t) < 0$;

  i.e. relative weight shifts from the endowment of capital toward skills.

- Difference between an agent's relative capital and the mean is

$$k_i(t) - 1 = \frac{1+\theta(t)}{1+\theta(0)}(k_{i,0} - 1) + \frac{\theta(0) - \theta(t)}{1+\theta(0)}(a_i - 1) \qquad (49)$$

$$\tilde{k}_i - 1 = \frac{1}{1+\theta(0)}(k_{i,0} - 1) + \left(\frac{\theta(0)}{1+\theta(0)}\right)(a_i - 1) = \frac{1}{1+\theta(t)}\left[(k_i(t) - 1) + \theta(t)(a_i - 1)\right]$$

- Expressions illustrate potential for agents to change relative wealth positions.
- If agent begins with above-average capital ($k_{i,0} > 1$), but is endowed with below-average skills ($a_i < 1$), he may end up with below-average capital.
- Two offsetting forces driving the accumulation of capital.
    1. Those with large initial wealth accumulate capital more slowly (during an expansion), which tends to deteriorate their relative position.
    2. Those with more ability have higher incomes, and hence accumulate more capital, which tends to improve their relative position.
- This contrasts with the previous model where ($a_i = 1$).

## The dynamics of relative income

- Agent *i*'s relative income can be expressed as

$$y_i(t) = \varphi(t)k_i(t) + (1-\varphi(t))a_i \tag{50}$$

where

$$\varphi(t) \equiv \left[ s_K(t) + s_L(t)\frac{l(t)}{\tilde{l}L(t)}\left(\tilde{L} - \frac{1}{1+\eta}\right)\frac{1}{1+\theta(t)} \right] \tag{51}$$

represents weight in current relative income due to *current* relative capital.

- Dynamics of $y_i(t)$ are driven by those of the aggregate variables, $K(t), L(t)$, (both directly and through effect on factor shares), as well as by agent's relative rate of capital accumulation, $k_i(t)$.
- We can express current relative income as a weighted average of $k_{i,0}$ and $a_i$

$$y_i(t) = \varphi(t)\frac{1+\theta(t)}{1+\theta(0)}k_{i,0} + \left(1-\varphi(t)\frac{1+\theta(t)}{1+\theta(0)}\right)a_i \tag{52}$$

### 4.3 Wealth and income mobility

- Compare two individuals *i*, *j*, and express their wealth gap at time *t* as

$$k_i(t) - k_j(t) = \frac{1+\theta(t)}{1+\theta(0)}\Delta k + \frac{\theta(0)-\theta(t)}{1+\theta(0)}\Delta a \qquad (53)$$

where $\Delta a \equiv a_i - a_j$ and $\Delta k \equiv k_{i,0} - k_{j,0}$.

- Two offsetting forces influencing this gap, the differences in initial capital and the differences in ability.
- In a growing economy, $\theta(0) > \theta(t)$, $\dot{\theta}(t) < 0$, implying that the term multiplying the capital gap is less than one and declining over time.

- **Wealth mobility**: Possibility that individual having an initial small wealth endowment overtakes some other initially richer agent.

  **Proposition:** In an economy that is accumulating capital [$\theta(0) > 0$],

  **(i)** if individual $j$ is initially endowed with less wealth than is individual $i$, the poorer agent will catch up in wealth if and only if $-\Delta a \cdot \theta(0) > \Delta k$;

  **(ii)** if individual $j$ is initially endowed with both less wealth and less ability than individual $i$, the poorer agent will never catch up.

- Poorer agent will catch up in wealth if and only if he has sufficiently superior ability.

- **Income mobility:** May occur on impact, or along subsequent transitional path.

  **Proposition:** Individual *i* may initially be richer than *j* because of higher initial wealth, higher ability, or both. If that is the case, then

  (i) if *i* has a larger endowment of both ability and wealth, *j* cannot catch up to *i*'s income level;

  (ii) if *j* is initially endowed with less wealth than is *i*, the poorer agent will catch up in income along the transitional path if and only if

$$\frac{\tilde{\varphi}}{1+\theta(0)-\tilde{\varphi}} < -\frac{\Delta a}{\Delta k} < \frac{\varphi(0)}{1-\varphi(0)} \tag{54a}$$

  and the economy satisfies $\varphi(0) > \tilde{\varphi}/(1+\theta(0))$;

  (iii) if *j* is initially endowed with less skill than is *i*, the poorer agent will catch up in income if and only

$$\frac{\tilde{\varphi}}{1+\theta(0)-\tilde{\varphi}} > -\frac{\Delta a}{\Delta k} > \frac{\varphi(0)}{1-\varphi(0)} \tag{54b}$$

  and the economy satisfies $\varphi(0) < \tilde{\varphi}/(1+\theta(0))$.

- As the economy converges to new steady state, income mobility is possible only for one type of agent, either skill-rich or the capital-rich, but not both.
- If wages are growing fast, then skill-rich agents will be able to catch-up but capital-rich individuals will not, and vice versa.
- The behavior of factor prices depend on both the structure of aggregate economy and the nature of the shock, which is captured by sgn $\left[\varphi(0)-\tilde{\varphi}/(1+\theta(0))\right]$.
- For Cobb-Douglas production function, in growing economy $\varphi(0)>\tilde{\varphi}/(1+\theta(0))$ always holds. Then skill-rich that may catch up in income; in contrast to contracting economy it is capital-rich for whom this is possible.

**Measure of wealth mobility**

- **Definition:** Let $\Delta\hat{a}$ be the minimum ability gap required for $j$ to catch up to $i$'s wealth, given their initial wealth gap, $\Delta k$. We then define the extent of wealth mobility, denoted $\omega_k$, as $\omega_k \equiv -\left(\Delta\hat{a} / \Delta k\right)^{-1}$.

- Measure of wealth mobility is inverse of the minimum ability gap for catch-up.

$$\omega_k = \theta(0) = \frac{F_L(\tilde{K})(1-\tau_w)/\tilde{K}}{\frac{F_L(\tilde{K})(1-\tau_w)}{\tilde{K}}\left(\frac{1+\eta}{\eta}\right)\left(\frac{1}{1+\eta}-\tilde{L}\right)-\mu}\left(\frac{L(0)-\tilde{L}}{1-\tilde{L}}\right) \qquad (55)$$

- The degree of wealth mobility depends on both structural characteristics of the aggregate economy and specific change generating the initial jump in aggregate labor supply.

## Measure of income mobility

- **Definition**: Let $\Delta\bar{a}$ ($\Delta\bar{k}$) be the minimum ability (wealth) gap required for $j$ to catch-up in income when it is the ability-rich (capital-rich) that may experience income mobility. Whenever the skill-rich can catch-up with the capital-rich, we define measure of income mobility to be $\omega_y^a \equiv -\left(\Delta\bar{a} / \Delta k\right)^{-1}$; whenever it is the capital-rich that are catching up, we define it to be $\omega_y^k \equiv -\left(\Delta\bar{k} / \Delta a\right)^{-1}$.

- We measure degree of income mobility by endowment gap required for poorer agent to be able to catch up to the richer one during the transition, where income mobility depends on which agent is doing the catching up. From the definitions

$$\omega_y^a = \frac{1+\theta(0)-\tilde{\varphi}}{\tilde{\varphi}} \tag{56a}$$

$$\omega_y^k = \frac{\tilde{\varphi}}{1+\theta(0)-\tilde{\varphi}} \tag{56b}$$

- Higher value of $\omega_y^a$ or $\omega_y^k$ implies that, for given distributions of initial wealth and skills, a greater fraction of the population will change their relative position along the distribution of income.

**Proposition:** In a growing economy if agent $i$ catches up to agent $j$'s level of wealth he will do so only after he has caught up to agent $j$'s level of income. It is also possible that he will catch up to his level of income, but not to his level of wealth.

- Since agents save a fraction of their income strictly less than one and given that $i$ had a higher initial stock of capital, $j$ will manage to accumulate as much wealth as $i$ only if he has a higher level of income. Hence, he must catch up $i$'s income level before he can catch-up to his wealth.

## 5. Wealth and income inequality

- Linearity of expressions permits us to transform them into measures of aggregate wealth inequality, expressed either as CV, $[\sigma_k(t)]$, or SCV, $[\sigma_k^2(t)]$.

- Both have qualitatively similar implications, they have different advantages; $\sigma_k(t)$ is dimensionally equivalent to Gini coefficient, $\sigma_k^2(t)$ is decomposable.

$$\sigma_k^2(t) = \frac{1}{[1+\theta(0)]^2}\left(\left[1+\theta(t)\right]^2 \sigma_{k,0}^2 + \left[\theta(0)-\theta(t)\right]^2 \sigma_a^2 + 2(1+\theta(t))[\theta(0)-\theta(t)]\sigma_{k,0}\sigma_a\chi\right)$$

where $\chi$ is the correlation coefficient between initial capital endowments and skills.

$$\tilde{\sigma}_k^2 = \frac{1}{[1+\theta(0)]^2}\left(\sigma_{k,0}^2 + \theta^2(0)\sigma_a^2 + 2\theta(0)\sigma_{k,0}\sigma_a\chi\right)$$

- Consider an economy that is accumulating capital as a result of an expansionary external shock.

- In general, this can be associated with an increase or decrease in wealth inequality, depending upon the relative dispersions of the initial endowments of capital and skills and their correlation.

- Wealth inequality can emerge from differences in skill endowments alone

- Analogously, we can express income inequality in terms of its SCV

$$\sigma_y^2(t) = \varphi(t)^2 \sigma_{k,0}^2 + (1-\varphi(t))^2 \sigma_a^2 + 2\varphi(t)(1-\varphi(t))\sigma_{k,0}\sigma_a \chi$$

- Consider an economy with a Cobb-Douglas production technology. If the economy experiences an expansionary external shock that leads to an accumulation of capital and does not cause a long-run decline in employment, we obtain the following:

(i) If $\sigma_a = 0$, income inequality, as measured by its SCV, initially increases and then declines unambiguously during the transitional phase.

(ii) For $\sigma_{k,0} = 0$, income inequality, as measured by its SCV, initially declines and then increases unambiguously during the transitional phase.

- First term captures **equalizing** effect of transition: capital-rich accumulate more slowly than capital-poor, then inequality in wealth is reduced.
- The second result captures **unequalizing** effect of the transition, and reflects two forces.
  1. Accumulating capital implies that wage increases, magnifying effect of unequal abilities.
  2. Because those with high ability have higher incomes they will also save more, adding to the inequality in ability an inequality in capital. As a result inequality increases during the transition.

- Relationship between income inequality and mobility.
- We can express the change in income inequality following a shock as

$$\tilde{\sigma}_y^2 - \tilde{\sigma}_{y,0}^2 = \left( \frac{1}{1+\omega_y^a} - \tilde{\phi}_0 \right) \left\{ \left( \frac{1}{1+\omega_y^a} + \tilde{\phi}_0 \right) \left[ \sigma_{k,0}^2 + \sigma_a^2 - 2\sigma_{k,0}\sigma_a \chi \right] - 2\left[ \sigma_a^2 - 2\sigma_{k,0}\sigma_a \chi \right] \right\}$$

  - Inequality and mobility need not move together.
  - Possible for a shock to generate substantial income mobility (i.e. large value of $\omega_y^a$) and yet engender small changes in inequality, $\left[ 1/(1+\omega_y^a) - \tilde{\phi}_0 \right]$ close to zero.

- Garcia-Penalosa and Turnovsky perform numerical simulations on mobility in response to two structural changes

  (i) Increase in productivity

  (ii) Changes in tax structure

- Critical element is elasticity of labor supply

**Table 1: Increase in productivity**

Baseline: Cobb-Douglas ( $\rho = 0, \varepsilon = 1$ ) and labor supply elasticity of $\eta = 1.75$

| | Labor | $\tilde{K}$ | $\tilde{Y}$ | $\tilde{\sigma}_k^2$ | $\tilde{\sigma}_e^2$ | $\tilde{\sigma}_y^2$ | $\omega_k$ | $\omega_y^a$ |
|---|---|---|---|---|---|---|---|---|
| **Base:** $A = 1.5$ | **0.277** | **1.804** | **0.771** | **14** | **1.422** | **0.676** | **-** | **-** |
| $A = 2$ | $L(0)$ 0.280<br>$\tilde{L}$ 0.277 (0%) | 2.771 (+53.6%) | 1.184 (+53.6%) | 13.575 (-3.03%) | 1.386 (-2.57%) | 0.669 (-1.10%) | 0.016 | 7.471 |

Low elasticity of the labor supply: $\eta = 1.0$ (and Cobb-Douglas production)

| | Labor | $\tilde{K}$ | $\tilde{Y}$ | $\tilde{\sigma}_k^2$ | $\tilde{\sigma}_e^2$ | $\tilde{\sigma}_y^2$ | $\omega_k$ | $\omega_y^a$ |
|---|---|---|---|---|---|---|---|---|
| **Base:** $A = 1.5$ | **0.401** | **2.614** | **1.117** | **14** | **0.990** | **0.875** | **-** | **-** |
| $A = 2$ | $L(0)$ 0.405<br>$\tilde{L}$ 0.401 (0%) | 4.015 (+53.6%) | 1.716 (+53.6%) | 13.590 (-2.93%) | 0.970 (-2.05%) | 0.862 (-1.44%) | 0.016 | 5.157 |

High elasticity of substitution in production: $\rho = -0.13, \varepsilon = 1.15$ (and labor supply elasticity of $\eta = 1.75$ )

| | Labor | $\tilde{K}$ | $\tilde{Y}$ | $\tilde{\sigma}_k^2$ | $\tilde{\sigma}_e^2$ | $\tilde{\sigma}_y^2$ | $\omega_k$ | $\omega_y^a$ |
|---|---|---|---|---|---|---|---|---|
| **Base:** $A = 1.5$ | **0.256** | **2.472** | **0.876** | **14** | **2.354** | **0.779** | **-** | **-** |
| $A = 2$ | $L(0)$ 0.252<br>$\tilde{L}$ 0.250 (-2.18%) | 4.222 (+70.8%) | 1.433 (+63.6%) | 13.820 (-1.28%) | 2.651 (+12.6%) | 0.803 (+3.11%) | 0.007 | 5.662 |

**Table 2: Fiscal changes**

Baseline: Cobb-Douglas ( $\rho = 0, \varepsilon = 1$ ) and labor supply elasticity of $\eta = 1.75$

| | Labor | $\tilde{K}$ | $\tilde{Y}$ | $\tilde{\sigma}_k^2$ | $\tilde{\sigma}_e^2$ | $\tilde{\sigma}_y^2$ | $\omega_k$ | $\omega_y^a$ |
|---|---|---|---|---|---|---|---|---|
| **Base:** $\tau_w = \tau_k = g = 0.22$ | **0.277** | **1.804** | **0.771** | **14** | **1.422** | **0.676** | **-** | **-** |
| Expenditure/tax reduction $\tau_w = \tau_k = g = 0.17$ | $L(0)$ 0.278<br>$\tilde{L}$ 0.277 | 1.979 | 0.795 | 13.890 | 1.413 | 0.674 | 0.004 | 7.368 |
| Shift in the tax burden: Reduction in capital income tax $\tau_k = 0.17, \tau_w = 0.245, g = 0.22$ | $L(0)$ 0.271<br>$\tilde{L}$ 0.270 | 1.933 | 0.776 | 13.920 | 1.650 | 0.601 | 0.003 | 9.106 |
| Shift in the tax burden: Reduction in labor income tax $\tau_k = 0.322, \tau_w = 0.17, g = 0.22$ | $L(0)$ 0.288<br>$\tilde{L}$ 0.289 | 1.536 | 0.753 | 14.240 | 1.066 | 0.847 | -0.009 | 5.281 |

Low elasticity of the labor supply: $\eta = 1.0$ (and Cobb-Douglas production)

| | Labor | $\tilde{K}$ | $\tilde{Y}$ | $\tilde{\sigma}_k^2$ | $\tilde{\sigma}_e^2$ | $\tilde{\sigma}_y^2$ | $\omega_k$ | $\omega_y^a$ |
|---|---|---|---|---|---|---|---|---|
| **Base:** $\tau_w = \tau_k = g = 0.22$ | **0.401** | **2.614** | **1.117** | **14** | **0.991** | **0.875** | **-** | **-** |
| Expenditure/tax reduction $\tau_w = \tau_k = g = 0.17$ | $L(0)$ 0.402<br>$\tilde{L}$ 0.401 | 2.868 | 1.152 | 13.894 | 0.985 | 0.872 | 0.004 | 5.085 |
| Shift in the tax burden: Reduction in capital income tax $\tau_k = 0.17, \tau_w = 0.245, g = 0.22$ | $L(0)$ 0.394<br>$\tilde{L}$ 0.394 | 2.813 | 1.130 | 13.919 | 1.129 | 0.795 | 0.003 | 5.746 |
| Shift in the tax burden: Reduction in labor income tax $\tau_k = 0.321, \tau_w = 0.17, g = 0.22$ | $L(0)$ 0.414<br>$\tilde{L}$ 0.416 | 2.202 | 1.082 | 14.250 | 0.775 | 1.048 | -0.009 | 4.077 |

**Table 2 (continued): Fiscal changes**

High elasticity of substitution in production: $\rho = -0.13, \varepsilon = 1.15$ (and labor supply elasticity of $\eta = 1.75$)

| | Labor | $\tilde{K}$ | $\tilde{Y}$ | $\tilde{\sigma}_k^2$ | $\tilde{\sigma}_e^2$ | $\tilde{\sigma}_y^2$ | $\omega_k$ | $\omega_y^a$ |
|---|---|---|---|---|---|---|---|---|
| **Base:** $\tau_w = \tau_k = g = 0.22$ | **0.256** | **2.472** | **0.876** | **14** | **2.354** | **0.779** | **-** | **-** |
| Expenditure/tax reduction $\tau_w = \tau_k = g = 0.17$ | $L(0)$ 0.255<br>$\tilde{L}$ 0.255 | 2.771 | 0.914 | 13.926 | 2.408 | 0.783 | 0.003 | 5.863 |
| Shift in the tax burden: Reduction in capital income tax $\tau_k = 0.17, \tau_w = 0.254, g = 0.22$ | $L(0)$ 0.247<br>$\tilde{L}$ 0.246 | 2.681 | 0.884 | 13.953 | 2.929 | 0.666 | 0.002 | 7.527 |
| Shift in the tax burden: Reduction in labor income tax $\tau_k = 0.298, \tau_w = 0.17, g = 0.22$ | $L(0)$ 0.269<br>$\tilde{L}$ 0.270 | 2.136 | 0.854 | 14.170 | 1.701 | 0.964 | -0.006 | 4.494 |

# Conclusions

- Relationship between growth and inequality/distribution is complicated!
- Multidimensional
- This discussion has dealt with just one aspect and under special conditions
- Hopefully has presented how one might study the issue in a tractable way that provides some insights