Bond premia, monetary policy and exchange rate dynamics

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[DRAFT]

Abstract

I show that the exchange rate response to unconventional monetary policy, via the bond premium, is qualitatively similar to the response to conventional monetary policy, via the risk-free interest rate. In a model with risk, monetary policy smoothing provides an explanation of why exchange rates are "too smooth" relative to volatile risk-free rates implied by asset pricing, irrespective of the degree of risk-sharing. The monetary policy trilemma can be understood as a tradeoff in the expression of risk.

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1 Introduction

A common assumption is that monetary policy moves the risk-free interest rate. That assumption sits uneasily with the result that risk-free interest rates implied by equity prices are highly volatile while policy interest rates are relatively smooth(Brandt et al. 2006, Cochrane and Hansen 1992, Hansen and Jagannathan 1991). Moreover, there is evidence that short-term premiums are empirically important,[1] and are empirically linked to the stance of monetary policy (Nagel 2014, Canzoneri et al. 2007).

The presence of a short-term bond premium raises the question: Does monetary policy intervention in short-term money markets move the risk-free rate, or the bond premium or both? The traditional answer in the risk-free rate. However, Canzoneri et al. (2007) show that observed policy rates are negatively correlated with risk-free rates constructed using common specifications of preferences and consumption data, implying that a monetary policy tightening raises the short-term bond premium. Canzoneri et al. argue that a rise in the policy rate slows activity and expected consumption growth, *reduces* the risk-free rate and translates into a higher short-term premium. While unconventional, that interpretation of monetary policy intervention in short-term money markets, is consistent with recent papers that link intervention in longer-term bond markets to movements in the term premium (Bernanke 2013, and references therein). If monetary policy potentially works through the bond premium, what are the implications?

In this paper, I show that the exchange rate response to unconventional monetary policy, via the short-term bond premium, is qualitatively similar to the response to conventional monetary policy, via the risk-free interest rate. In a model with risk, monetary policy smoothing provides an interpretation of why exchange rates are "too smooth" relative to volatile risk-free rates implied by asset pricing; and the monetary policy trilemma can be understood as a tradeoff in the expression of risk. The paper builds on the application of risk corrections to exchange rate models employed by Munro (2014), Lustig and Verdelhan (2007) and Backus et al. (2001). Munro (2014) derives an exchange rate model in which uncertainty about the value of ex-post bond payoffs generates a bond premium, that drives a wedge between the risk-free rate and the observed interest rate. Here, I augment that model with a role for monetary policy.

When monetary policy tightening raises the home risk-free interest rate, the exchange rate response follows Dornbusch (1976): the home currency initially appreciates to eliminate all future excess returns on home bonds, and then depreciates to offset the higher home interest return, period by period. When monetary tightening raises the observed interest rate, while the risk-free rate and the risk characteristics of the home bond are unchanged,

[1] Although government default risk is usually low relative to other rates, Della Corte et al. (2015) find relative default risk to be significant in explaining exchange rate behaviour. Krishnamurthy and Vissing-Jorgensen (2012), Amihud et al. (2005) and Duffie (1996) show that short-term safety premia reflected in US Treasuries interest rates are empirically important. (Lustig and Verdelhan 2007) show that high interest rate currencies tend to perform poorly in bad times, and argue that investors demand higher yields on bonds denominated in high interest currencies because holding them makes consumption more volatile.

the exchange rate response follows a similar pattern. The currency initially appreciates to eliminate all future *risk-adjusted* excess returns on home bonds, and subsequently depreciates to offset the higher *risk-adjusted* return, period by period.

The adjustment can also be understood in terms of risk. Policy intervention creates an asymmetry in the pricing of risk in the local money market market, relative to the pricing of risk in the currency market. The foreign currency 'excess return', that reflects the pricing of home and foreign bond risk in the currency market, is unchanged. The foreign 'currency premium', defined as the foreign currency 'excess return' net of the relative foreign bond market premium, reflects asymmetries in the pricing of risk in the the bond markets relative to currency market.[2] A policy-induced rise in the foreign bond market premium depresses the foreign currency premium. For the global investor's no-arbitrage condition to hold, the foreign currency must appreciate until foreign bond risk or a higher foreign risk-free match the higher yield. Canzoneri et al. 2007 argue that the risk-free rate falls, which would imply that foreign bond risk rises. An adjustment mechanism that involve the building and shedding of risk, has implications for the nature of international spillovers.

This framework also helps to explain why exchange rates are 'too smooth' relative to risk-free rates (Brandt et al. 2006). Risk-free rates implied by equities are very volatile (Hansen and Jagannathan 1991). Brandt et al. (2006) show that, if exchange rates reflect relative risk-free rates, then either exchange rates should be considerably more volatile than they are, or home and foreign risk-free rates must be correlated, implying a higher degree of risk-sharing than is typically estimated.[3]

When policy intervention stabilises the observed short-term interest rate, but the underlying risk-free rate is volatile, then the wedge between the two - the bond premium - must be volatile and negatively correlated with the risk-free rate. Taking the volatile risk-free rate as given, volatility in the bond premium reflects policy intervention to stabilise the observed rate, rather than a change in the risk characteristics of the bond. The currency 'excess return' reflects the unchanged risk characteristics of the bonds. That leaves the foreign currency premium (the unchanged foreign currency 'excess return' net of a volatile foreign bond premium) volatile and positively correlated with the foreign risk-free rate. Volatility in the foreign risk-free rate is offset by volatility in the foreign currency premium, leaving the exchange rate stable.

While monetary policy stabilisation of the observed rate isolates the exchange rate from variation in relative risk-free rates, it shifts variation in the underlying riskiness of the short-term bonds to the exchange rate. Intuitively, when policy intervention prevents bond risk from being priced in the local money market premium, it is reflected in the currency pre-

[2] In Lustig and Verdelhan (2007) and Backus et al. (2001) the currency premium reflects asymmetries in the pricing of currency revaluation risk.

[3] Complete risk-sharing is rejected empirically (Backus and Smith 1993). Kose et al. (2003) show that cross-country consumption correlations did not increase in the 1990s, despite financial integration. More recently, employing a different empirical approach, Flood and Matsumoto (2009) find that consumption growth rates have converged, suggesting that international risk sharing has improved during a period of globalization.

mium. That tradeoff between interest rate volatility and exchange rate volatility is in keeping with the monetary policy trilemma. In a financially open economy, policy can stabilise the interest rate (independent monetary policy) or the exchange rate rate, but not both. In this framework, the trilemma can be viewed as a tradeoff between bond risk being reflected in bond yields or in the the currency market.

This interpretation of the exchange rate volatility puzzle is similar to that of Chien et al. (2015) in that it involves a deviation from the complete complete markets solution. In that paper, non-participation in financial markets drives down the cross-country correlation in aggregate consumption, implying a low degree of risk-sharing. In contrast, Brandt et al. (2006) interpret the puzzle as evidence for a relatively high degree of risk sharing. Here the deviation from complete markets is associated with monetary policy intervention in short-term money markets, and is independent of assumptions about the degree of risk sharing.

The next section sets out a partial equilibrium model that allows for uncertainty about the value of short-term bond payoffs and monetary policy stabilisation of observed interest rates. Section 3 examines the exchange rate response to conventional and unconventional views of monetary policy. Section 4 considers the case when monetary policy stabilises the observed interest rate but the risk-free rate is volatile. Section 6 concludes.

2 An exchange rate asset price model with risk

This section sets out the 2-equation exchange rate asset price model derived in Munro (2014) that incorporates short-term bond premia, and augments it with monetary policy control of the observed interest rate. The model encompasses the standard exchange rate asset price model (Engel and West 2010, Engel and West 2005, Dornbusch 1976), but allows for a broader set of interest rate-exchange rate dynamics.

Exchange rate dynamics are driven by uncovered interest parity (UIP). UIP equates the expected returns on home and foreign short-term bonds:

$$q_t = -r_t^d - \lambda_t + E_t q_{t+1} \tag{1}$$

where q_t is the logarithm of the real exchange rate, defined as the value of the foreign currency in units of home currency, r_t^d is the home-foreign real short-term interest differential and E_t indicates expectations at time, t. The expected foreign currency 'excess return' λ_t is defined as $\lambda_t = E_t \Delta q_{t+1} - r_t^d$. Empirically, currencies with high interest rates (relative to other currencies and relative to average) tend to have expected excess returns on their short-term bonds (Engel 2016, Fama 1984). On average, they do not depreciate by enough to offset the higher interest return, and often appreciate. Here the 'excess return' is interpreted in terms of premia generated by uncertainty about the value of bond returns. Equation (1) can be read: the expected depreciation of the currency, $E_t \Delta q_{t+1}$, is equal to relative *risk-adjusted* returns ($r_t^d + \lambda_t$).

The second equation expresses the observed interest rate differential as relative risk-free returns, net of a foreign bond premium:

$$r_t^d \;=\; (r_t^f - r_t^{f*}) - \lambda_t^R \tag{2}$$

where r_t^f and r_t^{f*} are the home and foreign risk-free interest rates respectively, and λ_t^R is the foreign (relative to home) short-term bond market premium. The home and foreign risk-free rates are defined as the home and foreign investors' rates of time preference, respectively. The foreign bond premium is defined as the observed foreign interest rate net of the foreign risk-free rate, and reflects the pricing of risk in the bond markets. In the absence of policy intervention, the bond premium compensates for relative risk characteristics of the home and foreign bond, such as relative default risk (Della Corte et al. 2015), relative term premia, relative liquidity risk (Nagel 2014, Duffie 1996, Amihud and Mendelson 1991) and currency revaluation risk (Lustig and Verdelhan 2007). Policy intervention in short-term bond markets may also affect the pricing of risk in the bond market (Nagel 2014, Canzoneri et al. 2007).

When foreign bond risk is priced symmetrically in the foreign bond market and in the currency market (for example, when markets are complete), the relative bond market premium, λ_t^R and the currency excess return, λ_t, are equal. In this paper, I consider the effects of monetary policy intervention in the foreign bond market. A monetary tightening that raises the observed interest rate, can raise the risk-free rate (the traditional assumption) or the foreign bond premium (Canzoneri et al. 2007). The rest of this section provides a brief derivation of equations (1) and (2). Detailed derivations are included as Appendix A. Sections 3 to 5 discuss interpretations and implications of policy intervention.

2.1 One-period risk-free bonds

The risk-free rate, r_t^f, is defined by the home investor's rate of time preference: his willingness to give up a unit of consumption today to consume $(1+r_t^f)$ units of consumption next period. The 'stochastic discount factor' (SDF), M_{t+1}, of the home investor is:

$$M_{t+1} = E_t \beta U_{C,t+1}/U_{C,t} = \tfrac{1}{1+r_t^f} \tag{3}$$

where β is the home investor's subjective discount factor, $U'_{C,t}$ is the marginal utility of consumption, and E_t indicates expectations at time t. The results do not depend on a particular specification of the SDF, so there is no reason to specify a utility function, but simply to postulate that it exists. The risk-free rate is lower when people save more because they are patient (β), they are averse to varying consumption across time (inter-temporal substitution), they are averse to varying consumption across states (risk aversion), or they expect consumption growth to be volatile (precautionary savings).

The SDF and gross asset returns are assumed to be conditionally log-normal. Taking the logarithm of equation (3),

$$log M_{t+1} \;=\; -r_t^f$$

Similarly, the foreign real, risk-free interest rate, r_t^{f*}, is defined by the foreign investor's rate of time preference:

$$logM_{t+1}^* = -r_t^{f*} \tag{4}$$

where M_{t+1}^* is the foreign investor's SDF, β^* is her subjective discount factor.

2.2 The bond premium

The home investor's pricing (Euler) equation for the home bond is:

$$1 \;=\; E_t[M_{t+1}(1+r_t^c)Z_{t+1}] \tag{5}$$

where r_t^c is the observed interest rate, contracted at time t, and Z_{t+1} reflects factors that affect the value of the payoff at $t+1$. Those may include expected losses from default or from selling the bond before maturity, uncertainty regarding the evolution of the risk-free rate over the holding period and covariances of those factors with consumption utility. The log pricing equation is:

$$logE_t(1+r_t^c) \sim r_t^c \;=\; r_t^f - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1}), \tag{6}$$

where $m_t = log(M_t)$, and $z_t = log(Z_t)$. The second term on the right hand side captures expected losses and the final term on the right hand side of (6) is a risk correction. The risk correction increases the yield on assets with payoffs that are positively correlated with consumption growth (negatively correlated with consumption utility growth). Holding such assets assets makes consumption more volatile (Cochrane 2001). Appendix A presents the examples where default risk, a term premium and liquidity risk drive a wedge between the contracted rate and the risk-free rate.

Similarly, the contracted return in the foreign bond market can be expressed as the foreign investor's risk-free rate, r_t^{f*}, plus expected losses and a risk correction that is priced according to her SDF, M_t^*:

$$logE_t(1+r_t^{c*}) \sim r_t^{c*} \;=\; r_t^{f*} - logE_t(Z_{t+1}^*) - cov_t(m_{t+1}^*, z_{t+1}^*) \tag{7}$$

where $m_t^* = log(M_t^*)$. Combining (6) and (7), and assuming that contracted rates reflect the expected value of payoffs, the observed short-term home-foreign interest differential can be expressed as the relative risk-free return plus a relative bond premium, λ_t^R:

$$\begin{aligned} r_t^d &= r_t^c - r_t^{c*} \\ &= (r_t^f - r_t^{f*}) - \underbrace{(logE_t(Z_{t+1}) - logE_t(Z_{t+1}^*) + cov_t(m_{t+1}, z_{t+1}) - cov_t(m_{t+1}^*, z_{t+1}^*))}_{\text{bond premium, } \lambda_t^R} \end{aligned} \tag{2'}$$

The bond premium is the wedge between the observed, contracted rate on the bond and the unobserved risk-free rate. It reflects the pricing of bonds in the home and foreign bond markets.

2.3 Uncovered interest parity and currency revaluation risk

The currency is determined by the home investor. The home investor's pricing equation for the foreign short-term bond is:

$$Q_t \;=\; E_t[M_{t+1}(1+r_t^{c*})Z^*_{t+1}Q_{t+1}], \tag{8}$$

where Q_t is the real exchange rate (value of the foreign currency in units of home currency). Equation (8) equates the cost of buying one unit of the foreign bond this period, Q_t, to the expected, discounted return on the foreign bond at $t+1$, in home currency terms. The log pricing equation is:

$$r_t^{c*} = r_t^f - logE_t(Z^*_{t+1}) - E_t\Delta q_{t+1} - cov_t(m_{t+1}, z^*_{t+1}) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var(\Delta q_{t+1}), \tag{9}$$

where $q_t = log(Q_t)$. From the perspective of the home investor, the foreign bond premium reflects expected losses, including expected depreciation of the foreign currency, $E_t\Delta q_{t+1} + \frac{1}{2}var(\Delta q_{t+1})$, and risk corrections that reduce the yield on bonds that are expected to perform well when the marginal utility of consumption rises (eg safe-haven currencies). Holding such bonds makes consumption less volatile (Lustig and Verdelhan 2007).

Combining the home investor's pricing equation for the home short-term bond (6) with the home investor's pricing equation for foreign bonds (9), gives the UIP condition that equates the expected return on the home bond to the expected return on the foreign bond:

$$\begin{aligned} q_t \;=\; & -r_t^d \\ & -\underbrace{(logE_tZ_{t+1} - logE_tZ^*_{t+1} + (cov_t(m_{t+1}, (z_{t+1}-z^*_{t+1})) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var(\Delta q_{t+1}))}_{\text{`Excess return', } \lambda_t} \\ & +E_t(q_{t+1}) \end{aligned} \tag{1'}$$

The foreign currency 'excess return',λ_t, reflects the home investor's pricing of home and foreign bond risk. The home investor is assumed to dominate the currency market, eg. the foreign country is small and has a floating exchange rate.

Equation (1') encompasses the standard asset price model of the exchange rate. If the home risk-adjusted interest rate rises, or is expected to rise, relative to the foreign risk-adjusted rate, the home currency should immediately appreciate à la Dornbusch (1976) and then depreciate to its equilibrium value, over the period of higher home returns. The initial appreciation eliminates all future excess risk-adjusted returns, and the subsequent depreciation offsets the higher interest risk-adjusted payoffs each period, so there is no excess return to holding the home or the foreign asset.

2.4 The currency premium

It is useful to divide the currency 'excess return', λ_t, into two parts: the bond premium, λ_t^R, and a currency premium, λ_t^{FX}. Accordingly, the currency premium is the currency 'excess return' (1'), net of the bond premium (2'):

$$\begin{aligned}
\lambda_t^{FX} &= \lambda_t - \lambda_t^R \\
&= cov_t(m_{t+1}^*, z_{t+1}^*) - cov_t(m_{t+1}, z_{t+1}^*) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2} var(\Delta q_{t+1}) \\
&= cov_t\big((m_{t+1}^* - m_{t+1}), z_{t+1}^*\big) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2} var(\Delta q_{t+1}) \qquad (10)
\end{aligned}$$

Defined this way, the currency risk premium reflects the difference between the home and foreign investors' pricing of foreign bond risk. As in Lustig and Verdelhan (2007), Backus et al. (2001) and Chien et al. (2015), the currency premium reflects asymmetric pricing of risk. The last two terms on the right-hand side of (10) reflect asymmetric pricing of currency revaluation risk (Lustig and Verdelhan 2007, Obstfeld and Rogoff 1996). The first term on the right-hand side reflects asymmetric pricing of other foreign bond risks (Munro 2014).

Using this definition of the currency premium, the exchange rate (1) can be expressed as its expected future value, the observed interest differential and an excess return that includes the relative bond premium and a currency premium:

$$q_t = -\underbrace{((r_t^f - r_t^{f*}) - \lambda_t^R)}_{\text{Observed interest differential}} - \underbrace{(\lambda_t^R + \lambda_t^{FX})}_{\text{`Excess return'}\lambda_t} + E_t q_{t+1}$$

or as its expected future value, the risk-free interest differential and the currency premium:

$$q_t = -\underbrace{(r_t^f - r_t^{f*})}_{\text{Risk-adjusted differential, } r_t^d} - \underbrace{\lambda_t^{FX}}_{\text{Currency premium, } (\lambda_t - \lambda_t^R)} + E_t q_{t+1} \qquad (11)$$

Foreign bond risk can be reflected in the bond premium (priced symmetrically) or reflected in the currency premium (priced asymmetrically), but not in both. When markets are complete, the risk characteristics of the foreign bond are priced symmetrically, and are fully reflected in the relative bond premium Munro (2014). In that case, the currency 'excess return' is equal to the bond premium $\lambda_t = \lambda_t^R$, and the currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R = 0$ is zero.

Asymmetries in the pricing of risk may reflect differences in the pricing kernels ($m_{t+1} \neq m_{t+1}^*$) of the representative investors (Chien et al. 2015), or may reflect monetary policy intervention in the short-term bond markets. The case of interest in this paper is the latter.

3 The exchange rate response to monetary policy

This section considers two variants of the model that reflect two views of monetary policy. The first model makes the traditional assumption that a monetary tightening raises the risk-free interest rate. The second model turns that assumption on its head, and considers the case when monetary policy affects the bond premium component of the short-term interest rate. Perhaps surprisingly, the exchange rate responses to monetary policy policy are qualitatively similar.

The exchange responses to monetary policy and risk are illustrated using impulse response functions. In this partial equilibrium setting, the driving variables are assumed to be independent and to follow AR(1) processes with AR(1) coefficients of 0.9 and unit variance shocks. In both cases, the home investor rationally prices risk in the home bond market and in the currency market. That is, we assume a floating exchange rate regime and no policy intervention in the currency market. In the first model, the other driving factors are foreign monetary policy (the foreign risk-free rate) and the foreign bond risk. In the second model, the other driving factors are the, potentially volatile, foreign risk-free rate and monetary policy (the observed foreign short-term interest rate). It will be useful to think of the exchange rate response in terms of the change in relative risk-free returns and the change in the currency premium, as in equation (11).

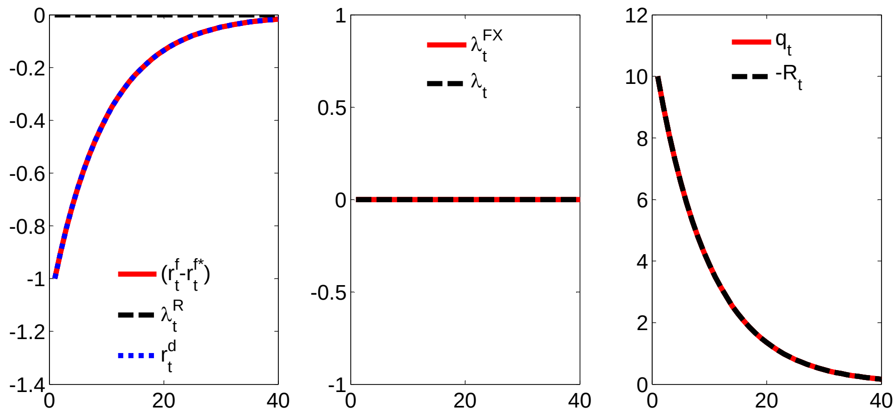
3.1 The traditional view

The first model reflects the traditional view of monetary policy that assumes risk-free interest rates to be set by the monetary authority. As in most New Keynesian DSGE models, that assumption equates the inter-temporal rate of substitution and the observed central bank rate (or other short-term reference rate). I consider the case when risk is priced symmetrically in the foreign bond market and in the currency market. For example, the same global investors participate in both markets. We will examine the responses to two driving factors: a foreign monetary tightening that raises r_t^{f*}, and an increase in foreign bond risk that raises both the foreign bond market premium λ_t^R and the foreign currency excess return λ_t.

The traditional exchange rate response to a rise in the foreign risk-free rate is illustrated in Figure 1. When the foreign risk-free interest rate, r_t^{f*}, rises, the risk-free home-foreign interest differential $r_t^f - r_t^{f*}$ and the observed interest differential r_t^d fall (left-hand panel). The foreign currency immediately appreciates (q_t rises, red line in right hand panel) to reflect the higher expected future path of the foreign interest rate, $R_t = E_t \sum_{k=0}^{\infty} r_{t+k}^d \sim \frac{1}{1-0.9}\Delta r_t^d$ (right hand panel). The initial appreciation eliminates all future excess returns relative to the long-run equilibrium value of the currency. The foreign currency subsequently depreciates to offset the higher foreign return, period by period. The magnitude of the initial appreciation reflects both the magnitude of the rise in r_t^{f*} and its expected persistence.

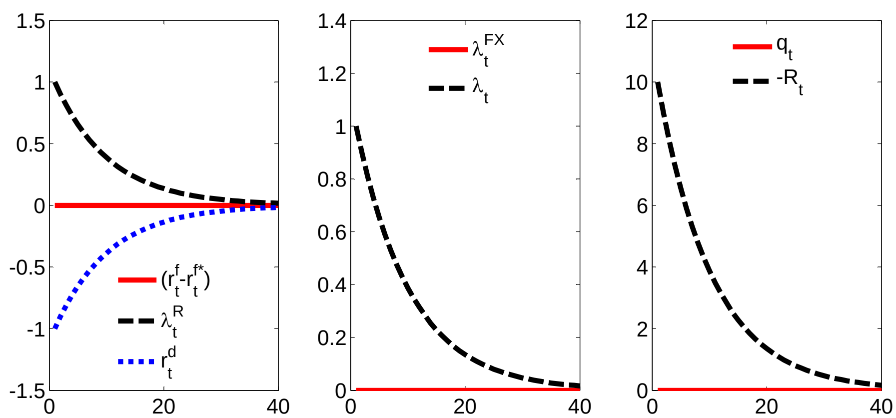
In contrast, in this version of the model, there is no exchange rate response to an exogenous rise in foreign bond risk. A rise in foreign bond risk is reflected in both the foreign bond market (λ_t^R rises, left hand panel of Figure 2) and in the currency market (λ_t rises, centre panel). The higher foreign bond premium increases the observed foreign interest rate, so reduces the observed home-foreign interest rate differential, r_t^d (left hand panel), and increases the expected path of relative returns $-R_t = E_t \sum_{k=0}^{\infty} \lambda_{t+k}^R$ (right hand panel). The currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ is unchanged because risk is priced symmetrically. Since the bond premium compensates the holder for risk, the foreign currency does not depreciate to offset the higher foreign bond premium, nor does it initially appreciate. The foreign currency exhibits 'excess returns' equal to the higher foreign bond payoff and the exchange rate is disconnected from observed relative interest returns.

Figure 1: Response to a foreign monetary tightening (via the risk-free rate)



Notes: The vertical axis is in percent per period. The horizonal axis is in units of time. Variables: $r_t^f - r_t^{f*}$ is the home-foreign risk-free interest rate differential; λ_t^R is the foreign bond premium relative to the home premium; $r_t^d = r_t^f - r_t^{f*} - \lambda_t^R$ is the home-foreign observed interest rate differential; λ_t is the foreign currency 'excess return'; $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ is the currency premium; $R_t = E_t \sum_{k=0}^{\infty} r_{t+k}^d$ is the expected relative interest rate path; q_t is the real exchange rate (the value of the foreign currency in units of home currency).

Figure 2: Response to a rise in foreign bond risk (priced symmetrically)



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

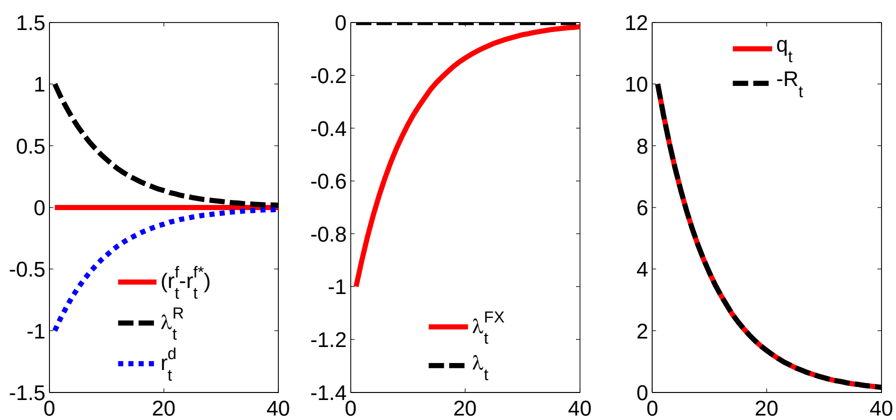
3.2 An unconventional view of monetary policy

The second model, while unconventional, is consistent with several empirical regularities, including: (i) the implication of the equity premium literature that, unless risk aversion is implausibly high, risk-free rates are very volatile (Hansen and Jagannathan 1991); (ii) positive correlation between short-term bond premia and the stance of monetary policy Canzoneri et al. (2007) and Nagel (2014); and (iii) the finding that purchases of long-term bonds compress risk premia rather than lower expectations of future short-term interest rates

(Bernanke (2013), and references therein). The risk-free rate is determined by expectations about future consumption, or other factors affecting utility, that are exogenous in this partial equilibrium setting. The observed interest rate is determined by monetary policy.

The response to a foreign monetary tightening is shown in Figure 3. Monetary intervention in the foreign short-term bond market raises the observed foreign interest rate, reducing the home-foreign observed interest differential, r_t^d (left-hand panel). With the foreign risk-free rate initially unchanged,[4] the foreign bond market premium, λ_t^R, rises (left hand panel of Figure 3). The rise in the foreign bond market premium reflects policy intervention in short term bond markets, rather than a change in the riskiness of the foreign bond. As the price of risk in the currency market, λ_t, unchanged (centre panel), policy intervention results in asymmetry in the pricing of risk and gives rise to a currency premium ($\lambda_t^{FX} = \lambda_t - \lambda_t^R$). For no-arbitrage conditions to hold in the currency market, the foreign currency premium falls (red line in centre panel), reflecting the asymmetric pricing of risk in the bond market relative to the currency market. The fall in the foreign currency premium appreciates the foreign currency, q_t rises (red line in the right hand panel). In contrast to the response to symmetrically-priced bond risk in Figure 2, where the high interest rate currency was weak relative to the expected interest rate path $-R_t$, here the high interest rate currency follows the expected interest rate path.

Figure 3: Response to a foreign monetary tightening (via the bond premium)



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

The response to the conventional and unconventional views of monetary policy are qualitatively similar. In both cases, the response can be thought of as a price response to a higher

[4] Canzoneri et al. (2007) argue that, the foreign risk-free rate falls, implying a rise in the foreign bond premium of more than the rise in the observed rate. In a very flexible model, after an immediate fall in consumption, expected consumption growth and the risk-free rate may rise. The exchange rate response is qualitatively the same in those cases. The higher foreign interest rate is reflected either in a higher foreign risk-free rate or in a lower currency premium.

risk-adjusted foreign return. However, there may also be important differences in a model with risk. When the home investor is offered a higher foreign return, but his assessment of foreign economic fundamentals, reflected in r_t^{f*}, and foreign relative bond risk, λ_t, are unchanged, then adjustment is needed to equalise expected returns on the home and foreign bonds. The foreign currency must appreciate until either a higher foreign risk-free rate (see footnote 4) or greater foreign bond risk offsets the higher return. The adjustment mechanism may involve a build-up or shedding of risk, and the degree of appreciation is not obvious. For example, what degree of appreciation is required to generate a given measure of currency revaluation risk? Do capital inflows generate a build up of other types of foreign bond risk? The answers have implications for the nature of international spillovers.

4 Exchange rates are "too smooth": a monetary policy explanation

Brandt et al. (2006) show that, although floating exchange rates are volatile, risk-free rates implied by asset prices are much more volatile (Hansen and Jagannathan 1991). If exchange rates reflect relative risk-free interest rates, then either home and foreign risk-free rates are correlated, implying a higher degree of risk sharing than standard estimates (see footnote 3), or exchange rates are "too smooth".

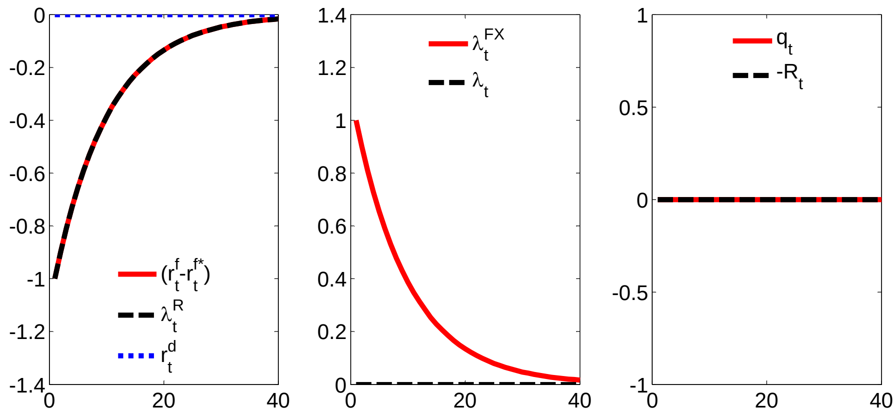
The combination of a volatile underlying risk-free rate and monetary policy stabilisation of the observed interest rate provides an additional explanation. Variation in the foreign risk-free rate does not change the observed policy rate, which is stabilised by policy intervention in the short-term bond market. Therefore the observed interest rate differential, r_t^d, is unchanged. With the observed rate held steady, a rise in the foreign risk-free rate compresses the foreign bond premium, λ_t^R (Figure 4, left panel). A volatile foreign risk-free rate is associated with a volatile foreign bond market premium and the two are negatively correlated (the home-foreign risk-free differential and the foreign bond premium are positively correlated, Figure 4, left panel).

The compressed foreign bond premium reflects monetary policy intervention to stabilise the observed rate, rather than a change in the riskiness of the foreign bond, therefore the home investor's pricing of foreign bond risk, λ_t, is unchanged (centre panel). For no-arbitrage to hold in the currency market, the foreign currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ must rise (centre panel of Figure 4).

The opposing effects of the higher foreign risk-free rate and the higher foreign currency premium leave the exchange rate smooth relative to the path implied the risk-free rate. That response is in contrast to the traditional Dornbusch (1976) response to a rise in the risk-free rate illustrated in Figure 1: a flat exchange rate path compared to an immediate foreign currency appreciation followed by a gradual depreciation.

That result is also independent of the degree of risk-sharing. In contrast, Brandt et al. (2006) et al interpret the puzzle as evidence for a high degree of risk sharing so that home and foreign risk-free rates are correlated; and Chien et al. (2015) provide a segmented markets explanation that implies a low degree of risk sharing.

Figure 4: Response to a rise in the foreign risk-free rate, with a stable policy rate



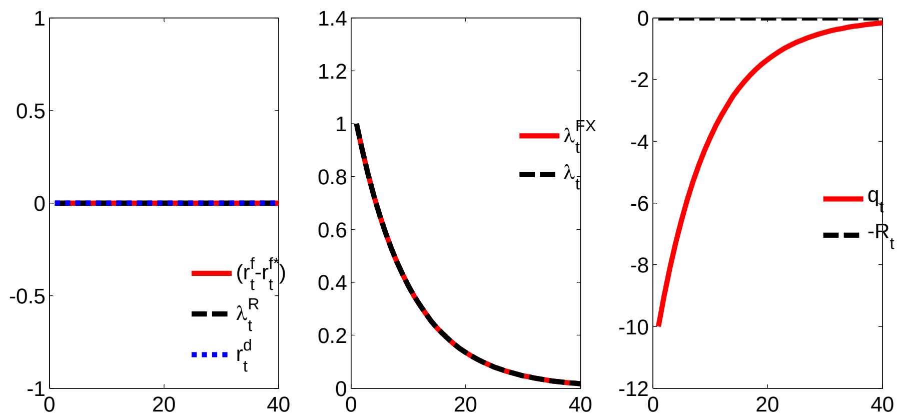
The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 2 for variable definitions.

5 The trilemma and risk

While monetary policy stabilisation of the observed rate isolates the exchange rate from variation in relative risk-free rates, it shifts variation in the underlying riskiness of the short-term bond to the exchange rate. Figure 5 shows the response to a rise in foreign bond risk, while the policy rate is held steady. Taking the underlying risk-free rate as initially unchanged, with a steady policy rate the foreign bond market premium ,λ_t, is initially unchanged (left hand panel). The rise in risk is reflected in the currency 'excess return' , λ_t, but not in the bond market. Therefore the currency premium, $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ rises, depreciating the foreign currency (right hand panel). Intuitively, when policy intervention prevents foreign bond risk from being reflected in the foreign bond market premium, it appears in the currency premium. Currency movements are driven by the relative risk characteristics of home and foreign bonds (λ_t, figure 5 by the relative stance of monetary policy (Figure 3), and by fundamentals such as the terms of trade and relative productivity Benigno and Thoenissen (2008) that affect the purchasing power parity equilibrium exchange rate.

That tradeoff between interest rate volatility and exchange rate volatility is in keeping with the monetary policy trilemma. In a financially open economy, policy can stabilise the interest rate (independent monetary policy) or the exchange rate rate, but not both. It is useful to think of the trilemma through in terms of uncovered interest rate parity: taking the foreign interest rate as given, policy can stabilise the home interest rate, or the exchange rate, but not both. Arbitrage in large fixed-income and foreign exchange markets (UIP) determines the other. In the framework employed here, the tradeoff between exchange rate volatility and interest rate volatility is a tradeoff between relative bond risk being reflected in bond yields or in the currency.

Figure 5: Response to a rise in foreign bond risk, with stable policy rate



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 2 for variable definitions.

6 Conclusion

Can we equate the monetary policy rate and the intertemporal rate of substitution? In an inflation targeting regime, the policy rate is a function of expected inflation and other Taylor-type rule variables. In contrast, the intertemporal rate of substitution is a function of expected consumption growth and other utility function variables. The equity premium literature implies that, under plausible levels of risk aversion, the risk free rate is highly volatile. In contrast, the observed policy interest rate reflects a high degree of smoothing. This paper builds on a growing literature that challenges the assumption that observed short-term interest rates are risk-free (for example, Nagel 2014, Canzoneri et al. 2007, Duffie 1996 and Amihud and Mendelson 1991).

Allowing for a bond premium wedge between the observed policy rate and the 'risk-free' intertemporal rate of substitution, I show that the exchange rate response to unconventional monetary policy, via the bond premium, is qualitatively similar to the response to conventional monetary policy, via the risk-free interest rate. In a model with risk, monetary policy smoothing provides an explanation of why exchange rates are "too smooth" relative to volatile risk-free rates implied by asset pricing, irrespective of the degree of risk-sharing. Finally, the paper links the monetary policy trilemma to risk. The tradeoff between exchange rate stabilisation and interest rate can be viewed as a tradeoff between the expression of bond risk being in bond yields or in the exchange rate.

For tractability, the model is presented in partial equilibrium. What are the implications of incorporating endogenous risk premia in a general equilibrium open economy model? The interest rate and exchange rate responses to monetary policy are qualitatively the same. However, when the rate of intertemporal substitution is determined by consumption data and thee specification of preferences, rather than equated to observed short-term interest rates, how agents discount the future may be very different. That has implication for variables, such as investment, that depend strongly on the discounting of future returns.

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A Derivations

This appendix is from the latest version of Munro (2014). It is repeated here for convenience, and because the model in the current version of that paper differs slightly from the cited version. Munro (2014) derives the model and discusses the forward premium puzzle.

The risk-free rate The home investor's real stochastic discount factor (SDF), M_{t+1}, between period t and $t+1$ is defined as:

$$M_{t+1} = \beta E_t \frac{U'_{C,t+1}}{U'_{C,t}}$$

where β is the subjective discount factor, and $U'_{C,t}$ is the marginal utility of consumption. Appealing to no-arbitrage and the Fundamental Theorem of Asset Pricing, there is no need to specify the form of the SDF, but simply to postulate that it exists. The home gross risk-free interest rate $1+r_t^f$ is defined by:

$$\frac{1}{1+r_t^f} = M_{t+1} \tag{A.1}$$

Following Lustig and Verdelhan (2007), I assume the stochastic discount factor and asset returns to be conditionally log-normal.[5] Define $x_{t+1} = log(X_{t+1})$. If x_{t+1} is normally distributed, then $X_{t+1} = e^{(x_{t+1})}$ is log-normally distributed and $E_t(X_{t+1}) = e^{(E_t(x_{t+1})+\frac{1}{2}\sigma^2_{x,t})}$.

Taking logs, (A.1) becomes:

$$-r_t^f = logM_{t+1} = E_t m_{t+1} + \tfrac{1}{2} var_t(m_{t+1}) \tag{A.2}$$

Risk corrections The home investor's pricing equation (Euler equation) for the home short-term bond is:

$$1 = E_t\big(\, M_{t+1}(1+r_t^c)Z_{t+1}\,\big)$$

where r_t^c is the observed, contracted rate on the bond, and Z_t, captures uncertainty about the ex-post payoff at time $t+1$. Taking logs,

$$\begin{aligned} 0 &= logE_t\big(M_{t+1}(1+r_t^c)Z_{t+1}\big) \\ &= log\big(e^{(E_t m_{t+1}+E_t r_t^c+E_t z_{t+1}+\frac{1}{2}var(m_{t+1}+r_t^c+z_{t+1}))}\big) \\ &= E_t m_{t+1} + \tfrac{1}{2}var(m_{t+1}) + E_t r_t^c + \tfrac{1}{2}var(r_t^c) + E_t z_{t+1} + \tfrac{1}{2}var(z_{t+1}) \\ &\quad +cov_t(m_{t+1}, r_t^c) + cov_t(m_{t+1}, z_{t+1}) \end{aligned} \tag{A.3}$$

Since the contracted rate, r_t^c is known with certainty ex-ante, the term $cov_t(m_{t+1}, r_t^c)$ is zero, and $logE_t(1+r_t^c) \sim r_t^c$. Only systematic risk (covariances with m_{t+k}) is priced, since idiosyncratic risk can be diversified away.

[5] Alternatively, one can use the definition of covariance, $cov(M, X) = E(MX) - E(M)E(X)$, and take a log approximation.

Combining (A.2) and (A.3) the home investor's pricing equation for the home short-term bond is:

$$r_t^c \;=\; r_t^f - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1}) \tag{A.4}$$

Allowing for incomplete markets ($m_t \neq m_t^*$), the equivalent log pricing equation for the foreign bond, from the foreign investor's perspective is:

$$r_t^{c*} = r_t^{f*} - logE_t(Z_{t+1}^*) - cov_t(m_{t+1}^*, z_{t+1}^*) \tag{A.5}$$

where r_t^{c*} is the coupon rate on the foreign bond, paid at time $t+1$ in foreign currency, Z_{t+1}^* captures uncertainty about payoffs on the foreign bond, and $r_t^{f*} = -log(E_t M_{t+1}^*)$ is the foreign risk-free rate. Subtracting (A.5) from (A.4) gives equation 2), with the relative bond market premium defined as in **??**.

Examples in which ex-post payoffs are not known with certainty follow in the next three sections.

Default premium Consider a 1-period bond that is contracted at a gross rate $(1+r_t^c)$, payable at $t+1$, but that defaults with a non-zero probability. The pricing equation is

$$1 = E_t[M_{t+1}(1+r_t^c)(1-d_{t+1})]$$

where $d_t \in [0,1]$ captures both the probability of default and loss in the event of default. The log pricing equation is:

$$\begin{aligned}
0 &= log[E_t\big(M_{t+1}(1+r_t^c)(1-d_{t+1})\big)] \\
&= log\big(e^{(E_t m_{t+1} + E_t r_t^c - E_t d_{t+1} + \frac{1}{2}var(m_{t+1} + r_t^c - d_{t+1}))}\big) \\
&= -r_t^f + r_t^c + logE_t(1-d_{t+1}) - cov_t(m_{t+1}, d_{t+1})) \\
r_t^c &= r_t^f - logE_t(1-d_{t+1}) + cov_t(m_{t+1}, d_{t+1})
\end{aligned}$$

The contracted rate is known ex-ante, so $logE_t(1+r_t^c) \sim r_t^c$ and $cov_t(r_t^c, m_{t+k}) = 0$. Defining the default premium as the difference between the contracted rate and the risk-free rate,

$$\begin{aligned}
\text{default premium} &\equiv r_t^c - r_t^f \\
&= -log(E_t(1-d_{t+1})) + cov_t(m_{t+1}, d_{t+1}),
\end{aligned}$$

The default premium reflects the expected loss and a risk correction that increases the contracted yield if losses from default are expected to be higher when the marginal utility of consumption is expected to rise. The default premium is part of the bond premium, λ_t^R (equation 2).

Term premium While the assumption that the term premium is small is relevant for overnight securities, for the monthly or quarterly returns that are typically used in empirical exchange rate analysis, term premia may be material. Consider a two-period fixed-rate bond that pays a certain $(1+r_{2,t}^c)^2$ at $t+2$. The pricing equation, $1 = E_t[M_{t+1}M_{t+2}(1+r_{2,t}^c)^2]$, equates the cost of buying the bond today with the expected value of the payoff at $t+2$. The log of the pricing equation is:

$$
\begin{aligned}
0 &= logE_t[M_{t+1}M_{t+2}(1+r^c_{2,t})^2] \\
&= log\left(e^{(E_t m_{t+1}+E_t m_{t+2}+2E_t r^c_{2,t}+\frac{1}{2}var(m_{t+1}+m_{t+2}+2r^c_{2,t}))}\right) \\
&= -E_t r^f_t - E_t r^f_{t+1} + 2E_t r^c_{2,t} + cov_t(m_{t+1}, m_{t+2}) \\
2r^c_{2,t} &= E_t r^f_t + E_t r^f_{t+1} - cov_t(m_{t+1}, m_{t+2}) \qquad \text{(A.6)}
\end{aligned}
$$

Since the contracted rate $r^c_{2,t}$ is known with certainty at time t, $cov_t(r^c_{2,t}, m_{t+k}) = 0$. However, the state of the economy, and so the marginal utility of consumption in subsequent periods is not known with certainty, so $cov_t(m_{t+1}, m_{t+2}) \neq 0$. Defining the term premium as the holding-period return on the 2-period bond net of the return on rolling over a 1-period risk-free bond,

$$
\begin{aligned}
\text{term premium} &\equiv 2r^c_{2,t} - r^f_t - r^f_{t+1} \\
&= -cov_t(m_{t+1}, m_{t+2})
\end{aligned}
$$

The term premium compensates the holder for uncertainty about the marginal utility of consumption in the future. To generate a positive term premium, the stochastic discount factor must have negative serial correlation: $cov_t(m_{t+1}, m_{t+2}) < 0$. Negative serial correlation in the risk-free rate means that holding a multi-period bond with a fixed nominal payoff makes consumption more volatile. If the payoff $r^c_{2,t}$ helps to smooth consumption today, the negative serial correlation between m_{t+1} and m_{t+2} means that it is unlikely to help to smooth consumption next period. The term premium is part of the bond premium, λ^R_t (equation 2).

Liquidity premium The price of a short-term bond can deviate considerably from its hold-to-maturity value because of collateral value, demand and supply, and short-term safety factors.

Consider holding the 2-period bond described above, but with a non-zero probability that the bond will be sold, at $t+1$, to smooth consumption, subject to a liquidation cost. The pricing equation is

$$
1 = E_t[M_{t+1}M_{t+2}(1+r^s_{2,t})^2(1-d_{t+1})]
$$

where $(1+r^s_{2,t})$ is the gross yield on the bond that may need to be sold, and d_{t+1} captures both the probability that the bond will be sold at $t+1$ and the expected discount if the bond is sold, relative its hold-to-maturity value. The log of the pricing equation is:

$$
\begin{aligned}
2r^s_{2,t} &= E_t r^f_t + E_t r^f_{t+1} - logE_t(1-d_{t+1}) - cov_t(m_{t+1}, m_{t+2}) \\
&\quad + cov_t(m_{t+1}, d_{t+1}) + cov_t(m_{t+2}, d_{t+1})
\end{aligned}
$$

The observed, contracted rate on the bond reflects expected risk-free returns, expected losses from selling the bond before maturity, a term premium, and a risk correction that increases the yield on the bond if losses are expected to be greater when the marginal utility of consumption is expected to rise.

Defining the liquidity premium as the yield on the bond that is sold at a discount at $t+1$, net of the yield on the 'liquid' bond (A.6),

$$\begin{aligned} \text{liquidity premium} &\equiv r^s_{2,t} - r^c_{2,t} \\ &= \tfrac{1}{2}(logE_t(1-d_{t+1}) + cov_t(m_{t+1}, d_{t+1})) \end{aligned}$$

the liquidity premium captures the expected loss from selling the bond, $logE_t(1-d_{t+1})$, and a risk correction.[6] If investors are more likely to liquidate bonds to smooth consumption when the marginal utility of consumption rises, the expected loss from selling illiquid bonds is likely to be positively correlated with m_{t+1}. The expected discount d_{t+1} can also be interpreted as a transaction cost associated with selling the bond or 'specialness' (Krishnamurthy and Vissing-Jorgensen (2012), Vayanos (1998) and Aiyagari and Gertler (1991)). If a bond is expected to sell at a premium in bad times $E_t d_{t+1} < 0$, for example when the market wants to hold high quality assets and collateral – a 'flight to quality' response – then the yield on the bond will be lower, reflecting its expected liquidity value. Feldhütter and Lando (2008), Duffie (1996) and Amihud and Mendelson (1991) estimate short-term safety factors, to be substantial for US Treasuries. (Nagel 2014) links liquidity premia to monetary policy. The liquidity premium is part of the bond premium, λ^R_t (equation 2).

Uncovered interest parity and currency revaluation risk The home (global) investor's pricing equation for the foreign short-term bond is:

$$Q_t = E_t M_{t+1}(1 + r^{c*}_t) Z^*_{t+1} Q_{t+1},$$

where Q_t is the real exchange rate (a rise is a depreciation of the home currency). Taking logs,

$$\begin{aligned} 0 &= logE_t\big(M_{t+1}(1+r^{c*}_t)Z^*_{t+1}\frac{Q_{t+1}}{Q_t}\big) \\ &= log\Big(e^{(E_t m_{t+1} + E_t r^{c*}_t + E_t z^*_{t+1} + E_t \Delta q_{t+1} + \frac{1}{2}var(m_{t+1} + r^{c*}_t + z^*_{t+1} + \Delta q_{t+1}))}\Big) \\ &= E_t m_{t+1} + \tfrac{1}{2}var(m_{t+1}) + E_t r^{c*}_t + \tfrac{1}{2}var(r^{c*}_t) + E_t z^*_{t+1} + \tfrac{1}{2}var(z^*_{t+1}) + E_t \Delta q_{t+1} \\ &\quad + cov_t(m_{t+1}, z^*_t) + cov_t(m_{t+1}, \Delta q_{t+1}) + \tfrac{1}{2}var(\Delta q_{t+1}) \\ q_t &= -r^f_t + r^{c*}_t + logE_t(Z^*_{t+1}) + E_t q_{t+1} + \tfrac{1}{2}var(\Delta q_{t+1}) \\ &\quad + cov_t(m_{t+1}, z^*_t) + cov_t(m_{t+1}, \Delta q_{t+1}) \end{aligned} \tag{A.7}$$

Subtracting, (A.4) from (A.7) gives the UIP condition (equation 1) and the currency 'excess returns' in (**??**):

$$\begin{aligned} \lambda_t &= (logE_t Z_{t+1} - logE_t Z^*_{t+1} + (cov_t(m_{t+1}, (z_{t+1} - z^*_{t+1})) - cov_t(m_{t+1}, \Delta q_{t+1}) \\ &\quad - \tfrac{1}{2}var(\Delta q_{t+1})) \end{aligned} \tag{A.8}$$

that equates the expected return on a home bond to expected return on a foreign bond, from

[6] See Amihud et al. (2005) for a discussion of different types of liquidity risk.

the home investor's perspective.

The currency premium is defined as:

$$\begin{aligned}
\lambda_t^{FX} &= \lambda_t - \lambda_t^R \\
&= (logE_t(Z_{t+1}) - logE_t(Z^*_{t+1}) \\
&\quad +cov_t(m_{t+1}, z_{t+1}) - cov_t(m_{t+1}, z^*_{t+1}) - cov_t(m_{t+1}, \Delta q_{t+1}) \\
&\quad -\tfrac{1}{2}var(\Delta q_{t+1}) - (logE_t(Z_{t+1}) - logE_t(Z^*_{t+1}) \\
&\quad +cov_t(m_{t+1}, z_{t+1}) - cov_t(m^*_{t+1}, z^*_{t+1})) \\
&= cov_t(m^*_{t+1}, z^*_{t+1}) - cov_t(m_{t+1}, z^*_{t+1}) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var(\Delta q_{t+1}) \\
&= cov_t\big((m^*_{t+1} - m_{t+1}), z^*_{t+1}\big) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var(\Delta q_{t+1}) \qquad \text{(A.9)}
\end{aligned}$$