

# Business Cycle Accounting

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## Abstract

We perform Business Cycle Accounting for New Zealand. A basic Real Business Cycle model with growth is first calibrated to New Zealand. This growth model is then combined with data to estimate four wedges using maximum likelihood estimation. These wedges measure distortions in efficiency, labour, investment and government spending. The wedges are then turned on and off to create a historical decomposition. The historical decomposition shows that labour wedge is most responsible for economic fluctuations such as booms and recessions in New Zealand. The Investment wedge plays almost no role in explaining any of the data and plays only a minor role in explain investment itself. An existing equivalence result relates the wedges to more advanced models, therefore knowing the importance of labour wedge here provides a guidance on how to model economic fluctuations in New Zealand to a greater extend.

**Keywords:** Business Cycle Accounting, Real Business Cycle model, Economic Fluctuations

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# 1 Introduction

We perform business cycle accounting for New Zealand. First a basic Real Business Cycle model with growth is calibrated to New Zealand. Using data with this growth model four type of wedges are estimated using the maximum likelihood estimation. Four type of wedges includes efficiency (productivity), labour, investment and government spending which measures distortions in each of these markets. We employed Dynare together with MATLAB to develop the growth model in order to estimate these wedges.

Each wedges alone or combination with each other are turned on and off to create historical decomposition. Through historical decomposition we would be able to see which wedges are most responsible for explaining economic fluctuation in New Zealand.

For both estimation and historical decomposition above we follow Chari, Kehoe, and McGrattan (2007) except in our method we use: (i) a more general utility function and (ii) use a quadratic, rather than linear, decision rules.

Using the equivalence results we are then able to relate these wedges to advanced economic models, and knowing which wedges are important will help us identify how to model economic fluctuations in New Zealand. It will also provide us with guidance on identifying which current economic models are most useful.

The main results shows that labour wedge is most useful for explaining fluctuations in output, labour and investment while efficiency wedge is moderately useful. Investment wedge is not very useful in explaining any of the data and only have a minor role in explaining investment itself. The government spending data can only be fully explained using the government wedge alone which is reassuring since it is an assumption of this model.

The equivalence results suggest that models adopting labour and efficiency wedges compared to those employing investment and government spending wedges are more helpful for in explaining economic fluctuations in New Zealand between 1991 to 2014.

## 2 Literature Review

The Business Cycle Accounting procedure was firstly introduced by Chari, Kehoe, and McGrattan (2007). Since then many authors have attempted to apply this procedure in many ways and in many nations.

Otsu (2010) applies this business cycle accounting procedure to a small open economy Neoclassical model. The author specifically looks at output and consumption crisis that took place in Hong Kong, Korea, Thailand and Singapore in mid 2000. The author finds that total factor productivity is the main cause of output drops in these countries. For Korea and Thailand, foreign debt frictions are important in explaining consumption fluctuations. For Singapore and Hong Kong, it is domestic financial frictions. Simonovska and Söderling (2008) applies business cycle accounting procedure to Chile using standard Neoclassical growth model for time period between 1998 to 2007. The authors find that different wedges played different roles but efficiency, labour and investment wedges played the main roles. They have further compared these results to business cycle accounting literature of both developing and developed countries. Iskrev (2013) applies business cycle accounting method for Portugal. The author finds that the total factor productivity wedge plays a large role in explaining output fluctuation from 1998 to 2012. Kersting (2008) applies business cycle accounting for UK to look at economic fluctuations between 1979-1989. The results suggest that frictions in labour-leisure played a major role in explaining recessions experienced in 1980's. Recovery experienced in 1984 can not be explained without improvements in the labour wedge. Kobayashi and Inaba (2006) applies business cycle accounting procedure to Japan from 1990's and to both Japan and US from inter-war period. The results suggest that labour frictions might be the main reason for long recession in Japan in 1990's. They also replace investment wedge with capital wedge to test the accuracy of business cycle accounting method. The results suggest that capital wedge might have played a major role in explaining depression faced by United States in 1930's.

Zhao (2012) uses two sector growth model and applies it to 44 countries. The author finds that human capital productivity wedge and human capital investment wedge are negatively correlated to each other. They can be used together to explain growth rates across countries more effectively than other wedges. Cho and Doblus-Madrid (2013) applies business cycle accounting to 23 crises around the world. The results suggest that East Asian Crises are mainly driven by efficiency and Investment wedges while other crises around the world are mainly due to distortions in efficiency and labour wedges. The authors say that their findings are consistent with other studies which looks at Asian financial markets. This is due to institutional differences in countries such as Japan. Chakraborty and Otsu (2013) applies business cycle accounting to BRIC countries (Brazil, Russia, India and China). The authors find that efficiency wedge plays a large role in explaining growth mainly in Brazil and Russia but investment wedge still plays an important role in China and India in late 2000's. The authors suggest that this is mainly due to different development stages faced by these countries in late 2000's.

Gali (1996) estimates conditional correlation between labour and productivity for G7 countries. The author further decompose the data into technology and non-technology components. The results suggest that technology shocks cause a negative co-movement between productivity and labour while demand shocks creates a positive co-movement between them. This produces counterbalancing results.

Šustek (2011) extends business cycle accounting method to monetary models to look at the impact of various frictions has on inflation as well as nominal interest rates adjustments. The results show that the wedge explaining total factor productivity plays a major role in explaining assets markets while wedges explaining frictions due to sticky prices only play a minor role.

Nutahara and Inaba (2012) test whether VAR(1) is a good approximation to use as the stochastic process of business cycle accounting procedure. The test applies business cycle accounting method to a medium size dynamic stochastic general equilibrium (DSGE) economy. It finds that VAR(1) is an good enough approximation of stochastic process to be used in business cycle accounting method.

Christiano and Davis (2006) looks at why results of business cycle accounting procedure suggested by Chari et al. (2007) might produce incorrect conclusions about importance of models with financial frictions. The authors argue that there are two reasons for these incorrect conclusions. One reason is that small changes in the implementations of business cycle accounting will change Chari et al. (2007) conclusions. Second reason is how shocks to wedges might affect the economy by their spillover effects on other wedges which is not considered in the business cycle accounting. Therefore authors say that Chari et al. (2007) understated the value of these shocks by assuming that spillover effects are zero. Authors says that lot of evidence in US and OECD countries establishes their view against business cycle procedure produced by Chari et al. (2007).

Generally the results produced by many business cycle accounting literature agree with our results. That is labour and efficiency wedges are most useful for explaining economic fluctuations while investment wedge plays almost no role. Although some results such as Chakraborty and Otsu (2013) and Cho and Doblus-Madrid (2013) contradict our results, these differences are mainly due to institutional variations in these Asian countries that authors have already suggested.

### 3 The Model

#### 3.1 Real Business Cycle Model

The prototype economy we use in the accounting procedure is the basic Real Business Cycle model with deterministic growth. In every period  $t$ , the economy experiences many events determined by the realization of a stochastic process  $\{s_t\}$ . The state,  $s^t = (s_0, \dots, s_t)$  then represents all the events that happens in the history including period  $t$ . The economy has four exogenous variables which are all a function of  $s^t$  and assumed to follow a Markov process. These includes the productivity wedge;  $A_t$ , labour wedge;  $1 - \tau_{lt}$ , the investment wedge;  $1/[1 + \tau_{xt}]$  and government expenditure wedge;  $g_t$ .

Model consists of a household and a representative firm. The markets for consumption goods, capital and labour are all perfectly competitive. Consumers can decide how

much to consume ( $c_t$ ) as well as how much to work ( $l_t$ ), where both  $c_t$  and  $l_t$  are in per-capita terms. Given these two choices households maximize their expected level of utility,

$$\sum_{i=0}^{\infty} E[\beta^i U(c_t, l_t), N_t] \quad (1)$$

with period utility function,

$$U(c_t, l_t) = \frac{(c_t^\psi (1 - l_t)^{1-\psi})^{1-\sigma} - 1}{1 - \sigma} \quad (2)$$

where  $\beta \equiv$  discount rate,  $E \equiv$  expectations in period zero of the shocks at time  $t$  and  $N_t \equiv$  total population<sup>1</sup>,

The household maximizes this utility subject to the budget constraint,

$$c_t + [1 + \tau_{xt}]x_t = [1 - \tau_{lt}]w_t l_t + r_t k_t + T_t \quad (3)$$

as well as the capital accumulation equation and some other minor assumptions,

$$(1 + \gamma_n)k_{t+1} = (1 - \delta)k_t + x_t \quad (4)$$

$$k_t > 0, t = 0, 1, 2, \dots$$

$$0 \leq l_t \leq 1, t = 0, 1, 2, \dots$$

$$k_{-1} \text{ given}$$

$$\lim_{T \rightarrow \infty} E[\beta^T U(c_T, 1 - l_T), N_T] \text{ (Transversality Condition)}$$

where  $\psi \equiv$  share parameter for leisure in composite commodity,  $\sigma \equiv$  inter-temporal elasticity of substitution,  $x_t \equiv$  investment per-capita,  $k_t \equiv$  capital stock per-capita,  $\delta \equiv$

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<sup>1</sup>Utility function here was taken from (Cooley and Prescott, 1995) pp.16. Chari, Kehoe, and McGrattan (2007) use a log-log utility function, which is covered as a limiting subcase of the utility function used here.

depreciation rate of capital,  $w_t \equiv$  wage rate,  $r_t \equiv$  return on the capital,  $T_t \equiv$  lump sum transfer per-capita,  $\gamma_n \equiv$  population growth rate.

Firms choose the quantity to produced, how much labour to acquire and amount of capital to rent given the market wage rate;  $w_t$  and the rental rate of capital  $r_t$ . Firms use the production function below,

$$AF(k_t, (1 + \gamma)^t l_t) \tag{5}$$

Firms maximize profits, which is equal to revenues minus costs, and since the price of output is normalized to one and costs are the cost of the capital and labour inputs, this means maximizing

$$AF(k_t, (1 + \gamma)^t l_t) - r_t k_t - w_t l_t \tag{6}$$

Where  $A \equiv$  labour augmenting technology and  $1 + \gamma \equiv$  labour-augmenting technological growth rate. In this model, efficiency wedge is given by the technology parameter while the labour wedge and investment wedge are equivalent to labour and investment tax rates, respectively.

### 3.2 Equilibrium First Order Conditions with Growth

In this model, competitive equilibrium is given by the solution to the following system of stochastic difference equations; which include the technological growth rates  $\gamma$ .

$$c_t(1 + \gamma)^t + x_t(1 + \gamma)^t + g_t(1 + \gamma)^t = y_t(1 + \gamma)^t \tag{7}$$

$$y_t(1 + \gamma)^t = A_t(k_t(1 + \gamma)^t)^\alpha (l_t(1 + \gamma)^t)^{1-\alpha} \tag{8}$$

$$-\frac{\psi c_t (1 + \gamma)^t}{(1 - \psi) l_t (1 + \gamma)^t} = [1 - \tau_{lt}] (1 - \alpha) A_t (1 + \gamma)^t (k_t (1 + \gamma)^t)^\alpha (l_t (1 + \gamma)^t)^{-\alpha} \quad (9)$$

$$(c_t^{1-\psi} (1 + \gamma)^t l_t^\psi)^{-\sigma} c_t^{-\psi} (1 - \psi) l_t^\psi [1 + \tau_{xt}] = \beta E (c_{t+1}^{1-\psi} (1 + \gamma)^{t+1} l_{t+1}^\psi)^{-\sigma} c_{t+1}^{-\psi} (1 - \psi) l_{t+1}^\psi \quad (10)$$

$$\{A_{t+1} \alpha k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} + (1 - \delta) [1 + \tau_{xt+1}]\}$$

and the Transversality Condition.

### 3.3 Equilibrium First Order Conditions without Growth

For the purpose of implementing the accounting procedure the deterministic growth rate needs to be removed from the equilibrium conditions (7)-(10). To remove the growth from these equations we re-write the above equations in terms of the stationary variables  $\hat{c}_t = \frac{c_t}{(1+\gamma)^t}$ ,  $\hat{l}_t = \frac{l_t}{(1+\gamma)^t}$ ,  $\hat{y}_t = \frac{y_t}{(1+\gamma)^t}$  and  $\hat{g}_t = \frac{g_t}{(1+\gamma)^t}$ . This gives the equations (11)-(14) below,

$$\hat{c}_t + \hat{x}_t + \hat{g}_t = \hat{y}_t \quad (11)$$

$$\hat{y}_t = A_t k_t^\alpha \hat{l}_t^{1-\alpha} \quad (12)$$

$$-\frac{\psi \hat{c}_t}{(1 - \psi) \hat{l}_t} = [1 - \tau_{lt}] (1 - \alpha) A_t k_t^\alpha \hat{l}_t^{-\alpha} \quad (13)$$

$$(\hat{c}_t^{1-\psi} \hat{l}_t^\psi)^{-\sigma} \hat{c}_t^{-\psi} (1 - \psi) \hat{l}_t^\psi [1 + \tau_{xt}] = \beta E (\hat{c}_{t+1}^{1-\psi} (1 + \gamma) \hat{l}_{t+1}^\psi)^{-\sigma} \hat{c}_{t+1}^{-\psi} (1 - \psi) \hat{l}_{t+1}^\psi \quad (14)$$

$$\{A_{t+1} \alpha k_{t+1}^{\alpha-1} \hat{l}_{t+1}^{1-\alpha} + (1 - \delta) [1 + \tau_{xt+1}]\}$$

As part of the numerical solution of these equations we first need to calculate the deterministic steady-state, this is done in part using the steady-state conditions derived from these equations and described in Appendix A.

### 3.4 Model Calibrations

Before we can estimate the model and perform the business cycle accounting procedure we must calibrate certain parameters of the Real Business Cycle model to New Zealand. Based on New Zealand data we recalibrate  $\alpha$  to target the capital income share,  $\beta$  to target the discount rate, and  $\gamma$  to target the labour-augmenting technological growth rate. Parameters defining the depreciation rate,  $\delta$ , the share of leisure in the composite consumption commodity,  $\psi$ , and the inter-temporal elasticity of substitution,  $\sigma$ , were chosen following Chari et al. (2007). The population growth rate,  $\gamma_n$ , did not need to be calibrated since it does not appear in the system of stochastic equations describing competitive equilibrium; as seen in Section 3.2.

The calibrated parameters of the growth model for New Zealand are thus,

Parameter	$\alpha$	$\beta$	$\gamma$	$\gamma_n$	$\delta$	$\psi$	$\sigma$
New Zealand	0.22	0.94	0.004379	-	0.0464	0.64	1

Table 1: Calibrated Parameters

Data needed for the calibration were taken from FRED<sup>2</sup> and StatsNZ<sup>3</sup>. Refer to Appendix B for details on exactly which data series were used as well and for more detail on how the parameters were calibrated to these data series.

## 4 Business Cycle Accounting Method

The Business Cycle Accounting procedure looks at the marginal effects of individual wedges, or combinations of these wedges, on output,  $y$ , labour  $l$ , investment  $x$ , and government spending (and net exports)  $g$ . Rational expectations impose that the probability distribution of each wedge is same as the probability distribution of corresponding wedges in the model. When turning off the effect of a given wedge, households and firms

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<sup>2</sup><http://research.stlouisfed.org/fred2/>

<sup>3</sup><http://www.stats.govt.nz/>

expectations of how the underlying state,  $s_t$ , will evolve over time are left unchanged. The resulting counter-factual time series for output, labour, investment, and government spending with certain wedges turned off are then compared with their observed historical time series data. This allows us to see which wedges have historically been important for explaining each of output, labour, investment and government spending.

The Business Cycle Accounting procedure is described as follows. We first describe how the accounting decomposition is performed assuming we already know the values and probabilities of the stochastic process,  $\{s_t\}$  underlying the wedges. We then describe how we implement the stochastic process  $\{s_t\}$  and estimate its historical values and probabilities from the data.

#### 4.1 Details of Accounting procedure

Assume that for the stochastic process  $\{s_t\}$  the realization of each state  $s^t$  in a specific history, and their associated probabilities,  $\pi_t(s^t)$ , are known. The four wedges including productivity ( $A_t$ ), labour  $1 - \tau_{lt}$ , investment  $1/(1 + \tau_{xt})$ , government expenditure  $g_t$  and other stochastic variables are assumed to be a known function of the current state,  $s_t$ .

To measure, for example, the individual effects of efficiency wedge, an economy with same state  $s^t$  and  $\pi_t$  is considered where the efficiency wedge,  $A_t$  is set equal to  $s_{At}$  of the prototype model while all other wedges are set equal to their unconditional expected mean values  $\tau_{lt} = \bar{\tau}_l$ ,  $\tau_{xt} = \bar{\tau}_x$  and  $g_t = \bar{g}$ . This ensures that the probability distribution with all four wedges active of the efficiency wedge is equivalent to that of the model. Then for the efficiency wedge alone economy we can calculate the values of all the model's variables — including output, labour, investment, and government spending, based the historical realizations of the state,  $s_t$  and compare these counterfactuals with the data for output  $y$ , labour  $l$ , investment  $x$  and government spending  $g$ .

This process can then be repeated for each of the labour, investment and government spending wedges individually as well as in different combinations e.g, efficiency, labour and investment together.

## 4.2 Implementation of the Stochastic Process

Until now it was assumed that the stochastic process governing  $\{s_t\}$  is known — so  $\pi_t(s^t)$  is known — and that  $s^t$  is observed. To do this we need to specify a form for the stochastic process and then use data to uncover the state,  $s_t$ , for each time period.

Assume that the  $s^t$  follows a Markov process given by  $\pi(s_t|s_{t-1})$ , and that wedges in period  $t$  can be used to discover  $s_t$  given that the mapping from the event  $s_t$  to the wedges is one to one and onto. To ensure this, define state  $s_t \equiv (s_{At}, s_{lt}, s_{xt}, s_{gt})$  and let the wedges as a function of the state be given by  $A_t = s_{At}$ ,  $\tau_{lt} = s_{lt}$ ,  $\tau_{xt} = s_{xt}$  and  $g_t = s_{gt}$ .

Estimation of the Markov process is by (simulated) maximum likelihood estimation from the data on output per-capita,  $y_t$ , labour per-capita,  $l_t$ , investment per-capita,  $x_t$ , and government spending (plus net exports) per-capita. To estimate the stochastic process for the state it is assumed that event  $s_t$  follows a first-order vector autoregressive, VAR(1), namely

$$s_{t+1} = P_0 + P_{s_t} + \varepsilon_{t+1} \quad (15)$$

where shock  $\varepsilon_t$  is i.i.d with mean zero and a covariance matrix  $V$ . To ensure a positive semi definite estimate of  $V$ ,  $Q$  the lower triangle matrix is estimated where  $V = QQ'$ . The parameters of  $P_0$ ,  $P$  and  $V$  of VAR(1) process underlying the wedges are estimated using Maximum Likelihood Method. To do so we use quadratic decision rules of the prototype model economy together with the data on  $y_t$ ,  $l_t$ ,  $x_t$  and  $g_t$  for New Zealand.

Having estimated the VAR(1) process governing  $\{s_t\}$ , we can then recover the value of  $s_t$  from measurements of the realized wedges. The government wedge is directly taken as the government spending plus net exports from New Zealand data. To recover the values of other wedges we need to use data together with model's decisions rules. Data is taken as  $y_t^d$ ,  $l_t^d$ ,  $x_t^d$  and  $g_t^d$  and  $k_0^d$  while model decision rules are taken as  $y(s_t, k_t)$ ,  $l(s_t, k_t)$  and  $x(s_t, k_t)$ , then realized wedge series  $s_t^d$  solve (16) below,

$$y_t^d = y(s_t, k_t), \quad l_t^d = l(s_t, k_t), \quad x_t^d = x(s_t, k_t) \quad (16)$$

with  $k_{t+1} = (1 - \delta)k_t + x_t^d$ ,  $k_0 = k_0^d$ , and  $g_t = g_t^d$ . We find solutions for three unknown values of the event  $s_t$  with equations (12)-(14) thereby recovering the state  $s^t$ .

We are now able to perform the Business Cycle Accounting decomposition previously described, looking at the marginal effect of each wedge alone, or in combination. To repeat, to do this we need to allow some wedges to vary as measured in the historical data while others are set constant. For example, to look at the marginal effects of the efficiency wedge, decision rules  $y^e(s_t, k_t)$ ,  $l^e(s_t, k_t)$  and  $x^e(s_t, k_t)$  are computed for the efficiency wedge alone economy where  $A_t = s_{At}$ ,  $\tau_{lt} = \bar{\tau}_l$ ,  $\tau_{xt} = \bar{\tau}_x$  and  $g_t = \bar{g}$ . Note that doing this will remove the direct effects of labour, investment and government expenditure wedges while maintaining their forecasting effects. Then  $k_0^d$ ,  $s_t^d$  together with these decision rules and capital accumulation equation (4) we can simulate model data for  $y_t^e$ ,  $l_t^e$  and  $x_t^e$  for efficiency wedge alone economy. We are able to compare these simulated historical counterfactuals with one or more wedges deactivated with the actual data for output per-capita,  $y_t$ , labour per-capita  $l_t$ , investment per-capita  $x_t$ , and government spending  $g_t$  for New Zealand.

## 5 Data for Estimation of wedges

New Zealand data needs to be adjusted accordingly so that it matches the theory. We do this by removing all the sale taxes, services flows from consumer durables and depreciation from consumer durables. Further we used the details provided by (Chari et al., 2006) technical appendix to adjust these data accordingly.

All the data used for estimation process were taken from two sources including FRED and statistics New Zealand. These data series include per-capita output, per-capita labour, per-capita investment and per-capita government spending starts on quarter one of 1991 and ends in quarter two of 2014.

Refer to Appendix B for additional information about which data series were used and how they were exactly adjusted in order to estimate each per-capita variables.

## 6 Results

### 6.1 General fluctuations in New Zealand

We graphed simulated counterfactuals for one or more combination of wedges with actual data for aggregated variables  $y_t, l_t, x_t$  from 1991-Q1 to 2014-Q2 for New Zealand. We can then see which wedges are most responsible for economic fluctuations in New Zealand that took place between the same time periods such as, 91-92 recession due to anti-inflationary policies, 1997-1999 recession due to the Asian Crisis and drought, (Reddell and Sleeman, 2008) and the great depression in 2007-2009.

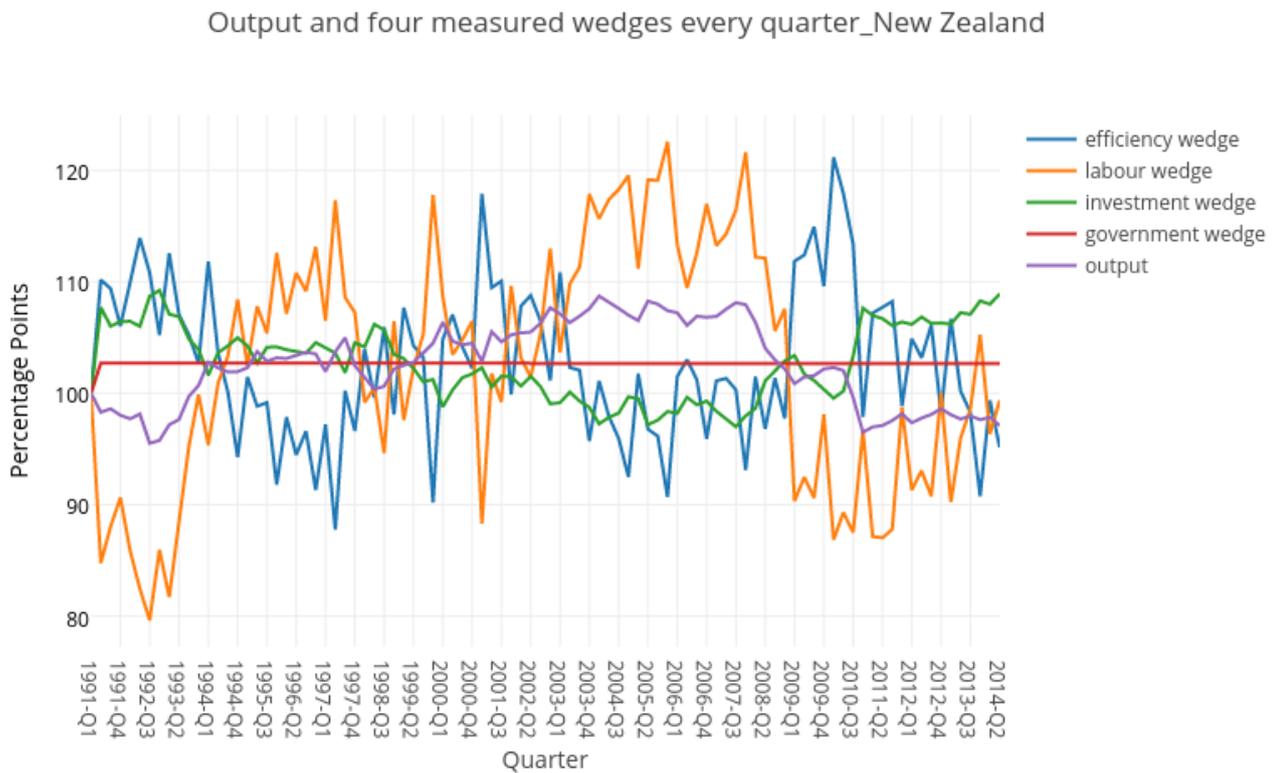


Figure 1: Output and prediction of the model for New Zealand with only one wedge per quarter

The simulated per-capita output for each of the single-wedge-alone economies are

graphed alongside per-capita output data for New Zealand in figure 1. We find that variations in output-per capita (purple line) between 1991:Q1 and 2014:Q2 are explained mainly by fluctuations in labour wedge (orange line). Further we can see that both simulated per-capita output with just the efficiency wedge (blue line) or the investment wedge (green line) alone display the opposite fluctuations to what we observed in real per-capita output data. The simulated data in terms of government wedge (red line) alone plays no role in explaining the fluctuations in per-capita output. We conclude that output fluctuations can be well explained by labour trade-offs in new Zealand between 1991 and 2014.

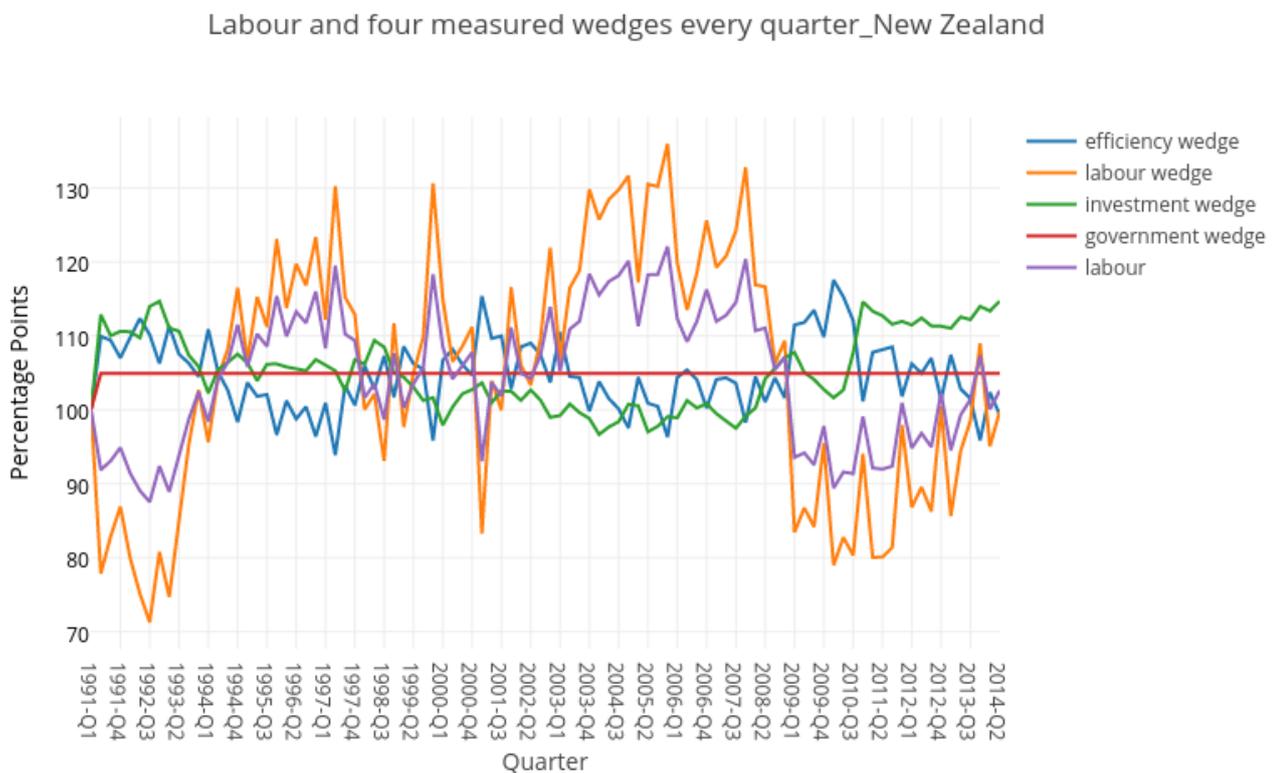


Figure 2: Labour and prediction of the model for New Zealand with only one wedge per quarter

The simulated labour per-capita for each of the single-wedge-alone economies are

graphed alongside labour per-capita data for New Zealand in figure 2. Again we see a similar scenario to figure 1. The simulated labour per-capita data using labour wedge (orange line) alone explains most of the fluctuations in labour-per capita data (purple line). The simulated labour per-capita data using either efficiency wedge (blue line) or investment wedge (green line) cause fluctuations in opposite directions to labour per-capita. The government wedge (red line) plays no role in explaining the labour-per capita as it stays constant through out the time series. In conclusion, labour fluctuations in New Zealand can be fully explained by employment and unemployment in the time period 1991-2014.

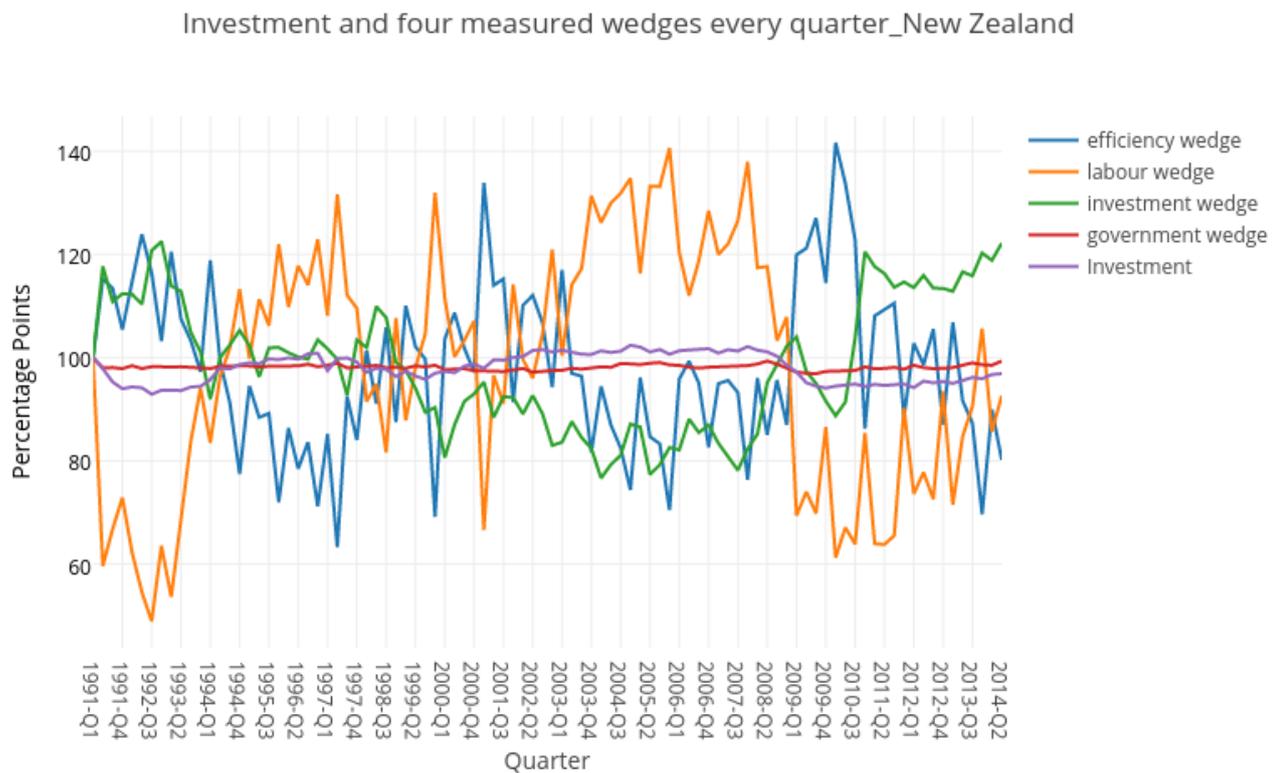


Figure 3: Investment and prediction of the model for New Zealand with only one wedge per quarter

Figure 3 above seems somewhat unclear at first. The simulated investment per-capita

for each of the single-wedge-alone economies are graphed alongside investment per-capita data for New Zealand. Taking a closer look it is clear that fluctuations in investment per-capita (purple line) data can somewhat be explained by either the labour wedge (orange line), investment wedge (green line) and efficiency wedge at different time periods. But the magnitudes of fluctuations due to each wedges alone seems much greater than the movements in investment per-capita. The per capita data simulated by the government wedge alone are useless in explaining investment fluctuations although although there are some fluctuations in the government wedge compared to cases in figure 1 and figure 2 above. The labour-leisure trade offs still plays a role in explaining investment per capita in New Zealand between 1991 and 2014.

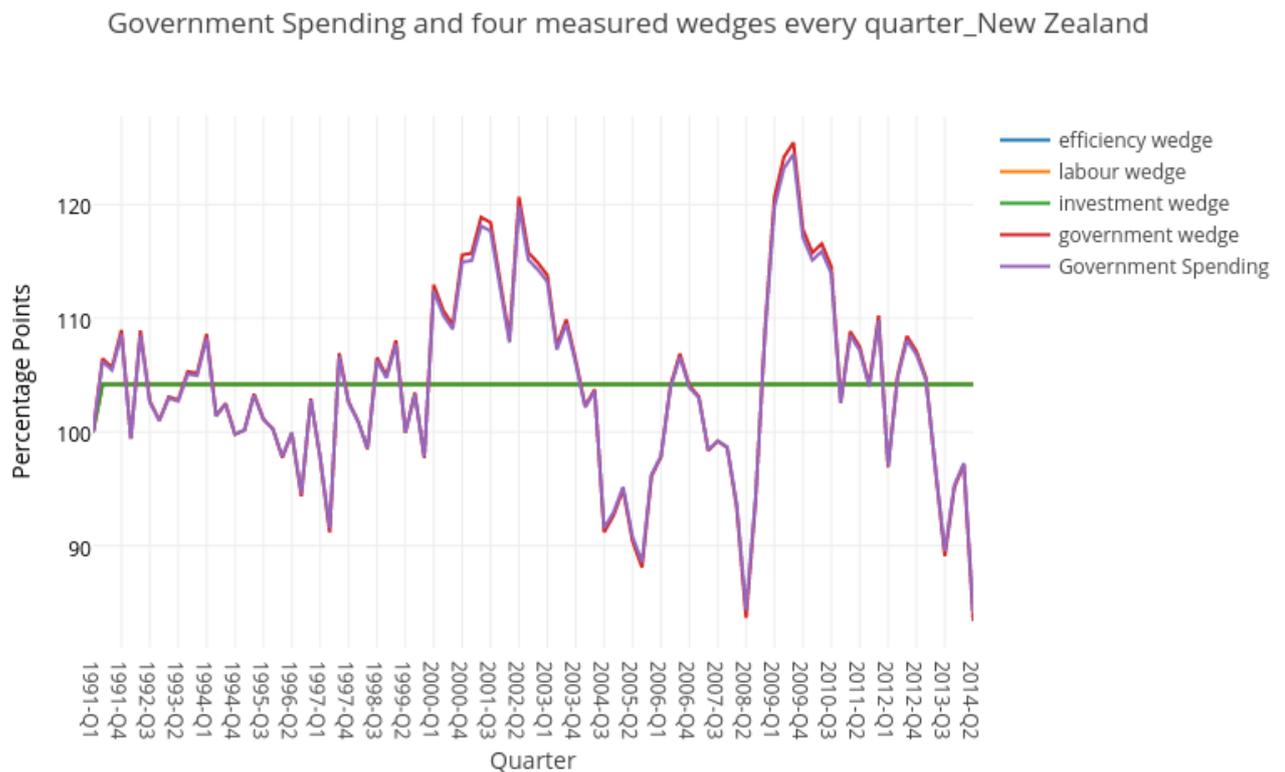


Figure 4: Government Spending and the prediction of the model for new Zealand with only one wedge per quarter

The simulated government per-capita for each of the single-wedge-alone economies is graphed alongside government per-capita data for New Zealand in Figure 4. It shows that the government per-capita (purple line) data are mainly explained by simulated government per-capita data (red line) when using government wedge alone. All the other wedges play no role in explaining government-per capita data. This is visible as simulated government per-capita data using either of the other three wedges alone stay constant throughout the time periods 1991 to 2014. This is what one should expect give the model assumption of government wedge been equivalent to government per-capita data.

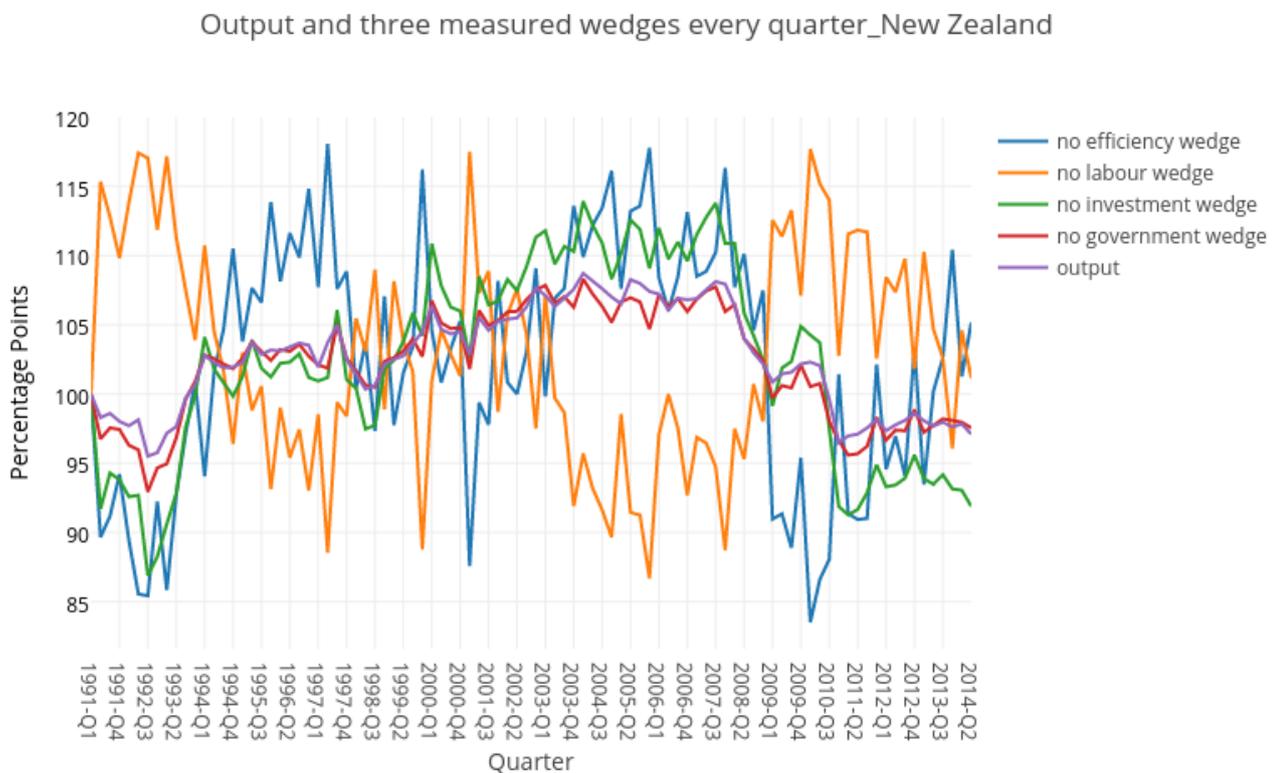


Figure 5: Output and the model prediction for New Zealand with all wedges but one per quarter

Model predictions when only one of the wedges are turned off with output per-capita

data (purple line) is shown on Figure 5 above. This is opposite to what we observed in figure 1. It is clear that the model prediction without the government wedge (red line) is most useful in explaining fluctuations in output per-capita. Then the model predictions without investment wedge (green line) and then efficiency wedge (blue line) seems to mostly explain output per-capita accordingly. But when the labour wedge (orange line) is turned off model seems to not follow output per-capita data at all. Further it shows that labour wedge is the extremely important for explaining output per-capita fluctuations, i.e, labour leisure trade-offs are important for output fluctuation in New Zealand.

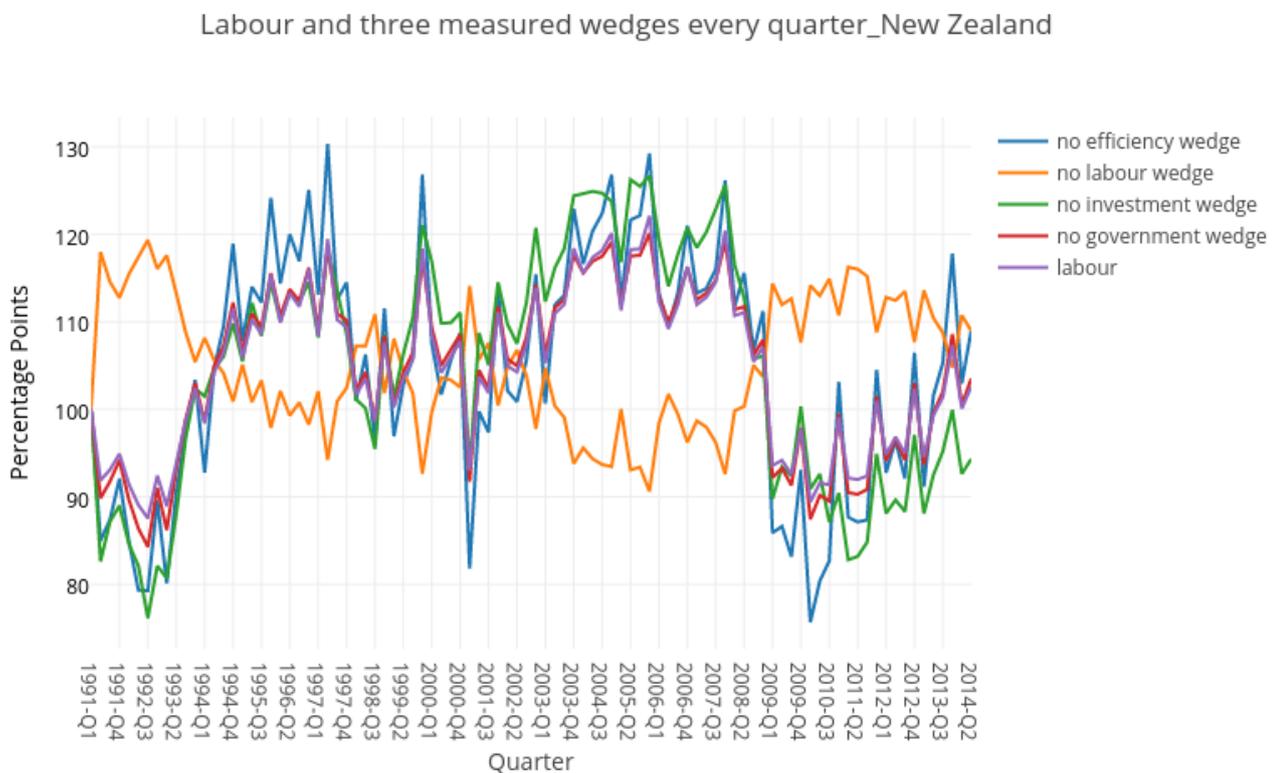


Figure 6: Labour and the model prediction for New Zealand with all wedges but one per quarter

Model predictions when only one of the wedges are turned off with labour per-capita

data (purple line) is shown on Figure 6. It repeats the similar scenario of figure 5. The labour per-capita (purple line) fluctuations are mainly explained by model predictions without government spending (red line), investment (green line) and efficiency (blue line) wedges respectively. But the model predictions without the labour wedge seems to give opposite predictions to what we see in fluctuations of labor per capita. This further confirms that the labour wedge is the most important wedge in explaining labour per-capita fluctuations.

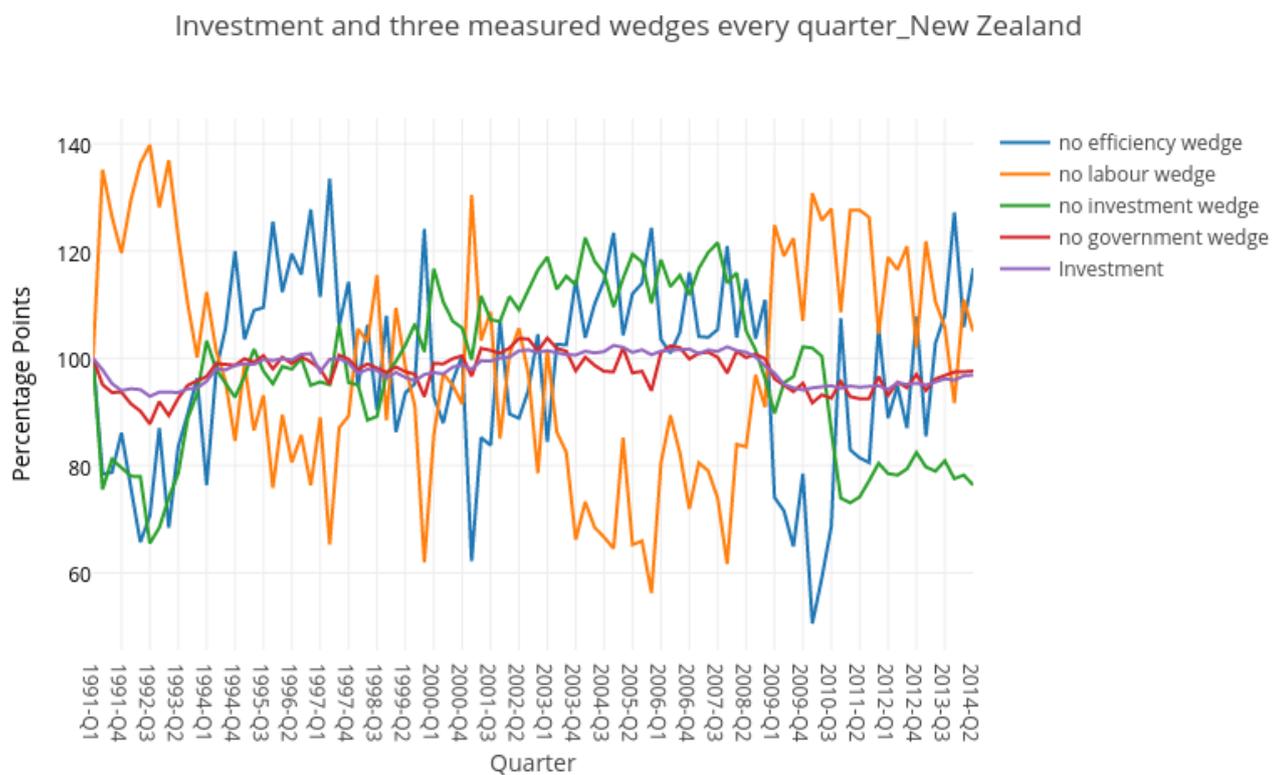


Figure 7: Investment and the model prediction for New Zealand with all wedges but one per quarter

Model predictions when only one of the wedges are turned off with investment per-capita data (purple line) is shown on Figure 7. It shows the same results on output and labour per-capita seen in figure 5 and 6. That is investment per-capita (purple

line) data are better explained by model predictions without the government wedge (red line), investment wedge (green line) and efficiency wedge (blue line) accordingly. Model predictions without the labour wedge (orange line) seems to be least useful for explaining fluctuations in investment-per capita. The most important wedge for explaining the investment per-capita is still the labour wedge.

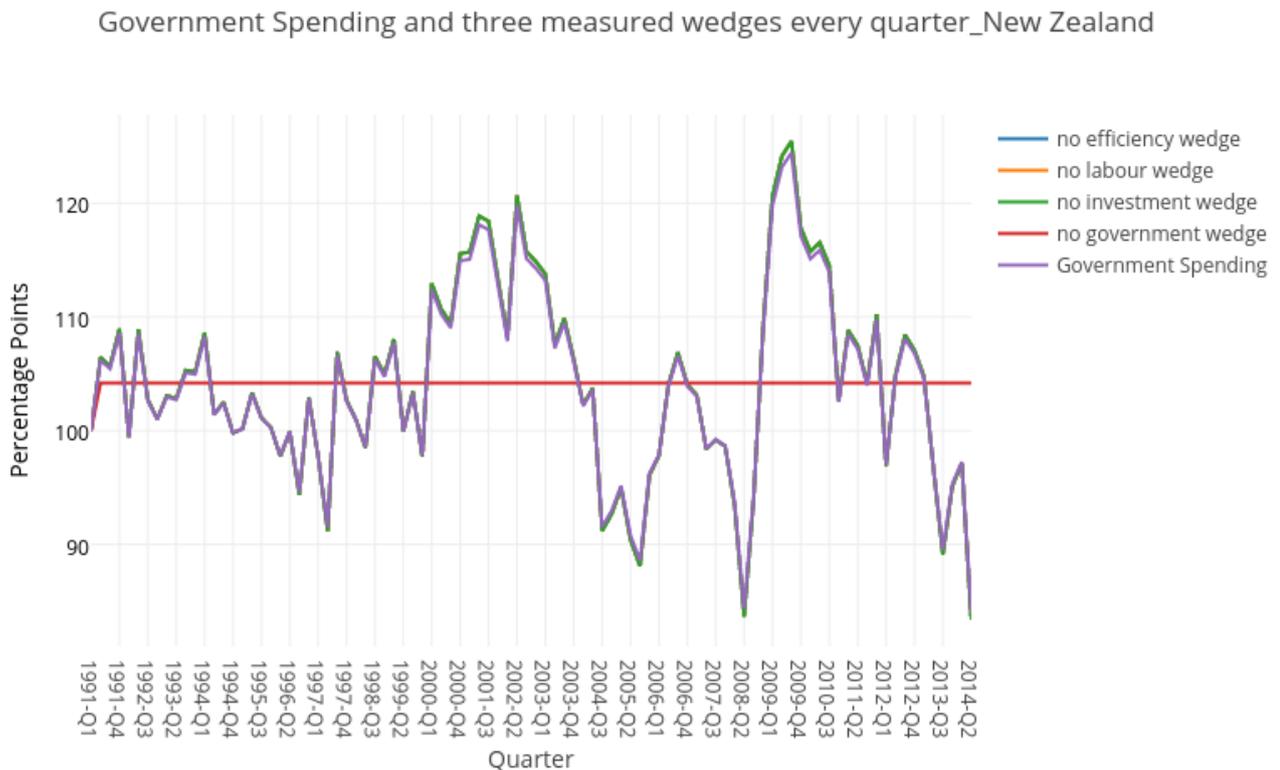


Figure 8: Government Spending and the model prediction for New Zealand with all wedges but one per quarter

Figure 8 is similar to what was seen in figure 4. The government wedge alone can mainly explain fluctuations in government-per capita (purple line) measures. When government wedge (red line) is turned off, the model prediction is least useful in explaining the fluctuations in the government per-capita. This is reassuring since this is one of the assumptions of the model, i.e,  $\hat{g}_t = g_t$ . When one of the other wedges are turned off

(shown in blue, yellow and green lines although blue and yellow lines are harder to see due to the overlap with the green line) model predictions still help to closely explain fluctuations in government per-capita measures.

Overall, the main results of this research shows that labour wedge mainly explains fluctuations in both output per-capita and labour per-capita in New Zealand between quarter one of 1991 and quarter two of 2014. Labour wedge also helps in explaining investment per-capita to some extent in the same period. The investment wedge seems to be only offer minor help in explaining investment per-capita fluctuations to minor extent. Whereas the government per-capita fluctuations seems to be mainly explained by government wedge itself.

These results seems to be closely matched with what Chari et al. (2007) found in their Business Cycle Accounting paper. Regardless of the fact that New Zealand can be considered as a small open economy while United States is a closed economy it does not seems to change the results of business cycle accounting.

Concentrating on equivalence results, it shows that the existing models which explains frictions in productivity, labour and financial markets through shocks process in labour wedges are more useful for explaining business cycle fluctuations in New Zealand (This is also somewhat true for efficiency wedge as well). Models which explain each of these market frictions using shock process through investment wedge and government wedge are somewhat less useful. For example Galí (2009) finds that government spending/ net exports plays an extremely important role in small open economy New Keynesian models. While this might well be the case, our results show that shock process that causes this in government spending/net exports should come through one of the other wedges. Another example is Bernanke and Gertler (1989) , Bernanke et al. (1999) and Kiyotaki and Moore (1997) where they claim that frictions in financial markets are important for explaining business cycle fluctuations. Again, this might well be the case, but our results suggest that for this to be true shock process that cause this should be taking place through one of the labour or efficiency wedges. Also, Bordo et al. (1997) sticky wages with monetary shocks models and Cole and Ohanian (2001) which

uses models with monopoly power are more useful since they employ labour wedges to explain market frictions.

## 7 Concluding Remarks

In this paper we applied a stochastic growth model to the New Zealand economy for the time period 1991 to 2014. By doing so we try to understand which market frictions (explained through wedges) are important for explaining business cycle fluctuations such as booms and recessions.

Looking at the main results it shows that labour wedge seems to explain most fluctuations in output, labour and investment per-capita measures during this time. On the contrary, investment wedge is useless in explaining economic fluctuations in New Zealand. The government wedge alone seems to explain most of the fluctuations in government per-capita measures.

This is consistent with what Chari et al. (2007) found when business cycle procedure was applied to closed US economy. We consider New Zealand to be a small open economy yet applying the same business cycle accounting procedure to New Zealand does not seem to make any difference. It gives the same predictions to Chari et al. (2007).

Looking at equivalence results, economic models that use shock process through labour and efficiency to explain economic fluctuations are the most useful models while those using investment and government spending are somewhat less useful. For example, the small open economy New Keynesian models used in Galí (2009) claim that the government spending/net exports are important in explaining business cycle fluctuations in Canada and US. This to be true these shock process should have impacted these economies through one of labour or efficiency wedge. This is similar to financial market distortions discussed by Bernanke and Gertler (1989), Bernanke et al. (1999) and Kiyotaki and Moore (1997) which authors claim that they are important in explaining business cycle fluctuations. The models that use sticky wages with monetary shocks such as Bordo et al. (1997) and Cole and Ohanian (2001) which uses models with monopoly

power are more useful since they employ labour wedges to explain market frictions.

## 8 Future Research

Business Cycle Accounting can be applied to any country around the world. To have a better understanding about this topic it would be useful to apply business cycle accounting to both developing and developed countries all around the world. We are planning to apply this business cycle accounting procedure to Australia, a large scale open economy to see whether results change from Chari et al. (2007) in US and our results for New Zealand in near future.

This research also suggest that we should pay more attention in improving current models by using labour and efficiency wedges instead of only depending on current models that use investment and government wedges to explain business cycle fluctuations in New Zealand. In that we have to use better models together with data to tell matching stories that explain data patterns observed in business cycle fluctuations.

## A The Model Extras

### A.1 Steady State Conditions

It is not possible to find a closed form steady state solution (17)-(21). Therefore, numerical methods are used to calculate the steady state values by guess varying (Note that it is not completely guess work since for e.g. we know that steady state investment,  $\bar{x}$  should equal depreciation rate,  $\delta$  times by steady state capital value,  $\bar{k}$ ).

$$\bar{x} = \delta \bar{k} \tag{17}$$

$$\bar{c} = \bar{y} - \bar{x} \tag{18}$$

$$\bar{y} = \bar{A} \bar{k}^\alpha \bar{l}^{1-\alpha} \tag{19}$$

$$\frac{\bar{k}}{\bar{y}} = \frac{\alpha}{\frac{1}{\beta(1+\gamma)^{-\sigma}} - (1-\delta)} \tag{20}$$

$$\frac{\bar{c}}{\bar{y}} = \frac{(1-\alpha)(1-\psi)}{-\psi} \tag{21}$$

Note that all the wedges apart from  $\bar{A}$  are set equal to zero at the steady state, whereas  $\bar{A}$  should be equal 1 at the steady state.

## B Data Sources

### B.1 Calibration of Parameters

Calibration of model parameters  $\alpha$  (capital income share),  $\beta$  (discount rate) and  $\gamma$  (technological growth rate) was done for New Zealand. Data needed for calibration purposes were taken from FRED <sup>4</sup> and Statistics New Zealand websites <sup>5</sup>.

#### **Alpha, $\alpha$**

$\alpha$  was estimated by dividing  $k/y$ , i.e, capital of New Zealand divided by it's total national income level.

For New Zealand data series, NAEXKP04NZQ189S (Gross Domestic Product by Expenditure in Constant Prices: Gross Fixed Capital Formation for New Zealand) divided by NAEXKP01NZQ189S (Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for New Zealand) seasonally adjusted quarterly estimates in New Zealand dollars from FRED gave the ratio of  $k/y$  from 1987-04-01 to 2014-07-01. Then the averages of these estimates from 1994-01-01 onward were used to calculate the  $\alpha$ . The reason to use  $k/y$  estimates from 1994-01-01 onward is that the thought of it would give a more accurate  $\alpha$  since  $k/y$  ratios prior to these days somewhat different from the recent estimates of  $k/y$ .

#### **Beta, $\beta$**

$\beta$  was estimated by taking the average of rate of 3-month/90 day bank bills.

For New Zealand data series, IR3TBB01NZQ156N (3-Month or 90-day Rates and Yields: Bank Bills for New Zealand) non-seasonally adjusted quarterly estimates in percentages from 1974-01-01 to 2014-07-01 from FRED were used to calculate the risk free rate of return on capital ( $r$ ). The Data prior to 1994-01-01 was discard to get a more accurate estimate since  $r$  values were substantially different prior to this period. The rest of the data were average to calculate  $r$ .

Then by approximating  $\beta = 1/(1 + r)$  from the wide literature, beta parameters for

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<sup>4</sup><http://research.stlouisfed.org/fred2/>

<sup>5</sup><http://www.stats.govt.nz/>

both countries were calculated.

### **Gamma, $\gamma$**

$\gamma$  was estimated by calculating the growth rate of real GDP per capita.

For New Zealand data series, NAEXKP01NZQ189S (Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for New Zealand) seasonally adjusted quarterly data in New Zealand dollars were gathered from FRED which starts from 1987-04-01 to 2014-07-01. New Zealand quarterly total population estimates were gathered from StatsNZ, data series DPE059AA (Estimated resident population). Then data series NAEXKP01NZQ189S was divided by the series DPE059AA to calculate the Real GDP per capita estimates. Then to get growth of technology per capita difference between real GDP per capita between each quarters were calculated and averaged. Again, the data before 1994-01-01 were discard to get more accurate estimates.

## **B.2 Time Series used in Estimation**

$y_t$  (per-capita GDP),  $l_t$  (per-capita labour),  $x_t$  (per-capita investment) and  $g_t$  (per-capita government expenditure) were calculated according to definition of each variable provided on the (Chari et al., 2006) technical appendix, p.33-34, B.2.2 Measures,

### **$y_t$ , Per-Capita Output**

For New Zealand, real GDP data series NAEXKP01NZQ189S(Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for New Zealand) was gathered from FRED which is a seasonally adjusted, quarterly estimate time series in New Zealand dollar units which starts from 1987-04-01 to 2014-07-01.

PCE data series, NZLPFCEQDSMEI (Private Final Consumption Expenditure in New Zealand) was also taken from FRED which is also seasonally adjusted, quarterly estimates in billions of New Zealand dollars which also starts from 1987-04-01. In order to keep the same dollar units across each time series this time series was needed to be adjusted in to dollar units of New Zealand dollars.

PCE durable data series, SND027AA (Final consumption expenditure durable goods) was taken from StatsNZ which is a seasonally adjusted, quarterly estimate which is in

millions of New Zealand dollars and goes from 1987Q2 onward 2014Q2.

Return on PCE durables were taken as 0.04 as given on charid paper. Depreciation on PCE durables were taken as 0 on the assumption that depreciation on PCE stays almost constant over the time. The reason to do this is that it was hard to find depreciation rate on durables in New Zealand.

NZ GST rates were applied accordingly. GST was first introduced to NZ in 1 of October 1986 which was increased to 12.5 percent in 1 of July 1989. Since 1st of October 2010 this rate was changed to 15 percent.

Non-institutional population 16-64 was taken from StatsNZ website specifically the data series DPE059AA (Estimated resident population). This was a quarterly estimate which goes form 1991Q1- 2014Q3.

Then for New Zealand using all these detailed data series per-capita output was calculated according to descriptions of Chari et al. (2006) on technical appendix, p.33-34, B.2.2. Only was able to calculate these estimates from 1991Q1 onwards for NZ and after 1960-01-01 for Australia since population data was only available after that for both countries.

#### **$l_t$ , Per Capita-labour**

For New Zealand total hours were taken from StatsNZ specifically the data series QEX024AA (QEX total paid hours). These were quarterly estimates which were also seasonally adjusted and goes from 1989Q1 to 2014Q3.

Non-institutional population 16-64 was taken from StatsNZ website specifically the data series DPE059AA (Estimated resident population). This was a quarterly estimate which goes form 1991Q1- 2014Q3.

Then for New Zealand using all these detailed data series per-capita labour was calculated according to descriptions of Chari et al. (2006) on technical appendix, p.33-34, B.2.2. Only was able to calculate these estimates from 1991Q1 onwards for NZ and after 1978-10-01. The reason for this in NZ case is that population data was only available after that for NZ where as for Australian case hours worked data was only available after that.

### $x_t$ , **Per Capita-investment**

For New Zealand real gross private domestic investment+ real government gross investment were taken from the FRED series NAEXKP04NZQ189S (Gross Domestic Product by Expenditure in Constant Prices: Gross Fixed Capital Formation for New Zealand). The reason to take the sum of these is that it is seasonally adjusted quarterly estimates in new Zealand dollars. Whereas, the data series available on StatsNZ is not seasonally adjusted.

Same data series that were used to calculate per-capita output ( $y_t$ ) was taken as for PCE, PCE durables. So were the GST rates that were used to calculate per-capita output ( $y_t$ ).

Non-institutional population also was taken as to be same as that was use to calculate per-capita  $y_t$  and  $l_t$ .

For New Zealand using all these detailed data series per-capita investment was calculated according to descriptions of Chari et al. (2006) on technical appendix, p.33-34, B.2.2t. Only was able to calculate these estimates from 1991Q1 onwards for NZ and after 1960-01-01. The reason for this is that population data was only available after these time periods for New Zealand.

### $g_t$ , **Per Capita-government**

For New Zealand real government consumption data was taken from FRED, specifically the series NAEXKP03NZQ189S (Gross Domestic Product by Expenditure in Constant Prices: Government Final Consumption Expenditure for New Zealand). These were seasonally adjusted quarterly estimates which were available in New Zealand dollars.

Real net exports were calculated using difference between two data series from FRED, NAEXKP06NZQ652S (Gross Domestic Product by Expenditure in Constant Prices: Exports of Goods and Services for New Zealand) and NAEXKP07NZQ652S (Gross Domestic Product by Expenditure in Constant Prices: Less: Imports of Goods and Services for New Zealand). Both these data series were seasonally adjusted quarterly estimates which were also chained into 2000 New Zealand currency units.

Non-institutional population also was taken as to be same as that was use to calculate per-capita  $y_t$  and  $l_t$ .

For New Zealand using all these detailed data series per-capita government was calculated according to descriptions of Chari et al. (2006) on technical appendix, p.33-34, B.2.2. These estimates were calculated from 1991Q1 onwards for NZ. The reason for this is that population data was only available after these time periods for New Zealand.

All per-capita  $y_t$ ,  $l_t$ ,  $x_t$  and  $g_t$  were de-trended at the end. To do this first each per-capita  $y_t$ ,  $l_t$ ,  $x_t$  and  $g_t$  were turned into log values for New Zealand. Then the de-trended  $y_t$ ,  $l_t$ ,  $x_t$  and  $g_t$  were calculated as the difference between the logged  $y_t$ ,  $l_t$ ,  $x_t$  and  $g_t$  and the logged values of the mean of each per-capita  $y_t$ ,  $l_t$ ,  $x_t$  and  $g_t$ .

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