Anti-takeover Defenses and their Role in Investment

Graeme Guthrie*
Victoria University of Wellington

Cameron Hobbs
Victoria University of Wellington

June 11, 2017

Abstract

We show how a board of directors can set the strength of a firm's anti-takeover defenses in order to influence the investment-timing decision of a CEO who receives private control benefits from a completed project. The strength of the firm's defenses determines when a hostile takeover occurs and the CEO's control benefits cease. The prospect of a future takeover in turn affects the CEO's willingness to invest in a low-value project. If takeover defenses are too strong then the market for corporate control imposes insufficient discipline on the CEO, who invests too soon. If they are too weak, then shareholders incur too many costs due to managerial distraction. The optimal strength of a firm's defenses depends on factors including the disruption costs associated with takeover fights, the CEO's planning horizon, and the volatility of the value of the firm's assets under its current and alternative management teams.

Keywords: market for corporate control, manager–shareholder conflict, investment incentives, real options

1 Introduction

The market for corporate control disciplines the managers of firms with publicly traded shares (Manne, 1965; Kini et al., 2004). Boards can influence the effect that the market for corporate control has on managerial behavior by choosing the strength of their firms' anti-takeover defenses. Maintaining weak defenses increases managers' exposure to the discipline of the market for corporate control, which has the potential to reduce manager–shareholder conflict. However, managers do not necessarily respond to the threat of a future takeover in ways that are best

*Corresponding author: School of Economics and Finance, PO Box 600, Victoria University of Wellington, Wellington, New Zealand. Ph: 64-4-4635763. Email: graeme.guthrie@vuw.ac.nz

for shareholders. If weak defenses mean that managers are distracted from their managerial responsibilities fighting hostile takeovers then weak defenses can actually exacerbate manager–shareholder conflict. In this paper we examine the disciplinary role of the market for corporate control with regards to managerial empire building. We show how boards can use anti-takeover defenses to optimize the incentives created by the market for corporate control and we identify the economic determinants of optimal defenses. Shareholders can benefit if their firm's board partly insulates management from the threat of a hostile takeover.

We carry out our analysis using a model of a firm with a single investment opportunity and a CEO who determines the firm's investment policy. The CEO in our model chooses the timing of investment in order to maximize the present value of the flow of private control benefits that he receives. Because all of these control benefits are received by the CEO, but the investment is entirely funded by the firm's owners, the CEO will invest earlier than is optimal for shareholders. However, our model also incorporates a raider, which has the option to launch a hostile takeover bid for the firm's asset. We investigate how the prospect of this takeover bid motivates the CEO to delay investment and we evaluate the extent to which this delay benefits shareholders. The target firm's board is able to influence the CEO's response to the takeover threat, and hence the effect of the takeover threat on shareholders, by choosing the strength of the firm's anti-takeover defenses.

In our model, all parties involved in a hostile takeover battle can incur costs. Consistent with the empirical evidence that CEOs usually lose their jobs—and the flow of private control benefits that go with them—following a successful takeover, we assume that the CEO is fired immediately if the raider's takeover bid succeeds. Confronted with this prospect, the CEO will resist the raider's takeover bid in an attempt to prolong the flow of private control benefits, forcing the potential acquirer to go hostile. Fighting a hostile takeover attempt is costly for the target's shareholders if the takeover bid fails. For example, management is distracted, costs climb, growth opportunities are missed, some workers will leave due to the prolonged uncertainty, and others will stay and resist the takeover as "white squires" (Pagano and Volpin, 2005). Lastly, having to make a bid hostile is costly to the raider if the takeover succeeds, because it means that the target will deploy anti-takeover defenses that reduce the value of the asset to the raider. For example, the target might implement a scorched earth policy using generous long-term labor contracts or other means, insert poison puts into its bonds, and so on.[1]

The target's CEO begins to receive a flow of private control benefits as soon as investment occurs, but shareholders pay the full cost of investing. Therefore, in the absence of a market for corporate control, the CEO in our model invests too early for shareholders' liking. The threat of a future takeover delays the CEO's investment decision, but the extent of the delay depends on the costs associated with hostile takeovers. The CEO and the raider both base the timing of their decisions (investing and launching a takeover, respectively) on the ratio of the completed

[1]For example, see Cook and Easterwood (1994) and Pagano and Volpin (2005).

project's values when managed by the CEO and the raider. The CEO invests the first time that this ratio exceeds the threshold that maximizes the present value of his flow of control benefits, whereas the raider launches its takeover bid the first time that this ratio is less than the threshold that maximizes the present value of its takeover payoff. The "distraction costs" associated with a hostile takeover bid are factored into the price that the raider needs to pay to acquire the target. When these costs are high, the CEO sets a relatively tough threshold and the raider sets a relatively easy one; that is, high distraction costs delay investment and accelerate takeovers. In contrast, if anti-takeover defenses impose high costs on the raider after a successful takeover, then the CEO sets a relatively easy threshold and the raider sets a relatively tough one; that is, strong defenses accelerate investment and delay takeovers.

The target's board of directors can potentially use these relationships to benefit the firm's shareholders. We know that bargaining power shifts between a firm's board and its CEO over time, with the CEO's position becoming stronger following periods of strong financial performance (Hermalin and Weisbach, 1998; Guthrie, 2017). A board will typically be strongest at the time a new CEO is appointed, so one possible action that a board working in shareholders' best interests can take is to impose relatively weak anti-takeover defenses before the CEO becomes sufficiently powerful that he dominates the board. This will delay investment and—if the strength of the firm's defenses is chosen appropriately—better align the CEO's investment incentives with shareholders' interests.

We use our model to investigate the economic determinants of the strength of firms' value-maximizing anti-takeover defenses. The optimal defensive strength depends on the values the firm's project would have under the management of the CEO and the raider, where those values are measured immediately before the board effectively hands control of the target firm over to its CEO. If the asset's value under the CEO's management is sufficiently large, then shareholders actually want the CEO to invest immediately. As there is nothing to be gained from a future takeover in this situation—just future distraction costs to be incurred—the board will erect the strongest possible anti-takeover defenses. In other situations, shareholders want the CEO to delay investment, which the board will ensure by making the firm vulnerable to a future hostile takeover. However, the board will weigh the benefits of the discipline provided by the market for corporate control against the future distraction costs that will be incurred in the event of a hostile takeover bid eventuating. Introducing some basic defensive structures will weaken the discipline imposed on the CEO, but reduce the present value of the distraction costs. We find that the board will choose stronger defenses when the asset's value to the raider is greater (and the takeover threat is greater). The precise strength chosen by the board depends on the firm's circumstances: anti-takeover defenses should be stronger when the distraction costs associated with fighting a takeover are higher, the CEO has a longer planning horizon, and the completed project has a higher implicit dividend yield.

Of course, the board's ability to use the market for corporate control like this relies on it being able to largely "lock in" the strength of the firm's anti-takeover defenses. If it cannot do so, then a dominant CEO will simply strengthen the defenses once he has effective control of the firm, eliminating the discipline the board tried to impose. Fortunately for shareholders, many of the board's decisions regarding anti-takeover defenses will be difficult for the CEO to change. Boards can use the corporate charter (which shareholders cannot amend) or the firm's bylaws (which shareholders can amend, with difficulty) to influence how the firm will respond to future hostile takeovers. For example, the board can set the rules regarding the extent to which shareholder approval is needed to create and maintain poison pills (Guthrie, 2017, Chapter 13). If a future board can create and maintain a poison pill without shareholder approval, then the CEO's ability to fight off a hostile takeover will be significant, but the CEO will be much more vulnerable if shareholder approval is needed to maintain a poison pill introduced by a future board. Similarly, the board can choose to incorporate the firm in a state with relatively weak anti-takeover laws (Cain et al., 2017). It can enhance the ability of workers to act as "white squires" by creating employee stock ownership plans (Pagano and Volpin, 2005).

Our main contribution is to the literature on manager–shareholder conflict over investment policies, especially investment timing. Several conflicts have been examined in this literature. Grenadier and Wang (2005) examine investment timing when the investment decision is delegated to a manager who can exert effort to increase a project's payoff, but can also divert some of that payoff to himself. They show that the manager waits too long to invest, especially for relatively poor projects.[2] Subsequent authors have investigated various ways in which firms can accelerate investment. For example, Shibata (2009) show that allowing shareholders to audit the completed project and punish the manager for misreporting his private information accelerates investment, but not always by enough to offset the direct costs of an audit; Shibata and Nishihara (2010) show that debt financing accelerates investment. As well as waiting too long to invest, managers can also divest too late. Lambrecht and Myers (2007) show that the threat of a hostile takeover can accelerate divestment by managers who maximize the present value of the cash flows they can extract from the firm, subject to keeping payouts to investors at a level that allows managers to maintain control; Lambrecht and Myers (2008) show that debt financing accelerates divestment. In contrast to these papers, our model examines managers' incentives to invest in a search for personal benefits of control that are ultimately funded by shareholders' capital—that is, empire building.

Our second contribution is to the real-options literature on mergers. We focus on hostile takeovers and show how the need to overcome a free-rider problem constrains the timing and terms chosen by the raider. In contrast, the existing literature concentrates on mergers where both firms agree on the timing and terms of the merger. Lambrecht (2004) pioneered this approach, interpreting a merger as a perpetual option to exploit economies of scale by combining

[2] Hori and Osano (2014) examine investment-timing decisions made by a self-interested manager and compare the ability of restricted stock and stock options to reduce manager–shareholder conflict over investment timing.

two firms' operations and sharing ownership of the new entity. The merging firms choose the post-merger ownership allocation that results in them both choosing to exercise the merger option at the same time.[3] This approach has been used to investigate mergers with competing bidders (Morellec and Zhdanov, 2005, 2008), mergers that alter oligopolistic industry structures (Hackbarth and Miao, 2012), and the behavior of stock returns during merger episodes (Hackbarth and Morellec, 2008). Other papers in this literature examine takeovers motivated by diversification (Thijssen, 2008), divestment opportunities (Alvarez and Stenbacka, 2006; Lambrecht and Myers, 2007), increasing market power (Bernile et al., 2012), and slow internal growth (Margsiri et al., 2008).

The rest of the paper is organized as follows. We set up our model in Section 2 and derive the optimal timing and terms of the conditional tender offer that the raider uses to take control of the asset in Section 3. Next, in Section 4, we derive the investment policy chosen by the target firm's CEO, before deriving the strength of the target board's preferred anti-takeover defenses in Section 5. We use numerical analysis to investigate the economic determinants of the strength of firms' anti-takeover defenses in Section 6 and conclude the paper in Section 7.

2 Model setup

A firm owns a perpetual option to invest in an asset that is of value to a raider. In turn, the raider has a perpetual option to attempt a hostile takeover of the firm after the firm has invested in the asset. Time is continuous, but events occur at three separate dates.

Step 1 At date $t_0 = 0$, the target firm's board of directors fixes the strength of its anti-takeover defenses.

Step 2 At a date $t_1 \geq t_0$ chosen by the target firm's CEO, the firm invests in the project.

Step 3 At a date $t_2 \geq t_1$ chosen by the raider, the raider attempts a hostile takeover of the target firm.[4]

Investment is instantaneous, irreversible, and requires capital expenditure of I, which is funded entirely by shareholders. As soon as investment occurs, the asset begins to generate a continuous cash flow of $\delta_x x$ that is paid out to shareholders, where δ_x is a constant and x is stochastic, with risk-neutral process

$$dx = (r - \delta_x)xdt + \sigma_x x d\xi.$$

[3] Lambrecht (2004) also modifies his basic model so that the target sets the post-merger ownership shares and the raider then chooses the timing of the merger. This approach, which Lambrecht interprets as a hostile takeover, is different from the one developed in our paper.

[4] The raider is not allowed to launch a takeover bid until the CEO has invested. This restriction might be due to the raider being unable to evaluate the value in launching a takeover until the asset is actually in place.

Here r is the (constant) risk-free interest rate, σ_x is a constant, and ξ is a Wiener process. The cash flow continues as long as the asset is owned by the firm. After a change in control, the asset can potentially generate a perpetual continuous cash flow of $\delta_y y$, where δ_y is a constant and y is stochastic, with risk-neutral process

$$dy = (r - \delta_y)ydt + \sigma_y y d\zeta.$$

Here σ_y is a constant and ζ is a Weiner process. Note that a perpetual cash flow of $\delta_x x$ has a present value of x and one of $\delta_y y$ has a present value of y. This allows us to interpret x as the value of the asset to the target firm's shareholders if a takeover is impossible and y as the potential value of the asset following a successful takeover attempt. We allow the two asset values to be correlated by assuming that $(d\xi)(d\zeta) = \rho dt$, for some constant ρ.[5]

The hostile takeover bid takes the form of a conditional tender offer. We assume that the target's ownership is diffuse, the raider can attempt a takeover at most once, a successful takeover is irreversible, and the hostile takeover affects the value of the asset in two ways. Firstly, fighting a hostile takeover attempt is costly for the target firm even if the takeover bid fails. For example, management is distracted, costs climb, growth opportunities are missed, worker turnover increases due to the prolonged uncertainty, and so on. We model these possibilities by assuming that if the takeover fails then the asset will be worth $(1 - \phi)x$ to the firm's shareholders, rather than x, which is the asset's value if a takeover is impossible. Secondly, as part of the board's defensive strategy, it reduces the value of the asset to the raider in the event that the tender offer succeeds. For example, the board might implement a scorched earth policy or insert poison puts into its bonds. We model such possibilities by assuming that the asset is worth $(1 - \gamma)y$ if it is controlled by the raider. The value of γ is determined by the strength of takeover defences chosen by the target's board at date t_0. Making these defenses stronger will increase γ.

We also allow for the separation of ownership and control. The variables x and y determine the cash flows that go to the asset's owners. There are also private benefits that flow to the party that controls the asset, which do not have to be shared with the asset's owners. After investment and before the takeover, the target's CEO receives a continuous stream of private benefits at the rate of $\omega_C \delta_x x$ per unit of time as long as he is CEO, for some constant ω_C; after the takeover, the raider receives a stream of private benefits at the rate $\omega_R \delta_y (1 - \gamma)y$, for some constant ω_R. The different coefficients (ω_C and ω_R) reflect the possibility that, for example, the raider can exploit synergies not available without a change in control. Finally, we allow for the possibility that the manager is more impatient than shareholders. Specifically, the manager leaves the firm at a date determined by a Poisson process with constant exogenous intensity θ.[6]

[5] Morellec and Zhdanov (2005) and Thijssen (2008) analyze the real options involved in the market for corporate control using models that also assume that the merging firms have correlated (but not perfectly correlated) cash flows.

[6] This approach is also used by Grenadier and Wang (2005) in their real-options model of manager–shareholder conflict.

If the CEO leaves before the takeover, then he is replaced by an identical CEO, who receives the same control benefits.

When we solve the model we will use the following functions of the model's parameters:

$$\beta_1 = \frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2} + \sqrt{\frac{2(\delta_x + \theta)}{\psi^2} + \left(\frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2}\right)^2},$$

$$\beta_2 = \frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2} - \sqrt{\frac{2(\delta_x + \theta)}{\psi^2} + \left(\frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2}\right)^2},$$

$$\beta_3 = \frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2} + \sqrt{\frac{2\delta_x}{\psi^2} + \left(\frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2}\right)^2},$$

$$\beta_4 = \frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2} - \sqrt{\frac{2\delta_x}{\psi^2} + \left(\frac{1}{2} - \frac{\delta_x - \delta_y}{\psi^2}\right)^2},$$

$$\beta_5 = \frac{1}{2} - \frac{\delta_x - \delta_y + \sigma_x(\sigma_x - \rho\sigma_y)}{\psi^2} - \sqrt{\frac{2r}{\psi^2} + \left(\frac{1}{2} - \frac{\delta_x - \delta_y + \sigma_x(\sigma_x - \rho\sigma_y)}{\psi^2}\right)^2},$$

where

$$\psi^2 = \sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2. \tag{1}$$

We derive equilibrium investment and takeover policies in the next three sections, along with the present values of the payoffs flowing to the raider, the CEO, and shareholders. We illustrate our results with numerical examples using the following parameter values: $r = 0.05$, $\delta_x = \delta_y = 0.03$, $\sigma_x = \sigma_y = 0.2$, $\rho = 0.5$, $\theta = 0.05$, $\omega_C = \omega_R = 0.1$, $\phi = 0.1$, and $I = 1$. In some of the examples, we assume that $\gamma = 0.1$, but in others we allow this parameter to vary.

3 The raider's choice of takeover terms and timing

In Step 3, the raider chooses the timing of its takeover bid and the price it offers to pay the target firm's shareholders. In this section we calculate the raider's payoff from exercising the takeover option, and then use this payoff to derive the raider's optimal takeover policy.

Suppose the asset values are x and y when the raider launches its takeover attempt. We analyze the resulting conditional tender offer using the approach in Grossman and Hart (1980). Let p denote the tender price chosen by the raider. If $p \geq (1-\gamma)y$ then it is a weakly dominant strategy for an individual shareholder to tender their share rather than reject the raider's offer.[7] That is, "all-tender" is a Nash equilibrium; if shareholders coordinate on this equilibrium then the takeover succeeds and they receive a lump sum payout of p in exchange for their shares. However, "all-hold" is also a Nash equilibrium, because if nobody else is tendering their shares then there is no point in an individual shareholder tendering their share. In this case, if shareholders

[7] If the tender offer fails, then the target's shares are worth $(1-\phi)x$ whether the individual shareholder tenders their shares or not. If it succeeds, then they are worth p if the shareholder tenders their share and $(1-\gamma)y$ otherwise.

coordinate on this equilibrium then the takeover fails and they are left holding shares that—due to the costs involved in fighting the hostile takeover—are now worth just $(1-\phi)x$. There are thus two symmetric Nash equilibria in pure strategies: all-tender, in which shareholders receive a lump-sum payout of p; and all-hold, in which they continue to hold shares worth $(1-\phi)x$. We assume that shareholders coordinate around the Pareto-dominant equilibrium. This implies that the tender offer fails if $p < (1-\phi)x$, because then shareholders will coordinate on all-hold. The tender offer succeeds if $p \geq (1-\phi)x$.

The raider's payoff is maximized by setting p as low as possible, subject to the constraints that it must be high enough for all-tender to be a Nash equilibrium (that is, $p \geq (1-\gamma)y$) and to induce shareholders to coordinate on the all-tender equilibrium (that is, $p \geq (1-\phi)x$). The raider therefore offers to buy the target's shares for $p = \max\{(1-\gamma)y, (1-\phi)x\}$. If $(1-\gamma)y \geq (1-\phi)x$ then the raider pays $p = (1-\gamma)y$, the asset's market value under new management. The raider's only benefit from the acquisition is the flow of control benefits, which starts immediately after the takeover is completed and has present value $\omega_R(1-\gamma)y$. In contrast, if $(1-\gamma)y < (1-\phi)x$ then the raider pays $p = (1-\phi)x$, which exceeds the asset's market value under new management and partly offsets the benefit to the raider of the flow of control benefits. Overall, the raider's takeover payoff equals $(1+\omega_R)(1-\gamma)y - \max\{(1-\gamma)y, (1-\phi)x\}$. It times its takeover bid in order to maximize the present value of this payoff. The raider's optimal takeover policy is described in the following proposition.[8]

Proposition 1. *If the raider launches a hostile takeover the first time that the asset values satisfy $y/x \geq \hat{z}$, where $\hat{z}$ is an arbitrary constant, then the present value of the raider's takeover payoff equals*

$$V(x, y; \hat{z}) = \begin{cases} x\left((1+\omega_R)(1-\gamma)\hat{z} - \max\{(1-\gamma)\hat{z}, 1-\phi\}\right)\left(\frac{y/x}{\hat{z}}\right)^{\beta_3}, & \text{if } y/x < \hat{z}, \\ (1+\omega_R)(1-\gamma)y - \max\{(1-\gamma)y, (1-\phi)x\}, & \text{if } y/x \geq \hat{z}. \end{cases}$$

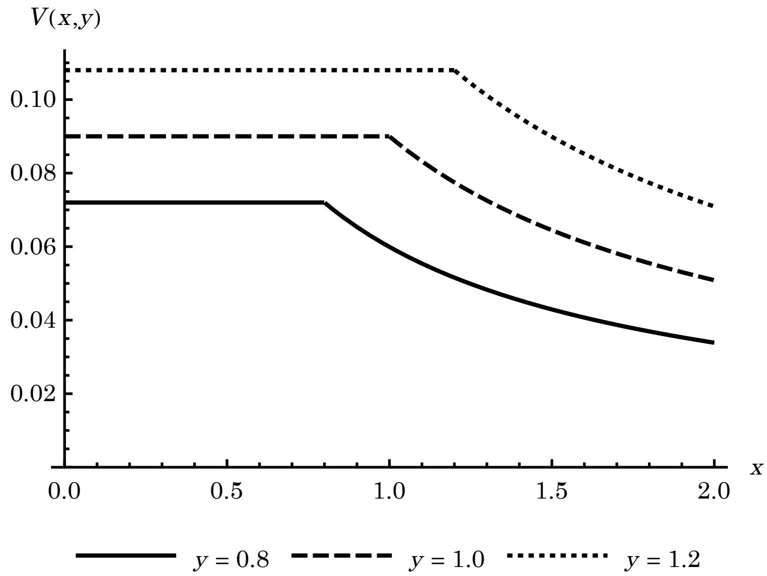
This present value is maximized if the raider chooses $\hat{z} = \hat{z}_R^*$, where

$$\hat{z}_R^* = \left(\frac{1-\phi}{1-\gamma}\right) \min\left\{1, \frac{\beta_3}{\beta_3 - 1} \cdot \frac{1}{1+\omega_R}\right\}.$$

Note that if $\omega_R > 1/(\beta_3 - 1)$ then $\hat{z}_R^* < (1-\phi)/(1-\gamma)$, so that when the raider launches the takeover the asset values satisfy $(1-\gamma)y < (1-\phi)x$. In this case the raider pays a price that exceeds the value of the asset under new management. That is, when the control benefits are sufficiently large, the raider will choose to pay the target firm's shareholders an amount for their shares that is strictly greater than the shares would be worth to them under free-riding. The raider is willing to do this—effectively "overpaying" for the target—in order to start receiving the flow of control benefits earlier. However, it will do so only if the control benefits are sufficiently large, otherwise it will wait and pay a "fair" price for the target firm.

[8] All proofs are contained in the appendix.

Figure 1: Present value of the raider's takeover payoff



Higher values of γ lead to a lower post-takeover asset value and less valuable private benefits of control to the raider, reducing the takeover payoff and inducing the raider to wait longer before launching a takeover. As the takeover occurs when y/x is greater than the threshold $\hat{z}_R^*$, the takeover is delayed by increasing the threshold. Higher values of ϕ will affect the takeover payoff only if the raider "overpays" for the target, when they will lower the purchase price and raise the takeover payoff, inducing the raider to launch a takeover earlier. This is achieved by lowering the takeover threshold. Thus, consistent with Proposition 1, $\hat{z}_R^*$ is increasing in γ and decreasing in ϕ.

Figure 1 plots the present value of the raider's takeover payoff, $V(x, y)$, as a function of x, for the three indicated values of y. All parameter values are given at the end of Section 2 and imply that $\hat{z}_R^* = 1$. The left-hand portion of each curve shows the raider's payoff from an immediate takeover, which, for the parameters adopted here, equals $\omega_R(1 - \gamma)y$. The right-hand portion shows the present value of the takeover payoff, given that the takeover will not occur until y/x climbs above $\hat{z}_R^*$. For high values of x, the raider will have to wait a long time to receive the payoff, so the present value is small. For values of x close to the takeover threshold, a takeover is imminent, so the present value is only slight less than the takeover payoff. All else equal, higher values of y lead to the earlier receipt of a larger takeover payoff, so that the present value of the payoff is increasing in y, as shown in Figure 1.

4 The CEO's choice of investment threshold

In Step 2 the CEO decides when the firm will invest in the project. The CEO chooses an investment policy that maximizes the present value of the flow of control benefits that he receives after the firm invests. Before deriving an optimal investment policy, we must therefore calculate the present value of the control benefits, measured after investment.

During the period after the firm invests and before the CEO leaves the firm, the CEO receives a continuous flow of control benefits at the rate $\omega_C \delta_x x$ per unit of time. As soon as the CEO leaves the firm, for whatever reason, this flow terminates. The following lemma gives the present value of this flow of benefits.

Lemma 1. *The present value of the CEO's flow of control benefits, measured after investment has occurred, equals*

$$G(x,y) = \begin{cases} \frac{\omega_C \delta_x x}{\delta_x + \theta} \left(1 - \left(\frac{y/x}{\hat{z}_R^*} \right)^{\beta_1} \right), & \text{if } y/x < \hat{z}_R^*, \\ 0, & \text{if } y/x \geq \hat{z}_R^*, \end{cases} \tag{2}$$

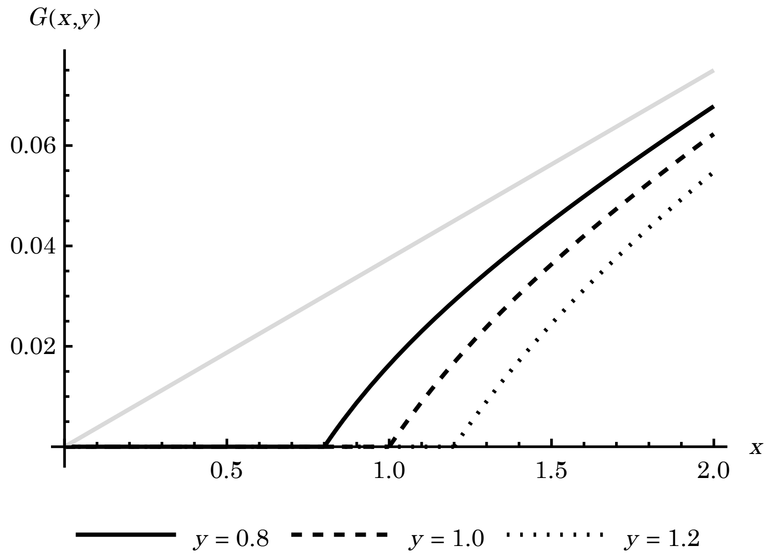
where the takeover threshold $\hat{z}_R^*$ is given in Proposition 1.

Consider the limiting case where y is very small relative to x, so that the probability of a takeover is extremely small. Equation (2) shows that in this case, the CEO values the flow of control benefits at $G(x,y) \approx \omega_C \delta_x x/(\delta_x + \theta)$. If, in addition, there is no possibility that the CEO will leave the firm for other reasons (that is, $\theta = 0$), then the CEO values the control benefits at $\omega_C x$. In the general case, this valuation is adjusted downwards due to the possibility that the flow of control benefits is terminated by the CEO leaving the firm, either before a takeover or as the result of an unsolicited takeover offer.

Continuing our numerical example, Figure 2 shows the present value of the CEO's flow of control benefits, measured after investment and before the takeover, for the same three values of y as in Figure 1. In all cases, when x is large, the prospect of a takeover is remote, so the present value approaches the no-takeover value of $\omega_C \delta_x x/(\delta_x + \theta)$, which corresponds to the light-gray straight line in the graph. When $x < y/\hat{z}_R^*$, which for this particular case means that $x < y$, the hostile takeover occurs immediately, so that the present value is zero. Lastly, higher values of y mean that the takeover threat is greater, so that the CEO's flow of control benefits will terminate sooner: the present value of this flow is lower for higher values of y.

The CEO chooses an investment policy that maximizes the present value of the flow of control benefits. If the CEO waits too long before investing, then he forgoes some of the flow of control benefits. However, investing too soon increases the possibility that the raider will launch a takeover bid that will terminate the CEO's flow of control benefits. A policy that is optimal from the CEO's point of view weighs the forgone control benefits against the threat of a hostile takeover. The following lemma shows that we can restrict attention to a particularly simple

Figure 2: Present value of the CEO's flow of control benefits, measured after investment



family of investment policies, safe in the knowledge that the best policy from this family is at least as good for the CEO as any policy from outside the family.

Lemma 2. *Consider the family of investment rules: invest in the project the first time that* $y/x \leq \hat{z}$ *for an arbitrary positive constant* $\hat{z}$*. The present value of the flow of control benefits is maximized for some member of this family.*

We can use Lemma 2 to restrict attention to a family of simple investment policies without losing any generality. Specifically, we restrict attention to investment policies of the form: invest the first time that $y/x \leq \hat{z}$, for some positive constant $\hat{z}$ to be determined. The following lemma values the CEO's investment option for an arbitrary investment policy from this family.

Lemma 3. *If the CEO launches the project the first time that* $y/x \leq \hat{z}$*, for an arbitrary positive constant* $\hat{z}$*, then the present value of the flow of control benefits, measured before investment, equals*

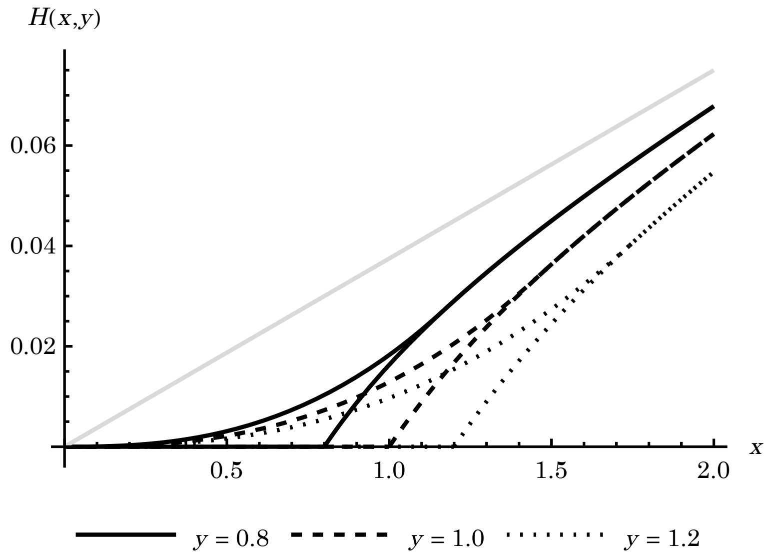
$$H(x, y; \hat{z}) = \begin{cases} G(x, y), & \text{if } y/x \leq \hat{z}, \\ xG(1, \hat{z}) \left(\frac{y/x}{\hat{z}} \right)^{\beta_2}, & \text{if } y/x > \hat{z}, \end{cases} \tag{3}$$

where the function $G()$ *is given in Lemma 1.*

The CEO chooses the investment threshold that maximizes the value he derives from the firm's investment option. From equation (3), the CEO chooses $\hat{z}$ in order to maximize

$$G(1, \hat{z})\hat{z}^{-\beta_2} = \frac{\omega_C \delta_x}{\delta_x + \theta} \left(1 - \left(\frac{\hat{z}}{\hat{z}_R^*} \right)^{\beta_1} \right) \hat{z}^{-\beta_2},$$

Figure 3: Present value of the CEO's flow of control benefits, measured before investment



provided he chooses an investment threshold that does not induce the raider to immediately launch a hostile takeover. Solving the associated first-order condition gives the CEO's optimal investment threshold.

Proposition 2. *If the raider launches a hostile takeover the first time that* $y/x \geq \hat{z}_R^*$*, then the CEO can maximize the value of his investment option by investing the first time that* $y/x \leq \hat{z}_C^*$*, where*

$$\hat{z}_C^* = \hat{z}_R^* \left(\frac{-\beta_2}{\beta_1 - \beta_2} \right)^{1/\beta_1}$$

and the takeover threshold $\hat{z}_R^*$ *is given in Proposition 1.*

Figure 3 continues our numerical example by plotting the present value of the CEO's flow of control benefits, measured before investment, assuming that the firm invests using the policy that maximizes this present value. Note that in this case $\hat{z}_C^* = 0.6845$, so that the CEO's investment thresholds for the three indicated levels of y are $x = 1.17$, $x = 1.46$, and $x = 1.75$, respectively. The CEO invests for values of x above these thresholds, so in this region the graph of $H(x, y)$ coincides with that of $G(x, y)$ in Figure 2. For smaller values of x, the CEO's value of waiting exceeds his investment payoff, so in this region the graph of $H(x, y)$ is above that of $G(x, y)$. As the CEO chooses the investment threshold optimally, the two curves are smoothly pasted at the investment threshold. Figure 3 exhibits the behavior we would expect: higher values of y mean a greater takeover threat, causing the CEO to choose a higher investment threshold and delay investment.

Note that as $\beta_1 > 0$ and $\beta_2 < 0$, the investment threshold, $\hat{z}_C^*$, is less than the takeover threshold, $\hat{z}_R^*$: the CEO will never invest if a takeover would occur immediately after investment.

The relationship between the two thresholds is such that the investment threshold is an increasing function of γ and a decreasing function of ϕ. Thus, changes in anti-takeover defenses that increase γ delay takeovers and accelerate investment.

In the absence of the market for corporate control, the CEO will always invest immediately as this maximizes the present value of the flow of control benefits—and the firm's shareholders bear all the cost of initiating this flow. The threat of a hostile takeover will lead the CEO to delay investment (unless $y_0/x_0 \leq \hat{z}_C^*$, where x_0 and y_0 denote the asset values at date 0), but the resulting investment policy will not typically be optimal from shareholders' point of view. The CEO will still undertake a bad investment (small x) if there is little prospect that any other firm will want to take control of that asset away from the CEO (small y/x). In contrast, the CEO will delay a good investment (large x) if a takeover is likely to occur soon afterwards (large y/x), depriving shareholders of a large positive-NPV investment opportunity. In order to evaluate these possibilities, we need to determine the effect of the CEO's investment policy on the value of the firm's shares, which is the purpose of the next section.

5 The board's choice of anti-takeover defenses

In Step 1 the board chooses the strength of the firm's anti-takeover defenses in order to maximize the current value of the firm's shares. In order to calculate the relationship between the strength of anti-takeover defenses and the value of the firm's shares prior to investment, we must first calculate their value *after* investment. During the period after the firm invests and before the takeover, shareholders receive a continuous cash flow at the rate $\delta_x x$ per unit of time, reflecting the project's implicit dividend yield δ_x. This cash flow terminates when the takeover occurs and shareholders receive a lump sum of $p = \max\{(1-\gamma)y, (1-\phi)x\}$. The following lemma gives the present value of this cash flow stream.

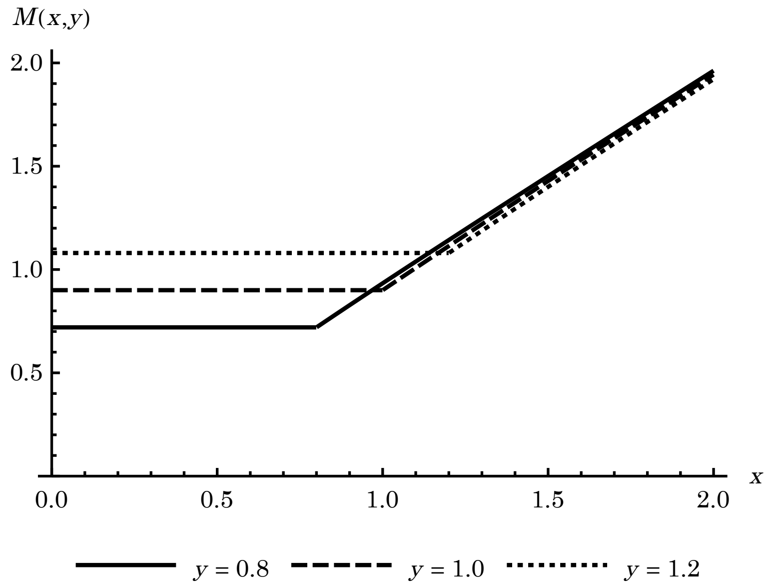
Lemma 4. *The firm's shares are worth*

$$M(x,y) = \begin{cases} x\left(1 - \phi\left(\frac{y/x}{\hat{z}_R^*}\right)^{\beta_3}\right), & \text{if } y/x < \hat{z}_R^*, \\ \max\{(1-\gamma)y, (1-\phi)x\}, & \text{if } y/x \geq \hat{z}_R^*, \end{cases} \tag{4}$$

to its shareholders after the CEO invests and before the takeover.

By definition, the asset is worth x if a takeover is impossible. Equation (4) shows that its value approaches this level when y/x is so small that the wait for a takeover will be long. However, as y/x approaches the takeover threshold $\hat{z}_R^*$, the asset's value approaches $(1-\phi)x$. The target firm's shareholders therefore bear the full distraction cost incurred in fighting the takeover, even though the takeover actually succeeds. Note that the value of the asset at the time of the takeover does not depend on its value in the alternative use, but y *does* affect the

Figure 4: Share value after investment



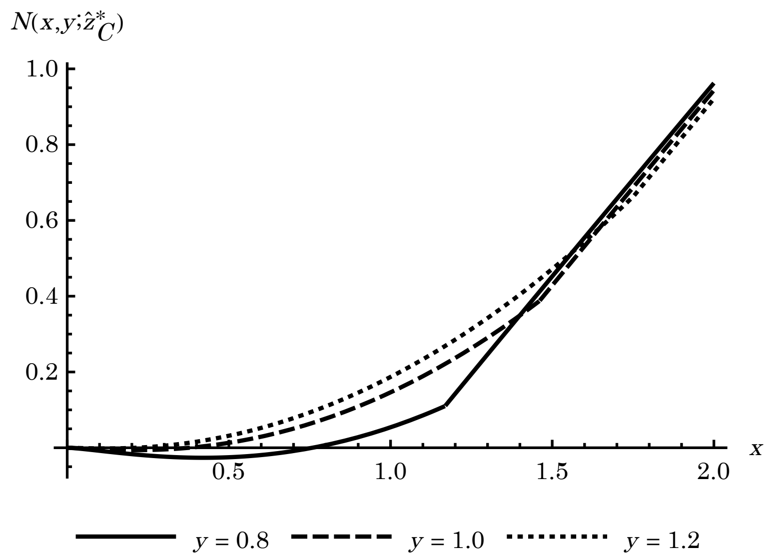
timing of the takeover.[9] Higher values of y accelerate the takeover, increasing the present value of the future distraction costs and thereby lowering the value of the firm's shares. This behavior is evident in Figure 4, which plots the value of the firm's shares during the period between the CEO investing and the takeover occurring. The horizontal part of each curve shows the value of the firm's shares in the region where a takeover occurs immediately. The upward sloping curves show the share value in the region where a takeover is delayed. In this region, higher values of y (meaning that a takeover will occur sooner) are associated with a lower share value.

Equation (4) and Figure 4 show that once the CEO has invested, the prospect of a future takeover is bad for the shareholders in our model. This happens because shareholders bear the cost of defending a takeover even though the takeover succeeds. From Section 3, these costs lower the amount that the raider has to pay in order for "all-tender" to Pareto dominate "all-hold" in the game played by shareholders faced with a tender offer for their shares. This reduces the amount that the raider has to pay for the shares, which means that the asset does not need to be as valuable in its alternative use for the takeover to go ahead and for the raider to start receiving the flow of control benefits earlier. This explains why a future takeover is bad for shareholders ex post. However, the prospect of a takeover can have benefits ex ante by affecting the CEO's choice of investment policy, as we now see.

Knowledge of the post-investment value of the firm's shares allows us to calculate their pre-investment value for an arbitrary member of the family of investment policies from which the CEO will choose. Consider the policy whereby the CEO chooses to invest the first time that

[9] The exception is the special case in which the takeover occurs immediately. For values of y/x above the takeover threshold $\hat{z}^*_R$, the asset is worth $\max\{(1-\gamma)y, (1-\phi)x\}$.

Figure 5: Share value before investment



$y/x \leq \hat{z}$, for some arbitrary constant $\hat{z}$. Prior to investment, the firm's shareholders effectively own a contingent claim that pays out the lump sum $M(x, y) - I$ the first time that $y/x \leq \hat{z}$. The following lemma gives the market value of this claim.

Lemma 5. *If the CEO launches the project the first time that $y/x \leq \hat{z}$, for an arbitrary positive constant $\hat{z}$, then the firm's shares are worth*

$$N(x, y; \hat{z}) = \begin{cases} M(x, y) - I, & \text{if } y/x \leq \hat{z}, \\ xM(1, \hat{z}) \left(\frac{y/x}{\hat{z}}\right)^{\beta_4} - I \left(\frac{y/x}{\hat{z}}\right)^{\beta_5}, & \text{if } y/x > \hat{z}, \end{cases} \tag{5}$$

to its shareholders before the CEO invests.

Figure 5 plots the pre-investment market value of the firm's shares for our baseline example, assuming that the CEO invests using his optimal policy, given in Proposition 2. Recall from Figure 3 that the CEO's investment thresholds for the three indicated levels of y are $x = 1.17$, $x = 1.46$, and $x = 1.75$. The CEO invests immediately if x is above the threshold corresponding to the current value of y, so in these regions the graph of $N(x, y; \hat{z}_C^*)$ equals shareholders' investment payoff $M(x, y) - I$. For smaller values of x, the graph shows shareholders' value of waiting. As it is the CEO, and not shareholders, who chooses the investment policy, the two curves are not smoothly pasted at the investment threshold. When the takeover threat is strong because y is large (that is, the dotted curve), the CEO chooses a high investment threshold, delaying investment and increasing the value of the firm's shares. In contrast, when the takeover threat is weak because y is small (the solid curve), the share value is low. Here the CEO invests early because there is little chance of a hostile takeover ending the CEO's tenure.[10]

[10] In this case, the share value actually become negative. In part, this reflects the fact that the discount rate

The two parameters that affect the outcome of a takeover, ϕ and γ, both affect the firm's pre-investment share value. High distraction costs ϕ affect the pre-investment share value in two ways. Firstly, avoided distraction costs are factored into the price that the raider needs to pay in a successful takeover so, as Lemma 4 shows, high values of ϕ lower the value of shares during the post-investment period. Secondly, by lowering the price the raider needs to pay, higher distraction costs increase the takeover threat (Proposition 1), causing the CEO to set a more demanding investment threshold (Proposition 2), which will have an additional effect on the pre-investment share value. However, weak defenses imply a low value of γ, which makes the control benefits the raider can extract from the asset more valuable. This also increases the takeover threat and causes the CEO to set a more demanding investment threshold.

In our model, date t_0 is the board's final opportunity to control the firm's decisions; after that date, the CEO controls the firm's decision making. However, the board can influence the CEO's decision-making by establishing the firm's future anti-takeover defenses at date t_0. We examine this issue further by assuming that the board chooses γ at date t_0 in order to maximize the firm's pre-investment share value.[11] In order to evaluate the effect of γ on the firm's pre-investment share value in more detail, we evaluate equation (5) at date 0, assuming that the CEO uses the investment threshold in Proposition 2. Using the solution for $M(x,y)$ in Lemma 4, we can write

$$N(x_0, y_0; \hat{z}_C^*) = \begin{cases} x_0 \left(1 - \phi \left(\frac{-\beta_2}{\beta_1 - \beta_2}\right)^{\beta_3/\beta_1} \left(\frac{y_0/x_0}{\hat{z}_C^*}\right)^{\beta_3}\right) - I, & \text{if } y_0/x_0 \le \hat{z}_C^*, \\ x_0 \left(1 - \phi \left(\frac{-\beta_2}{\beta_1 - \beta_2}\right)^{\beta_3/\beta_1}\right) \left(\frac{y_0/x_0}{\hat{z}_C^*}\right)^{\beta_4} - I \left(\frac{y_0/x_0}{\hat{z}_C^*}\right)^{\beta_5}, & \text{if } y_0/x_0 > \hat{z}_C^*, \end{cases}$$

where

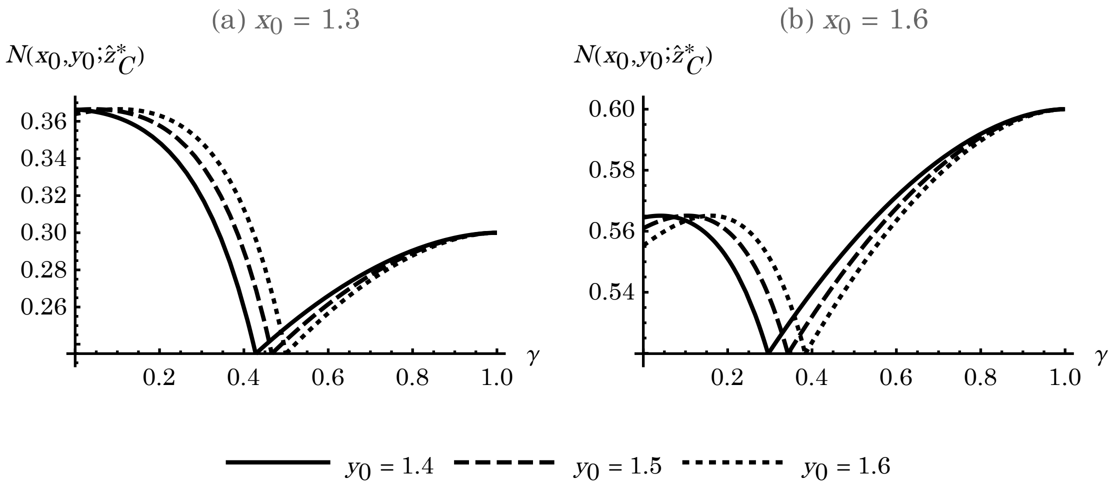
$$\hat{z}_C^* = \hat{z}_R^* \left(\frac{-\beta_2}{\beta_1 - \beta_2}\right)^{1/\beta_1} = \left(\frac{1-\phi}{1-\gamma}\right) \min\left\{1, \frac{\beta_3}{\beta_3 - 1} \cdot \frac{1}{1+\omega_R}\right\} \left(\frac{-\beta_2}{\beta_1 - \beta_2}\right)^{1/\beta_1}.$$

The two graphs in Figure 6 plot $N(x_0, y_0; \hat{z}_C^*)$ as a function of γ, for the indicated combinations of (x_0, y_0). In each of the six cases considered, there are two distinct regions. For low values of γ (weak anti-takeover defenses), the initial share value is a concave function of γ. These are values of γ which induce the CEO to delay investment. When the takeover threat is initially low (that is, y_0 is small), the local minimum of the share value is achieved by setting $\gamma = 0$, but if the threat is initially strong enough then it is locally optimal to introduce some anti-takeover defenses. In contrast, for high values of γ, the initial share value is an increasing function of γ. In this region, the defenses are sufficiently strong that the CEO will choose to invest immediately. In this case, the board will actually choose to make the firm invulnerable to takeover: setting $\gamma = 1$ has no effect on investment timing, but eliminates the possibility of future distraction costs if the raider launches a hostile takeover bid. Thus, the board's objective

applied to future profits reflects the risks associated with the asset's cash flow and the timing of investment, whereas the discount rate applied to future capital expenditure reflects only invest-timing risk.

[11] We also assume that the level of distraction costs, ϕ, is independent of the board's choice of γ.

Figure 6: Pre-investment share value as a function of γ



function has two local maxima. The left-hand graph in Figure 6 shows that when the asset value is initially relatively low the board will choose anti-takeover defenses that induce the CEO to delay investment; the right-hand graph shows that when the asset value is relatively high the board will fully protect the CEO from a hostile takeover.

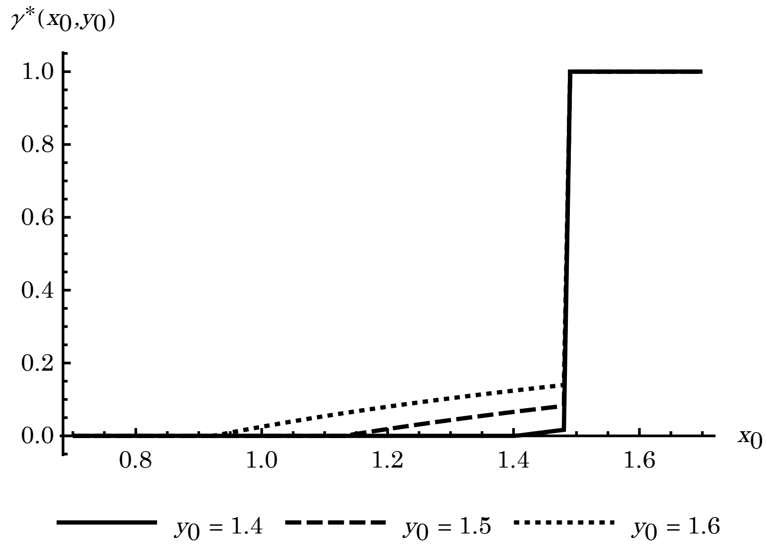
Figure 7 plots the board's optimal choice of γ as a function of x_0, for the indicated levels of y_0. Consistent with the objective functions in Figure 6, for each value of y_0, the board will choose to put no anti-takeover defenses in place if the asset value is sufficiently small and impregnable defenses in place if the asset value is sufficiently large. Between these two extremes, it will choose defensive settings such that $0 < \gamma^* < 1$. In this intermediate region, the board's choice of defensive strength is an increasing function of the asset's value. In all cases, there is a discontinuity in γ^* when it jumps to 1. Immediately to the left of this point in the graph, the board chooses defenses that are strong enough that the CEO will invest *almost* immediately. A slight increase in x_0 makes the board want to induce immediate investment, in which case it is optimal to make the firm's defenses complete (that is, choose $\gamma^* = 1$).

6 Sensitivity analysis

In this section we investigate the relationship between the model's exogenous parameters and shareholders' optimal takeover defensive settings.[12] Table 1 summarizes our results for the baseline case. A board that works in shareholders' best interests will observe x_0 and y_0, and then lock in the optimal defensive settings reported in the top panel. The board's choice of γ depends on

[12]We do not report results for ω_C and ω_R as they do not affect the board's choice of γ. In the case of ω_R this is because it requires extremely high values of the parameter to affect the raider's takeover threshold $\hat{z}_R^*$, given in Proposition 1.

Figure 7: Share-value maximizing level of γ



the state of nature at the time the board effectively hands control of the firm over to the CEO. The CEO will observe the board's choose of anti-takeover defenses and choose the investment policy given in the bottom panel of the table. Note that the CEO will invest the first time that $y/x \leq \hat{z}_C^*$, or, equivalently, the first time that $x/y \geq 1/\hat{z}_C^*$. We report the latter threshold in the table for ease of interpretation.

This shows three distinct regions. When x_0 is large, shareholders would like the CEO to invest immediately. In this region, shown by the dark gray cells in the table, the board should therefore set the strongest possible defenses (that is, set $\gamma = 1$). This eliminates the possibility of a future takeover and induces the CEO to invest immediately (that is, set $1/\hat{z}_C^* = 0$). In all other cases, shareholders would like the CEO to delay investment. The unshaded regions of the two panels show that if the board puts no anti-takeover defenses in place, then the CEO will invest only once $x \geq 1.623y$. In situations when the asset is highly valuable in the alternative use (that is, y is large), this will lead to long delays in investment. The board responds by introducing moderately strong anti-takeover defenses, shown by the light gray regions of the two panels. These defenses will be stronger when x_0 is larger (because excessive investment delays are more costly) and y_0 is larger (because the takeover threat is greater).

Table 2 repeats the format of Table 1 for the case where $\phi = 0.2$; that is, the cost to shareholders of resisting a hostile takeover are higher than in the baseline case. We hold all other parameters at their baseline levels. Recall from Proposition 1 that higher disruption costs lower the price that the raider needs to pay to takeover the firm, which accelerates the takeover. Proposition 2 shows that the CEO responds to the increased takeover threat by adopting a tougher investment threshold, delaying investment. The unshaded cells in the bottom panel

Table 1: Board and CEO policies for the baseline case

$y_0 \backslash x_0$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
	Optimal strength of anti-takeover defenses (γ^*)										
0.9	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000
1.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.5	0.000	0.000	0.000	0.000	0.000	0.019	0.043	0.066	1.000	1.000	1.000
1.6	0.000	0.000	0.000	0.025	0.054	0.080	0.103	0.124	1.000	1.000	1.000
1.7	0.000	0.015	0.051	0.082	0.110	0.134	0.156	0.176	1.000	1.000	1.000
	Minimum level of x/y needed to invest ($1/\hat{z}_C^*$)										
0.9	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000	0.000	0.000
1.0	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000	0.000
1.1	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.2	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.3	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.4	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.5	1.623	1.623	1.623	1.623	1.623	1.593	1.553	1.516	0.000	0.000	0.000
1.6	1.623	1.623	1.623	1.583	1.535	1.493	1.456	1.422	0.000	0.000	0.000
1.7	1.623	1.599	1.540	1.490	1.445	1.405	1.370	1.338	0.000	0.000	0.000

show that in the absence of any anti-takeover defenses, the CEO will now invest only once $x \geq 1.826y$; in the baseline case, investment occurred once $x \geq 1.623y$. The board responds by introducing anti-takeover defenses in some states where they did not appear in the baseline case, and strengthening defenses in states where they did appear. Investment is still delayed relative to the baseline case, but by less than would have been the case if the board did not give the CEO some more protection against hostile takeovers.

We continue our sensitivity analysis in Table 3, which shows our results for the case where $\theta = 0.1$; that is, we shorten the CEO's planning horizon. In the baseline case, the CEO leaves the firm according to a Poisson process with intensity $\theta = 0.05$, but here the intensity is increased to $\theta = 0.1$. That is, the CEO's expected tenure (ignoring termination triggered by a takeover) falls from 20 years to ten years. The CEO therefore finds delaying investment more costly than in the baseline case because of the higher probability that his employment ends before the flow of private control benefits begins. The unshaded cells in the bottom panel show that in the absence of any anti-takeover defenses, the CEO will now invest once $x \geq 1.489y$, compared to the baseline policy of investing once $x \geq 1.623y$. The board responds by weakening the firm's anti-takeover defenses in states where moderate defenses were in place in the baseline case. Investment is still accelerated relative to the baseline case, but by less than would have been the case if the board did not take away some of the CEO's protection against hostile takeovers.

Table 2: Board and CEO policies when $\phi = 0.2$

$y_0 \backslash x_0$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
	Optimal strength of anti-takeover defenses (γ^*)										
0.9	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000
1.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.021	1.000	1.000	1.000	1.000
1.5	0.000	0.000	0.000	0.007	0.037	0.063	0.087	1.000	1.000	1.000	1.000
1.6	0.000	0.000	0.037	0.069	0.097	0.122	0.144	1.000	1.000	1.000	1.000
1.7	0.018	0.059	0.094	0.124	0.150	0.173	0.194	1.000	1.000	1.000	1.000
	Minimum level of x/y needed to invest ($1/\hat{z}_C^*$)										
0.9	1.826	1.826	1.826	1.826	1.826	1.826	0.000	0.000	0.000	0.000	0.000
1.0	1.826	1.826	1.826	1.826	1.826	1.826	1.826	0.000	0.000	0.000	0.000
1.1	1.826	1.826	1.826	1.826	1.826	1.826	1.826	0.000	0.000	0.000	0.000
1.2	1.826	1.826	1.826	1.826	1.826	1.826	1.826	0.000	0.000	0.000	0.000
1.3	1.826	1.826	1.826	1.826	1.826	1.826	1.826	0.000	0.000	0.000	0.000
1.4	1.826	1.826	1.826	1.826	1.826	1.826	1.787	0.000	0.000	0.000	0.000
1.5	1.826	1.826	1.826	1.813	1.759	1.711	1.668	0.000	0.000	0.000	0.000
1.6	1.826	1.825	1.758	1.700	1.649	1.604	1.564	0.000	0.000	0.000	0.000
1.7	1.793	1.718	1.655	1.600	1.552	1.510	1.472	0.000	0.000	0.000	0.000

In Tables 4 and 5 we investigate the sensitivity of our results to the (risk-neutral) drifts in the two asset values. It might seem natural to alter the two drifts separately; that is, first vary δ_x and then vary δ_y. However, our theoretical results suggest that the picture will be clearer if we concentrate on the variables x (which determines the level of cash flows) and y/x (which determines the timing of events) separately. Therefore, we begin in Table 4 by summarizing the results if *both* implicit dividend yields are increased together, from $\delta_x = \delta_y = 0.03$ to 0.05. This alters the drift of x while holding the drift of y/x constant, enabling us to examine the effect of changes in the growth rate in the level of cash flows, while leaving the driver of the timing of events unchanged.[13] The value of delaying investment is relatively low when the dividend yield rises (or the expected growth rate falls), so the board and CEO will both want to accelerate investment relative to the baseline case. The unshaded entries in the bottom panel of the table show that in the absence of takeover defenses, the CEO's choice of investment threshold is indeed relaxed, from $x \geq 1.623y$ to $x \geq 1.556y$. Shareholders also benefit from earlier investment, so the board eliminates the takeover threat by setting $\gamma^* = 1$ for lower values of x_0 than in the baseline case. For moderate values of x_0, the board also strengthens defenses, causing the CEO to accelerate investment even more than would otherwise be the case. However, for very low values of x_0, the board actually weakens defenses relative to the baseline case, presumably because the

[13]Note that the (risk-neutral) drift in x has fallen from 0.02 to zero, whereas the drift of y/x is unaffected.

Table 3: Board and CEO policies when $\theta = 0.10$

$y_0 \backslash x_0$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
	Optimal strength of anti-takeover defenses (γ^*)										
0.9	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000
1.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.6	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.034	1.000	1.000	1.000
1.7	0.000	0.000	0.000	0.000	0.018	0.045	0.069	0.091	1.000	1.000	1.000
	Minimum level of x/y needed to invest ($1/\hat{z}_C^*$)										
0.9	1.489	1.489	1.489	1.489	1.489	0.000	0.000	0.000	0.000	0.000	0.000
1.0	1.489	1.489	1.489	1.489	1.489	1.489	0.000	0.000	0.000	0.000	0.000
1.1	1.489	1.489	1.489	1.489	1.489	1.489	1.489	0.000	0.000	0.000	0.000
1.2	1.489	1.489	1.489	1.489	1.489	1.489	1.489	0.000	0.000	0.000	0.000
1.3	1.489	1.489	1.489	1.489	1.489	1.489	1.489	1.489	0.000	0.000	0.000
1.4	1.489	1.489	1.489	1.489	1.489	1.489	1.489	1.489	0.000	0.000	0.000
1.5	1.489	1.489	1.489	1.489	1.489	1.489	1.489	1.489	0.000	0.000	0.000
1.6	1.489	1.489	1.489	1.489	1.489	1.489	1.473	1.439	0.000	0.000	0.000
1.7	1.489	1.489	1.489	1.489	1.462	1.422	1.386	1.354	0.000	0.000	0.000

new low average growth rate makes a long delay until investment optimal in this case.

Table 5 summarizes our results if the implicit dividend yield of y increases from $\delta_y = 0.03$ to 0.05. This time we hold δ_x constant, so that the average growth rate of the factor driving the level of cash flows is unaffected, but the average growth rate of the factor driving the timing of cash flows is lowered. In terms of the risk-neutral process, the value of the asset if it is used by the target firm has a higher average growth rate than its value if used by the raider. Compared to the baseline case, if the board does not change the firm's anti-takeover defenses then the threat from a hostile takeover will be relaxed. The board responds by weakening the firm's defenses, which boosts the discipline provided by the market for corporate control.

Finally, we consider the role played by the volatilities of the two asset values. We continue our approach of focussing on x and y/x, rather than on x and y. First, we increase σ_x and σ_y from their baseline values of 0.2 to 0.25, and simultaneously increase the correlation coefficient ρ from 0.5 to 0.68, which ensures that the volatility of y/x, given in equation (1), is still equal to its baseline value of 0.2.[14] That is, we increase the volatility of the level of the asset's cash flow, but do not change the volatility of the relative value, which is what drives the various timing decisions. The results, shown in Table 6, are identical to those in Table 1. That is, the volatilities

[14] Given σ_x and σ_y, we can achieve the desired level of ψ by setting the correlation coefficient equal to $\rho = (\sigma_x^2 + \sigma_y^2 - \psi^2)/(2\sigma_x\sigma_y)$.

Table 4: Board and CEO policies when $\delta_x = \delta_y = 0.05$

$y_0 \backslash x_0$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
	Optimal strength of anti-takeover defenses (γ^*)										
0.9	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
1.1	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
1.2	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
1.3	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
1.4	0.000	0.000	0.000	0.000	0.054	1.000	1.000	1.000	1.000	1.000	1.000
1.5	0.000	0.000	0.000	0.000	0.117	1.000	1.000	1.000	1.000	1.000	1.000
1.6	0.000	0.000	0.000	0.057	0.172	1.000	1.000	1.000	1.000	1.000	1.000
1.7	0.000	0.000	0.000	0.112	0.221	1.000	1.000	1.000	1.000	1.000	1.000
	Minimum level of x/y needed to invest ($1/\hat{z}_C^*$)										
0.9	1.556	1.556	1.556	1.556	1.556	0.000	0.000	0.000	0.000	0.000	0.000
1.0	1.556	1.556	1.556	1.556	1.556	0.000	0.000	0.000	0.000	0.000	0.000
1.1	1.556	1.556	1.556	1.556	1.556	0.000	0.000	0.000	0.000	0.000	0.000
1.2	1.556	1.556	1.556	1.556	1.556	0.000	0.000	0.000	0.000	0.000	0.000
1.3	1.556	1.556	1.556	1.556	1.556	0.000	0.000	0.000	0.000	0.000	0.000
1.4	1.556	1.556	1.556	1.556	1.472	0.000	0.000	0.000	0.000	0.000	0.000
1.5	1.556	1.556	1.556	1.556	1.374	0.000	0.000	0.000	0.000	0.000	0.000
1.6	1.556	1.556	1.556	1.467	1.288	0.000	0.000	0.000	0.000	0.000	0.000
1.7	1.556	1.556	1.556	1.381	1.212	0.000	0.000	0.000	0.000	0.000	0.000

of the individual asset values do not seem to affect the optimal strength of anti-takeover defenses: it is the volatility of the *relative* asset value that matters. We assess the role of the volatility of y/x by resetting σ_x and σ_y to their baseline values of 0.2 and setting the correlation coefficient to $\rho = 0.21875$. This has the effect of increasing ψ from 0.2 to 0.25 and leads to the results summarized in Table 7. Higher volatility in y/x increases the threat of a takeover to the CEO, holding defenses constant, which induces the CEO to wait until y/x is further from the takeover threshold before investing. This explains why, absent takeover defenses, the CEO sets a tougher investment threshold than in the baseline case, $x \geq 1.798y$ rather than $x \geq 1.623y$. The board compounds the effect by setting weaker takeover defenses than in the baseline case. It sets impregnable defenses only for a much higher value of x_0, and leaves the firm defenseless in some states where moderate defenses were chosen in the baseline case. In short, although the greater volatility of y/x makes the CEO want to delay investment longer, the CEO's response is not enough for the board's liking, so it increases the discipline provided by the market for corporate control.

Table 5: Board and CEO policies when $\delta_y = 0.05$

$y_0 \backslash x_0$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
	Optimal strength of anti-takeover defenses (γ^*)										
0.9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.011	1.000	1.000
1.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.021	0.070	1.000	1.000
	Minimum level of x/y needed to invest ($1/\hat{z}_C^*$)										
0.9	1.653	1.653	1.653	1.653	1.653	1.653	1.653	0.000	0.000	0.000	0.000
1.0	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	0.000	0.000	0.000
1.1	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	0.000	0.000	0.000
1.2	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	0.000	0.000
1.3	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	0.000	0.000
1.4	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	0.000	0.000
1.5	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	0.000	0.000
1.6	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.634	0.000	0.000
1.7	1.653	1.653	1.653	1.653	1.653	1.653	1.653	1.618	1.538	0.000	0.000

7 Concluding remarks

This paper develops a model of investment timing involving a firm with a CEO who chooses the firm's investment policy in order to maximize the present value of his flow of private control benefits. The only constraint on the CEO is the possibility of a future hostile takeover that will terminate his receipt of these benefits. We show how the market for corporate control influences the CEO's behavior and how the firm's board can erect anti-takeover defenses that maximize the value of the firm's shares. The prospect of future hostile takeover attempts causes the CEO to delay investment relative to the situation when there is no takeover threat. If shareholders would benefit from early investment then the firm's board should put anti-takeover defenses in place that are strong enough to eliminate the takeover threat. However, in other cases shareholders are better off if the board makes the firm vulnerable to a future takeover. If the takeover threat is relatively strong, then the board should give the CEO *some* protection from a future takeover, otherwise shareholders are better off if the firm is left defenseless.

Two parameters affect the outcome of a hostile takeover in our model. One (γ) determines the ability of the target to reduce the value of the asset to the raider if the takeover bid is successful, capturing possibilities such as scorched earth policies and poison puts. The other (ϕ) determines the value of the asset to the target's shareholders if the takeover attempt is defeated,

Table 6: Board and CEO policies when $\sigma_x = \sigma_y = 0.25$ and $\psi = 0.20$

$y_0 \backslash x_0$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
	Optimal strength of anti-takeover defenses (γ^*)										
0.9	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000
1.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.5	0.000	0.000	0.000	0.000	0.000	0.019	0.043	0.066	1.000	1.000	1.000
1.6	0.000	0.000	0.000	0.025	0.054	0.080	0.103	0.124	1.000	1.000	1.000
1.7	0.000	0.015	0.051	0.082	0.110	0.134	0.156	0.176	1.000	1.000	1.000
	Minimum level of x/y needed to invest ($1/\hat{z}_C^*$)										
0.9	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000	0.000	0.000
1.0	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000	0.000
1.1	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.2	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.3	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.4	1.623	1.623	1.623	1.623	1.623	1.623	1.623	1.623	0.000	0.000	0.000
1.5	1.623	1.623	1.623	1.623	1.623	1.593	1.553	1.516	0.000	0.000	0.000
1.6	1.623	1.623	1.623	1.583	1.535	1.493	1.456	1.422	0.000	0.000	0.000
1.7	1.623	1.599	1.540	1.490	1.445	1.405	1.370	1.338	0.000	0.000	0.000

capturing what we term the "distraction costs" associated with resisting a hostile takeover. In our model, if the board strengthens the firm's anti-takeover defenses then it increases γ (as the defenders will be able to do more damage to the raider's asset). To keep the analysis tractable, we treat ϕ as an exogenous constant. However, if the board erects stronger defenses, then it is likely that the defenders will be less distracted in resisting the takeover attempt, so that increases in γ will probably be accompanied by decreases in ϕ. In contrast, if the board erects weak defenses, then γ will be low and ϕ will be high. If this is the case then it is not appropriate to treat γ as an independent choice variable, holding ϕ constant, as we have done. However, in order to determine the board's choice of defensive measures, we would need to know the set of feasible (γ, ϕ) combinations, which is beyond the scope of the current model. That would require a formal model to determine distraction costs as a function of the strength of the firm's defenses, which is left for future research.

Table 7: Board and CEO policies when $\sigma_x = \sigma_y = 0.2$ and $\psi = 0.25$

$y_0 \backslash x_0$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
	Optimal strength of anti-takeover defenses (γ^*)										
0.9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000	1.000
1.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	1.000
1.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000
1.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	1.000	1.000
1.6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.033	0.066	1.000	1.000
1.7	0.000	0.000	0.000	0.000	0.000	0.016	0.055	0.090	0.121	1.000	1.000
	Minimum level of x/y needed to invest ($1/\hat{z}_C^*$)										
0.9	1.798	1.798	1.798	1.798	1.798	1.798	1.798	0.000	0.000	0.000	0.000
1.0	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	0.000	0.000	0.000
1.1	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	0.000	0.000
1.2	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	0.000	0.000
1.3	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	0.000	0.000
1.4	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	0.000	0.000
1.5	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.792	0.000	0.000
1.6	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.739	1.680	0.000	0.000
1.7	1.798	1.798	1.798	1.798	1.798	1.769	1.699	1.637	1.581	0.000	0.000

Appendix

A.1 Proof of Proposition 1

Denote the value of the raider's takeover option by $V(x, y)$. In the waiting region, $V(x, y)$ satisfies the PDE

$$0 = \frac{1}{2}\sigma_x^2 x^2 V_{xx} + \rho\sigma_x\sigma_y xyV_{xy} + \frac{1}{2}\sigma_y^2 y^2 V_{yy} + (r - \delta_x)xV_x + (r - \delta_y)yV_y - rV. \tag{A.1}$$

The takeover payoff function is homogeneous of degree 1 in (x, y), so we look for a solution of the form $V(x, y) = xv(y/x)$, for some function v. Substituting this expression into equation (A.1) shows that v must satisfy the ODE

$$0 = \frac{1}{2}\psi^2 z^2 v''(z) + (\delta_x - \delta_y)zv'(z) - \delta_x v(z)$$

in the waiting region. In the stopping region, we have

$$V(x, y) = (1 + \omega_R)(1 - \gamma)y - \max\{(1 - \gamma)y, (1 - \phi)x\},$$

which reduces to

$$v(z) = (1 + \omega_R)(1 - \gamma)z - \max\{(1 - \gamma)z, 1 - \phi\}.$$

If the raider initiates a hostile takeover the first time that $y/x \geq \hat{z}$, for some constant $\hat{z}$, then

$$v(z) = \begin{cases} \left((1+\omega_R)(1-\gamma)\hat{z} - \max\{(1-\gamma)\hat{z}, 1-\phi\}\right)\left(\frac{z}{\hat{z}}\right)^{\beta_3}, & \text{if } z < \hat{z}, \\ (1+\omega_R)(1-\gamma)z - \max\{(1-\gamma)z, 1-\phi\}, & \text{if } z \geq \hat{z}. \end{cases}$$

The raider's optimal takeover threshold maximizes

$$\Big((1+\omega_R)(1-\gamma)\hat{z} - \max\{(1-\gamma)\hat{z}, 1-\phi\}\Big)\, \hat{z}^{-\beta_3}.$$

Note that in the region where $\hat{z} \geq (1-\phi)/(1-\gamma)$, the objective function reduces to

$$f(\hat{z}) = \omega_R(1-\gamma)\hat{z}^{1-\beta_3},$$

which is decreasing in $\hat{z}$. We can therefore restrict attention to the region where $\hat{z} \leq (1-\phi)/(1-\gamma)$. In this region, the objective function is

$$f(\hat{z}) = \Big((1+\omega_R)(1-\gamma)\hat{z} - (1-\phi)\Big)\, \hat{z}^{-\beta_3},$$

which has derivative

$$f'(\hat{z}) = \left(\beta_3(1-\phi) - (\beta_3-1)(1+\omega_R)(1-\gamma)\hat{z}\right)\hat{z}^{-\beta_3-1}.$$

The raider's optimal takeover threshold is therefore

$$\hat{z}_R^* = \left(\frac{1-\phi}{1-\gamma}\right)\min\left\{1, \frac{\beta_3}{\beta_3-1}\cdot\frac{1}{1+\omega_R}\right\}.$$

A.2 Proof of Lemma 1

In the waiting region, $G(x,y)$ satisfies the PDE

$$0 = \frac{1}{2}\sigma_x^2 x^2 G_{xx} + \rho\sigma_x\sigma_y xy G_{xy} + \frac{1}{2}\sigma_y^2 y^2 G_{yy} + (r-\delta_x)xG_x + (r-\delta_y)yG_y - (r+\theta)G + \omega_C\delta_x x,$$

where the term $-(r+\theta)G$ reflects the possibility that the CEO leaves the firm before the takeover occurs. The cash flow is homogeneous of degree 1 in (x,y) and the timing, via the takeover threshold, is homogeneous of degree 0. Its present value must therefore also be homogeneous of degree 1, implying that $G(x,y) = xg(y/x)$ for some function g. Substituting this expression into the PDE for G shows that g satisfies the ODE

$$0 = \frac{1}{2}\left(\sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2\right)z^2 g''(z) + (\delta_x - \delta_y)zg'(z) - (\delta_x+\theta)g(z) + \omega_C\delta_x$$

in the region where $z < \hat{z}_R^*$. The stopping condition reduces to $g(z) = 0$ in the region where $z \geq \hat{z}_R^*$. The solution is

$$g(z) = \begin{cases} \frac{\omega_C\delta_x}{\delta_x+\theta}\left(1 - \left(\frac{z}{\hat{z}_R^*}\right)^{\beta_1}\right), & \text{if } z < \hat{z}_R^*, \\ 0, & \text{if } z \geq \hat{z}_R^*, \end{cases}$$

which leads to equation (2).

A.3 Proof of Lemma 2

In any state (x, y), the CEO has to choose between (i) investing and initiating a stream of control benefits with present value $G(x, y)$, and (ii) waiting and retaining the option to invest in the future, worth an amount that we denote by $H(x, y)$. Equation (2) implies that $G(ax, ay) = aG(x, y)$ for all $a > 0$. The same equation, combined with the properties of geometric Brownian motion, imply that $H(ax, ay) = aH(x, y)$ for all $a > 0$.[15]

It is optimal for the CEO to wait in state (x, y) if and only if $H(x, y) > G(x, y)$, which therefore holds if and only if $H(ax, ay) > G(ax, ay)$, which holds if and only if it is optimal for the CEO to wait in state (ax, ay). For example, suppose an optimal investment policy is to wait if and only if $x < f(y)$, for some function f. It follows that $x < f(y)$ if and only if $ax < f(ay)$ for all $a > 0$. That is, $x < f(y)$ if and only if $x < f(ay)/a$ for all $a > 0$, which implies that the policy function must satisfy $f(y) = f(ay)/a$ for all $a > 0$. In particular, this equation must hold when $a = 1/y$, which shows that $f(y) = yf(1) = y/\hat{z}$ for some constant $\hat{z}$.

A.4 Proof of Lemma 3

As the flow of control benefits does not begin until the firm invests in the project, the function $H(x, y)$ satisfies the PDE

$$0 = \frac{1}{2}\sigma_x^2 x^2 H_{xx} + \rho\sigma_x\sigma_y xyH_{xy} + \frac{1}{2}\sigma_y^2 y^2 H_{yy} + (r - \delta_x)xH_x + (r - \delta_y)yH_y - (r + \theta)H$$

in the waiting region. It satisfies $H(x, y) = G(x, y)$ in the stopping region. The (lump-sum) cash flow is therefore homogeneous of degree 1 in (x, y) and the timing, via the investment threshold, is homogeneous of degree 0. The present value of this lump-sum cash flow must therefore also be homogeneous of degree 1, implying that $H(x, y) = xh(y/x)$ for some function h. Substituting this expression into the PDE for H shows that h satisfies the ODE

$$0 = \frac{1}{2}\left(\sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2\right) z^2 h''(z) + (\delta_x - \delta_y)zh'(z) - (\delta_x + \theta)h(z)$$

in the region where $z > \hat{z}$ for some constant $\hat{z}$. Imposing the value-matching condition that $h(\hat{z}) = g(\hat{z})$ leads to the solution

$$h(z) = \begin{cases} g(z), & \text{if } z \leq \hat{z}, \\ g(\hat{z})\left(\frac{z}{\hat{z}}\right)^{\beta_2}, & \text{if } z > \hat{z}. \end{cases}$$

This implies the function in equation (3).

[15] For any point on any future path of the Wiener processes (ξ, ζ), the investment payoff if the state is currently (ax, ay) equals a multiplied by the investment payoff if the state is currently (x, y). A stopping rule that maximizes the present value of the former will maximize the present value of the later, and the two present values will differ by a factor of a.

A.5 Proof of Lemma 4

In the waiting region, the function $M(x,y)$ satisfies the PDE

$$0=\frac{1}{2}\sigma_x^2x^2M_{xx}+\rho\sigma_x\sigma_yxyM_{xy}+\frac{1}{2}\sigma_y^2y^2M_{yy}+(r-\delta_x)xM_x+(r-\delta_y)yM_y-rM+\delta_xx,$$

where the nonhomogeneous term reflects the project's implicit dividend yield. All cash flows are homogeneous of degree 1 in (x,y) and the timing, via the takeover threshold, is homogeneous of degree 0. The present value must therefore also be homogeneous of degree 1, implying that $M(x,y)=xm(y/x)$ for some function m. Substituting this expression into the PDE for M shows that m satisfies the ODE

$$0=\frac{1}{2}\left(\sigma_x^2-2\rho\sigma_x\sigma_y+\sigma_y^2\right)z^2m''(z)+(\delta_x-\delta_y)zm'(z)-\delta_xm(z)+\delta_x$$

in the region where $z<\hat{z}_R^*$. The stopping condition reduces to $m(z)=\max\{(1-\gamma)z,1-\phi\}$ in the region where $z\geq\hat{z}_R^*$. The solution is

$$m(z)=\begin{cases}1-(1-\max\{(1-\gamma)\hat{z}_R^*,1-\phi\})\left(\frac{z}{\hat{z}_R^*}\right)^{\beta_3}, & \text{if } z<\hat{z}_R^*,\\ \max\{(1-\gamma)z,1-\phi\}, & \text{if } z\geq\hat{z}_R^*.\end{cases}$$

Noting that

$$\max\{(1-\gamma)\hat{z}_R^*,1-\phi\}=(1-\phi)\max\left\{\min\left\{1,\frac{\beta_3}{\beta_3-1}\cdot\frac{1}{1+\omega_R}\right\},1\right\}=1-\phi$$

leads to the function in equation (4).

A.6 Proof of Lemma 5

Suppose that $y/x>\hat{z}$, so that investment will be delayed. We calculate the present values of future capital expenditure and profits separately. Starting with capital expenditure, shareholders incur lump-sum expenditure of I as soon as y/x falls to $\hat{z}$. As y/x evolves according to a geometric Brownian motion, the present value of the cash flow is easily shown to equal $I((y/x)/\hat{z})^{\beta_5}$. Turning to the subsequent payout to shareholders, they effectively receive a lump sum of $M(x,y)$ as soon as y/x falls to $\hat{z}$. Repeating the argument in the other proofs shows that the present value of this lump sum equals $xn(y/x)$, where $n(z)=m(\hat{z})(z/\hat{z})^{\beta_4}$. The pre-investment value of the firm's shares is therefore

$$xm(\hat{z})\left(\frac{y/x}{\hat{z}}\right)^{\beta_4}-I\left(\frac{y/x}{\hat{z}}\right)^{\beta_5}$$

in the waiting region.

References

Alvarez, L. H. R. and Stenbacka, R. (2006). Takeover timing, implementation uncertainty, and embedded divestment options. *Review of Finance*, 10(3):417–441.

Bernile, G., Lyandres, E., and Zhdanov, A. (2012). A theory of strategic mergers. *Review of Finance*, 16:517–575.

Cain, M. D., McKeon, S. B., and Davidoff Solomon, S. (2017). Do takeover laws matter? Evidence from five decades of hostile takeovers. *Journal of Financial Economics*, 124(3):464–485.

Cook, D. O. and Easterwood, J. C. (1994). Poison put bonds: An analysis of their economic role. *Journal of Finance*, 49(5):1905–1920.

Grenadier, S. R. and Wang, N. (2005). Investment timing, agency, and information. *Journal of Financial Economics*, 75:493–533.

Grossman, S. J. and Hart, O. D. (1980). Takeover bids, the free-rider problem, and the theory of the corporation. *Bell Journal of Economics*, 11(1):42–64.

Guthrie, G. (2017). *The Firm Divided*. Oxford University Press, New York, NY.

Hackbarth, D. and Miao, J. (2012). The dynamics of mergers and acquisitions in oligopolistic industries. *Journal of Economic Dynamics and Control*, 36:585–609.

Hackbarth, D. and Morellec, E. (2008). Stock returns in mergers and acquisitions. *Journal of Finance*, 58(3):1213–1252.

Hermalin, B. E. and Weisbach, M. S. (1998). Endogenously chosen boards of directors and their monitoring of the CEO. *American Economic Review*, 88(1):96–118.

Hori, K. and Osano, H. (2014). Investment timing decisions of managers under endogenous contracts. *Journal of Corporate Finance*, 29:607–627.

Kini, O., Kracaw, W., and Mian, S. (2004). The nature of discipline by corporate takeovers. *Journal of Finance*, 59(4):1511–1552.

Lambrecht, B. M. (2004). The timing and terms of mergers motivated by economies of scale. *Journal of Financial Economics*, 72(1):41–62.

Lambrecht, B. M. and Myers, S. C. (2007). A theory of takeovers and disinvestment. *Journal of Finance*, 62(2):809–845.

Lambrecht, B. M. and Myers, S. C. (2008). Debt and managerial rents in a real-options model of the firm. *Journal of Financial Economics*, 89:209–231.

Manne, H. G. (1965). Mergers and the market for corporate control. *Journal of Political Economy*, 73(2):110–120.

Margsiri, W., Mello, A. S., and Ruckes, M. E. (2008). A dynamic analysis of growth via acquisition. *Review of Finance*, 12(4):635–671.

Morellec, E. and Zhdanov, A. (2005). The dynamics of mergers and acquisitions. *Journal of Financial Economics*, 77(3):649–672.

Morellec, E. and Zhdanov, A. (2008). Financing and takeovers. *Journal of Financial Economics*, 87:556–581.

Pagano, M. and Volpin, P. F. (2005). Managers, workers, and corporate control. *Journal of Finance*, 60(2):841–868.

Shibata, T. (2009). Investment timing, asymmetric information, and audit structure: A real options framework. *Journal of Economic Dynamics and Control*, 33:903–921.

Shibata, T. and Nishihara, M. (2010). Dynamic investment and capital structure under manager–shareholder conflict. *Journal of Economic Dynamics and Control*, 34:158–178.

Thijssen, J. J. J. (2008). Optimal and strategic timing of mergers and acquisitions motivated by synergies and risk diversification. *Journal of Economic Dynamics and Control*, 32:1701–1720.