# Modelling New Zealand Dairy Production: The Impact of Traceability Between the Farm and the Factory

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**Abstract**

Traceability is the capability to trace goods throughout the distribution chain. Traceability has become an increasingly important research area in recent years. It has always been an important aspect of production, but recent contamination events have highlighted its significance. The Fonterra botulism scare of 2013 in particular exposed a need for fast accurate product tracing in the New Zealand dairy industry.

We present a Markov chain model for the flow of milk through the early stages of the dairy supply chain. The state of the Markov chain is the value of product at each location in the production chain, in this case the milk tanker, factory reception or processing.

The model incorporates parameters for product testing and tracing upon arrival in each state of the model. By varying these parameters we are able to alter the precision of the traceability system, and gain an understanding of where and when traceability has the greatest impact. By analysing the results of simulations under various scenarios we are able to estimate the value traceability can contribute to the output of the production chain. ***Keywords:*** Traceability, Dairy, Markov chains

## 1 Introduction

Traceability has become an increasingly important research area in recent years. It has always been an important aspect of production, but recent contamination events have highlighted its significance. Increasing complexity in widespread international supply chains, along with varying standards across countries, means traceability has become even more important in maintaining international reputations and trade. Ideally food safety standards would be such that contaminations did not occur at all, this is just not possible in reality. Traceability is required to manage defects and contaminations that do occur, reducing their impact, both economically and in regards to public health.

Milk and dairy products from domestic animals have been used by people for hundreds of years[1]. They have long been an important part of the western style diet and are increasingly being consumed as part of Asian diets as well[2]. Dairy products are a valuable source of essential nutrients, and are often a large component of young children's diets. The potential risk to young children means milk and its derivatives are particularly vulnerable to contamination scares. The 2008 melamine contamination of infant formula in China, the Fonterra botulism scare in 2013 and the 2015 poisoning threat to baby formula in New Zealand are all examples of this.

The 2013 Botulism scare in particular highlights the need for fast, accurate testing and traceability. The amount of time taken in the scare of 2013 to confirm the sources of the contamination, and the location of the contaminated batches, risked not only Fonterra's reputation but New Zealand's reputation as an exporter as well. New Zealand is a world leader when it comes to the production and export of dairy products[3]. The dairy industry forms a large part of the country's exports. Both of these aspects make the New Zealand dairy industry a good case to apply and develop our model.

This paper builds on a Markov chain model developed by Welsh et.al, (2016)[4]. We incorporate traceability parameters and effects into this model in order to investigate the potential value a good traceability system can contribute to the initial stages of the dairy supply chain. Following this introduction, section 2 gives an overview of the dairy industry, with a brief look at the history of dairy production and product importance. The second half of the section covers aspects of specific importance to the New Zealand dairy industry. We follow this with a review of traceability in Section 3. We cover the definition of traceability, why it is important and the approaches taken by previous literature. In Section 4 we discuss the use of discrete time Markov chains (DTMCs) to model the flow of value through the dairy supply chain. We introduce the stages of the model and develop the transition probabilities to include the effects of a traceability system. In the next section we discuss parameter values. Finally, we present simulation results and discuss implications of this work.

# 2 Dairy

Dairy products have been an important part of the western style diet for centuries. Milk is rich in a variety of essential nutrients[5], and the worldwide market for dairy and milk based products continues to grow[6]. Along with this growth come increasing food safety issues, with consumer perception becoming increasingly important[7].

Aside from it's value as milk, many derived dairy products are available. In particular, functional foods and health supplements made with milk proteins have proven to be of considerable value[8]. Steijns (2001) discusses various components of dairy products and their role in managing a variety of health concerns[5]. Research has also been done into how certain dairy products may be useful for cancer prevention[2,9].

As mentioned earlier, New Zealand is a world leader in the production and export of dairy products[3]. The dairy industry in this country is mainly pasture based[1]. The New Zealand dairy industry has earned a reputation for its low cost, high quality systems and technological expertise[3].

About 97% of New Zealand dairy farmers sell their milk through Fonterra Cooperative Group[3]. Cows are generally milked twice per day[10], and milk is collected from the farm in a tanker every 1-2 days[3]. Fonterra operates a national fleet of 525 tankers collecting from around 12,000 farms[11]. The frequency of collection is generally dependent on the time of year, as milk production is seasonal. The amount of milk a farmer is allowed to supply to Fonterra is limited by the number of shares they own in the cooperative. Because of this, output becomes targeted[3]. The cost efficiency of New Zealand dairy farms is examined by Jiang et al. (2014)[3], their results indicated that there is still room for improvement. Trends in developing high capacity milking parlours and automatic miking systems, have seen an increase in cow throughput, along with reduced manual labour on dairy farms. As these trends continue, further labour based barriers to farm expansion may be overcome[10].

Because dairy products are such a large part of so many diets, and especially for young children, they are particularly vulnerable to contamination scares. While reliable testing and quality standards are important for all consumable products, they are particularly important for dairy. The Fonterra supplier's handbook lists at least 9 contamination types to be tested for along with general quality grading and organoleptic assessment, though only two of these tests are conducted upon tanker collection every time[12]. While New Zealand is known for low cost dairy production, research by Jiang et al. indicates that there is still room for improvement. Traceability can be incorporated into testing and quality control systems to increase certainty of safe product, as well as potentially improve efficiency in the production chain.

# 3 Traceability

Traceability is the capability to trace goods throughout the distribution chain[13]. The study and implementation of traceability is an interdisciplinary field, spanning the natural and social sciences, it is a widely used concept, with various approaches studied over the last few decades[14]. The aim of a traceability system is to collect information relevant to the location of products along the supply chain[15], allowing the flow of material to be followed[14].

In the event of a product contamination or other fault, traceability becomes very important. Traceability makes selective recalls possible[13], with no traceability it is difficult to determine how far a contamination has gone, necessitating a widespread recall and the very real possibility of contaminated product being consumed by the public. To be effective in a recall situation, a traceability system must be able to trace back along the supply chain to the source of the contamination as well as forward to identify all the affected product[16]. An effective traceability system allows the fast and precise withdrawal of contaminated product. Such efficient product withdrawal mitigates costs associated with a contamination scare, and reduces the potential risk to consumers health[17]. The precision of a traceability system is also important. This can determine how much product is recalled and the value lost[15], as well as the time and effort required to locate all of the faulty product. Fast efficient location allows for reduced spread and therefore a reduced impact on consumer confidence. Good traceability is not about reducing the probability of a contamination event, but about reducing the consequences if contamination does occur[18]. Buhr (2003)[19] identifies traceability as crucial to a firm's ability to limit the size and spread of a recall.

While traceability is important for food safety and reducing the potential impact of contamination events, it can also be used to optimise production planning and scheduling, creating a competitive advantage[20]. Wang and Li (2006) propose frameworks to achieve business benefits through the integration of traceability and supply chain management processes. They provide a case study of a British meat processing company as illustration. Canavari et.al. discuss traceability as part of information management in supply chains[21] .

Within the U.S private sector there has been widespread voluntary adoption of food traceability systems in order to improve efficiency in the event of a recall, particularly in the grain sector where supply management and demand for high-value attributes lead firms to differentiate and track production[22]. The food industry as a whole, has also responded to food safety crises by implementing quality assurance and traceability systems of their own, beyond what is legislated[23]. Firms' reputations are an important consideration when providing incentives for them to deliver safe high-quality goods[24]. Good traceability reduces anonymity in the supply chain, it can also be useful in identifying who may be liable in the event of a contamination or fault[17]. This also provides an incentive for firms to improve their safety and quality standards. As the probability that they will be held accountable increases, firms seek to improve their own standards[25]. U.S. firms interviewed by Resende-Filho and Buhr (2010)[17], did not know the cost of a recall, but did view the resulting loss of product sales as the primary cost.

Resende-Filho and Buhr (2010)[17] develop conceptual and process simulation models to investigate the value of traceability for food recalls. Incorporating quality control, they identify key factors affecting the value of a traceability system. Their focus is on the economic modelling of traceability as a tool to reduce the extent and size of a recall. A case study of E. coli in ground beef is presented. It is suggested that the main value of a traceability system lies in its ability to improve the recall process though records management and verification.

It can be difficult to estimate the value of a traceability system, the return is essentially the loss avoided in the case of a contamination or other production fault[18]. Dupuy et.al[18] propose a mathematical mixed integer linear programming (MILP) model for a batch dispersion problem. Their model is applies to a sausage manufacturing process in a French food company. They aim to minimise the quantity of recalls and optimise traceability. While they conclude that their model is too large for daily industry use, they suggest their method could be applied to simpler models. Dabbene and Gay[15] build on the work done by Dupuy et. al.[18]. They introduce a modelling framework and optimisation strategy and use recall cost to measure and optimise the performance of traceability systems. As in Dupuy et.al.[18], they express the optimisation problem in the form of a MILP model. They model the flow of product batches through the supply chain via a directed graph. The capacity of the nodes is bounded by the amount of product that can be processed at that node at one time. An improvement made by this model is the ability to account for the quantities being moved, not just where and when[15]. They describe an approach to account for either the worst-case recall cost, or average recall cost. Numerical examples are provided based on the same sausage scenario presented in Dupuy et.al[15,18].

The literature outlined above is largely deterministic. This paper fills a gap in traceability literature by considering a stochastic model using Markov chains. A model for the flow of milk from the farm to the factory is developed by Welsh et al. (2016)[4]. Markov chains are used to simulate dairy production under different testing conditions and with various rates of product rejection.

Our model builds on that developed by Welsh et al. (2016) modifying it in order to asses the value of traceability to the dairy industry in the early stages of the production chain. Markov chain models have not previously been used in modelling or assessing traceability. Our model provides a framework for assessing the potential value of traceability in the dairy and similar industries, filling a need to be able to assign value to a difficult to quantify aspect of production.

# 4 Markov Chain Model

## 4.1 Problem Statement

Traceability is an important and useful addition to the quality control system of any supply chain. It is of particular interest and importance in the dairy industry, yet its value is difficult to quantify. We aim to develop a useful model, for the flow of value though the dairy supply chain, incorporating traceability in such a way as to allow estimation of its value. While traceability can contribute value to a firm or industry in other ways, our model focuses on the value contributed through the reduction of product loss in the event of contamination.

## 4.2 Dairy Product Flow DTMC Model

We model the flow of milk from the farm to the factory. Milk passes through three locations after leaving the farm: tanker, factory reception and processing. Upon arrival at each location testing can occur. Using a discrete Markov chain, in which the state of the chain is the value of milk in each stage, we are able to model the milk flow and assess the value of traceability associated with each stage.

When testing is undertaken, a number of tests are performed. The results of some of these tests area available instantaneously, however some may not be available for several hours, possibly days. Fonterra recently developed an new milk fingerprinting system, that allows them to get most relevant milk quality information on the same da the milk is collected. Instantaneous results available at the farm prior to collection are those generally organoleptic tests, regarding smell appearance and temperature[26]. The results of an "instantaneous" tests will determine a primary rejection at what ever stage of the process the test is performed. long term tests are those that require testing in a laboratory environment, such as tests for bacteria and chemical residues. Because the results of a "long term test" are not available immediately, they do not influence primary rejection in the current stage, rather they may result in a secondary rejection at a future stage.

We use discrete time Markov chains because the events where milk moves between stages are clearly defined. Beginning with each equation separately, we can derive the probability of each event happening in a discrete time period. Figure 4.1 shows the path milk takes from the farm to the factory, and where the event decisions occur.

Because we are using three stages, the tanker, factory reception and processing, the state of the Markov chain is described by a vector of three values. We denote these stages as $T$ for the milk tanker stage, $F$ for the factory reception stage and $P$ for the processing stage. The total number of possible states depends on the amount of product that is allowed to move between stages in each time step, and the maximum capacity of each stage. For example if the maximum capacity vector

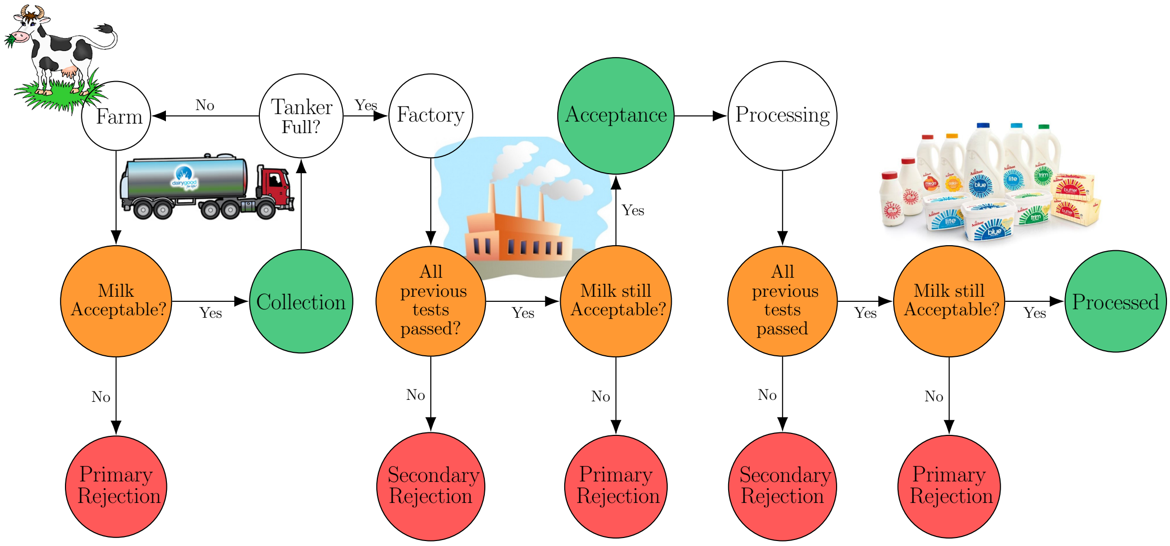

Figure 4.1: Flow chart, showing the path milk takes from the farm to the factory and the decisions that are made along the way.

is given by $(T_{\text{Max}}, F_{\text{Max}}, P_{\text{Max}}) = (130, 200, 880)$ using a constant movement value of 1, the total number of states is given by $130 \times 200 \times 880 = 22880000$. If however we increase the movement each time step to 10, we have $\frac{130}{10} \times \frac{200}{10} \times \frac{880}{10} = 22880$ possible states.The probability for transitioning from state $i$ to state $j$ is $P_{ij}$. We will explain the transition probabilities for the tanker, factory reception and processing stages in sections 4.4, 4.5 and 4.6 respectively.

A discrete time Markov Chain models the state of a system (here the value of milk in different stages of the dairy supply chain), from one time step to the next. In order to specify the transition probabilities, the time step must be small enough that in any given time step, at most one event can occur. For example, throughout a day, there are $\Phi$ collections made by milk tankers. To ensure that there is at most one collection or delivery per time step, the length of the time step must be less than $\frac{1}{\Phi}$ days. In this model the value of the time step must account for all events that can occur across the three stages (tanker, factory and processing). Possible values for the time step will be discussed throughout sections 4.4 to 4.6.

## 4.3 Developing the Model with Traceability

Starting with the multi-event model developed by Welsh et al.(2016)[4], we will incorporate terms and transition probabilities to represent various aspects of traceability. In this paper we make the assumption that any costs associated with the care and milking of the cows is the responsibility of the farmer and does not influence our model.

Figure 4.2 depicts the path the milk takes from the farm to the factory. There are three possible points for testing, each before milk from different sources is combined. These are prior to collection by a milk tanker, before a tanker deposits its load in a vat at the factory, and before entering processing. Raw milk will not keep for long, so cannot be held in each stage to wait for the results of any non-instantaneous (long term) test. Therefore there is an element of traceability

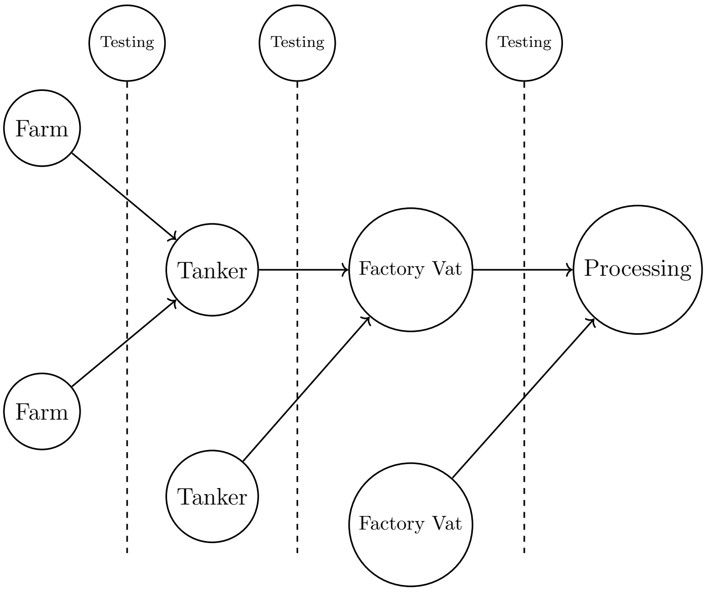

Figure 4.2: The path of milk, from the farm to the factory. Each time the milk enters a new stage, testing and potential rejection is depicted by vertical dashed lines.

required to keep track of where the milk has gone, should any of the tests come back with a poor result at any stage. Table 4.1 summarises the parameters we will use in this model. Frequencies are represented by the capital Greek letters $\Phi, \mathcal{X}, \Psi$ and $\Omega$. Probabilities are represented by lowercase Greek letters. Testing costs are represented by an $E$ with a subscript for the associated stage, similarly for the traceability costs using an $L$. The value of the vat on the farm, which the tankers collect from, is given by $V$, we assume this to be the same for all farms.

This model extends Welsh et al. (2016)[4] by including terms for the cost of traceability, and new probabilities for rejecting product based on previous tests, which we label as secondary rejection probabilities. The effects of traceability on product loss are also incorporated through 'traceability factors' and 'mixing errors' which we will explain in the relevant sections.

## 4.4 The Milk Tankers

There are four possible events that can occur in the milk tanker stage, one at a time. They are Milk collection, Milk rejection, Milk delivery, or Transporting. Figure 4.3 is a flow diagram of the value entering and leaving the tanker stage. This is essentially a close up view of the Tanker stage in figure 4.2

In any given period of time the tanker will make a certain number of collections and a certain number of deliveries. In figure 4.3 the frequency of collection, i.e. the number of collection attempts per day, is represented by $\Phi$ while delivery frequency is denoted by $\mathcal{X}$. $\alpha$ is the probability that the milk being collected passes all 'instantaneous' tests on site, and is mixed with the product already in the tanker, meaning $1-\alpha$ is the probability that an instantaneous test is failed and the milk is rejected. $T(t)$ is the value contained in all of the tankers at time $t$. To keep things simple, we will assume that all vats contain the same amount of milk, and

| **Parameter** | **Description** | **Units** |
|---|---|---|
| $V$ | Amount of milk collected from an on-farm vat | $ |
| $\Phi$ | Frequency of collection attempts | Vats per day |
| $\mathcal{X}$ | Frequency of delivery to factory | Tankers per day |
| $\Psi$ | Frequency with which milk enters processing | Silos per day |
| $\Omega$ | Frequency of process exit | Units per day |
| $E_T$ | Cost of testing milk at collection site | $ |
| $L_T$ | Cost of tracing collected milk$^*$ | $ |
| $E_F$ | Cost of testing milk upon delivery | $ |
| $L_F$ | Cost of tracing accepted milk$^*$ | $ |
| $D_F$ | Cost of disposing of unwanted milk | $ |
| $E_P$ | Cost of testing prior to processing | $ |
| $L_P$ | Cost of tracing milk accepted for processing$^*$ | $ |
| $D_P$ | Cost of disposing of unaccepted silo milk | $ |
| $\alpha$ | Probability of primary vat acceptance | N/A Probability |
| $\beta$ | Probability of primary tanker acceptance | N/A Probability |
| $\gamma$ | Probability of primary silo acceptance | N/A Probability |
| $\eta$ | Probability of secondary vat acceptance$^*$ | N/A Probability |
| $\theta$ | Probability of no secondary rejection at processing entry$^*$ | N/A Probability |
| $\varsigma$ | Probability of Partial primary tanker acceptance | N/A Probability |
| $\varpi$ | Probability of Partial secondary tanker acceptance$^*$ | N/A Probability |
| $\varrho$ | Probability of Partial silo acceptance$^*$ | N/A Probability |
| $\lambda$ | Tanker Traceability factor$^*$ | N/A 0 or 1 |
| $\varepsilon$ | Mixing error for a tanker load in a reception silo$^*$ | $ |
| $\ell$ | Factory reception traceability factor$^*$ | N/A $0 < \ell < 1$ |
| $C_T$ | Average capacity of one milk tanker | $ |
| $N_T$ | Total capacity of entire milk tanker fleet | $ |
| $C_F$ | Reception silo capacity | $ |
| $N_F$ | Total Factory reception stage capacity | $ |
| $C_P$ | Processing unit capacity | $ |
| $N_P$ | Total Processing capacity | $ |
| $Q$ | Value leaving Processing each time step | $ |

Table 4.1: A summary of each parameter used in our model and its units. $^*$Denotes parameters we use in out model that were not present in Welsh et al. (2016)[4]

therefore the same value. $V$ represents the value of the farm vat that is transferred to the milk tanker, this is a constant was we assume the same amount of material is collected from every farm, every collection. If there is no testing prior to collection by the tanker, $\alpha = 1$ as the milk cannot be rejected, and $E_T = 0$ as there is no cost of testing.

The tanker will make its delivery to the factory prior to receiving the results of some tests, we refer to these tests as long term tests. These are the tests that may lead to secondary rejections the entry to the factory reception or processing stages, depending on how long the results take. What happens to the milk after this is at the factory reception stage and does not affect the tanker. The cost of the test is incurred independent of the milk being collected or rejected, this cost is assumed to be constant and is represented by $E_T$. The cost of tracing the milk is

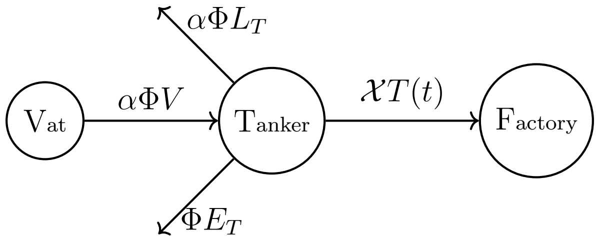

Figure 4.3: The flow of value into and out of the tanker stage. $T(t)$ is the value in the tanker stage at time $t$.

represented by $L_T$, this cost only applies if the milk passes immediate testing and is collected. Because this is the first stage, no previous testing has been done, so this cost is the only modification to the milk tanker stage from the model as it is presented in Welsh et al.(2016)[4].

If milk is rejected, the co-operative does not pay the farmer for that milk, it is considered as if the milk was never supplied. The responsibility of disposing of milk rejected at this stage lies with the farmer[12].

In our model we only allow one event, out of collection, rejection or delivery, to take place in any given time step. In reality, as there are many milk tankers, two or more tankers may collect milk from different farms simultaneously. These events also take time in reality however. Because, in our model, we assume instantaneous material transfer, and it is possible for two events to occur in two adjacent time steps, we can closely mimic simultaneous events for different tankers.

### 4.4.1 Collection:

The milk becomes the responsibility of the factory when it is collected by the tanker. This is also when they can apply their first test, before mixing with any previously acquired milk already in the tanker. For tests that yield instantaneous results, this is enough, but if any time is required to get the results, we need some ability to track where the milk associated with each test has gone. Because the nature of raw milk does not allow it to be kept long enough to wait for any 'long term' test results, the milk must be collected, unless any instantaneous tests suggest rejection. The results of any long term test results (when they come through) now apply to all of the milk in that tanker. The probability of an attempted milk collection in time step $\Delta t$ is given by

$$\left(1 - \frac{T(t)}{N_T}\right)\Phi\Delta t \tag{4.1}$$

The probability given in equation 4.1 is proportional to the unused capacity of the tanker stage at time $t$. In the event that all of the milk tankers are full, equation 4.1 reduces to 0, reflecting the fact that milk cannot be collected if there is no capacity to collect it. As the tanker stage gets close to capacity, equation 4.1 gets very small. In reality tankers are collecting milk and delivering it throughout their shift, and it is unlikely that the system approaches capacity at all. It does however, seem logical that the closer the system is to capacity, the fewer tankers have any space to collect milk, reducing the chances of a collection at that point in time.

If all the milk tankers are empty in a given time step $t$, equation 4.1 is reduced to $\Phi\Delta t$. It is possible $\Delta t$ will be such that $\Phi\Delta t = 1$ or very close. While this may seem a little unrealistic at first, in the scenario where all tankers are empty, none will be delivering to the factory, all will be at some point on their journey from the factory to a farm. It is plausible given a large fleet of tankers, that at least one tanker will be ready to collect milk in any given time step, particularly under the restrictions of this scenario. Given $\alpha$ is the probability the milk passes all of the instantaneous tests and is thus is accepted by a tanker, the probability that a milk tanker collects milk in a given time step $\Delta t$ is:

$$\alpha\left(1 - \frac{T(t)}{N_T}\right)\Phi\Delta t \tag{4.2}$$

In transferring the milk from the farm's vat to the tanker, the value of that milk is transferred to the tanker. The tanker gains the value of the milk, but loses the cost of any testing performed as well as the cost associated with tracing the milk in case of poor test results in the future. The value change in the tanker stage in the event of collection is thus $V - L_T - E_T$.

### 4.4.2 Rejection:

If an 'instantaneous' test is failed before the milk is added to the tanker, the milk is rejected and the tanker incurs the cost of testing, no value is gained. The probability of a rejection is given by:

$$(1-\alpha)\left(1 - \frac{T(t)}{N_T}\right)\Phi\Delta t \tag{4.3}$$

This results in a value change of $-E_T$. There is no traceability cost, as rejected milk leaves the supply chain, there is no need to trace it further.

### 4.4.3 Delivery:

After collecting milk from multiple farms, the tanker will deposit its load at a factory, along with all the value associated with it. The probability that a tanker will deliver to the factory in a given time step is dependent on the amount of milk in the tanker stage $T(t)$, the number of deliveries to the factory per day $\mathcal{X}$, and the available capacity in the factory reception stage $\left(1 - \frac{F(t)}{N_F}\right)$. The probability of a tanker delivering milk to the factory in time step $\Delta t$ is given by

$$\frac{T(t)}{N_T}\left(1 - \frac{F(t)}{N_F}\right)\mathcal{X}\Delta t \tag{4.4}$$

At most one tanker can deliver to the factory in a given time step, so the value of milk in the tanker stage will reduce by $C_T$, the average tanker capacity. Costs associated with testing and traceability are taken out at the factory reception stage

### 4.4.4 Transporting:

If a tanker is not collecting, rejecting or delivering, it is assumed to be in the process of travelling between farms or back to the factory. Therefore the probability that all tankers are transporting is

$$1-\left[\left(1-\frac{T(t)}{N_T}\right)\Phi+\frac{T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\mathcal{X}\right]\Delta t \tag{4.5}$$

When all milk tankers are transporting there is no value change in the milk tanker stage.

### 4.4.5 Summary equation:

Equation 4.6 summarises the transition probabilities and resulting value changes we have just described for the tanker stage.

$$p_{ij}(\Delta t)=\begin{cases}\alpha\left(1-\frac{T(t)}{N_T}\right)\Phi\Delta t & j=i+V-E_T-L_T \quad \text{Collection}\\ (1-\alpha)\left(1-\frac{T(t)}{N_T}\right)\Phi\Delta t & j=i-E_T \quad \text{Rejection}\\ \frac{T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\mathcal{X}\Delta t & j=i-C_T \quad \text{Delivery}\\ 1-\left[\left(1-\frac{T(t)}{N_T}\right)\Phi+\frac{T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\mathcal{X}\right]\Delta t & j=i \quad \text{Transporting}\end{cases} \tag{4.6}$$

The sum of the four transition probabilities equals one, because these transitions represent all possible changes in the tanker stage over the time interval $\Delta t$.

### 4.4.6 Time Step Size

The time step is chosen to ensure that all of the transition probabilities, within each stage, add one. Individually each probability must be between 0 and 1 in any given timestep. In the case of the tanker stage this means we require:

$$\left[\left(1-\frac{T(t)}{N_T}\right)\Phi+\frac{T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\mathcal{X}\right]\Delta t\leq 1 \tag{4.7}$$

There is no global maximum for the left hand side of 4.7 but depending on whether $\Phi<\mathcal{X}$, $\Phi>\mathcal{X}$ or they are equal we can estimate local maximums and then solve for the maximum allowable value of $\Delta t$. For $\Phi\geq\mathcal{X}$ the left hand side of equation 4.7 is maximised when $T(t)=0$ giving

$$\begin{aligned}\left[(1-0))\,\Phi+0\left(1-\frac{F(t)}{N_F}\right)\mathcal{X}\right]\Delta t&\leq 1\\ \implies \Phi\Delta t&\leq 1\\ \implies \Delta t&\leq\frac{1}{\Phi}\end{aligned} \tag{4.8}$$

If $\Phi<\mathcal{X}$ , the left hand side of equation 4.7 is maximised when $T(t)=N_T$

and $F(t) = 0$

$$\begin{aligned}\left[\left(1 - \frac{N_T}{N_T}\right)\Phi + \frac{N_T}{N_T}(1-0)\mathcal{X}\right]\Delta t \le 1 \\ \implies 0 + \mathcal{X}\Delta t \le 1 \\ \implies \Delta t \le \frac{1}{\mathcal{X}}\end{aligned} \tag{4.9}$$

Combining the two we can write

$$\Delta t \le \frac{1}{\max\{\Phi, \mathcal{X}\}} \tag{4.10}$$

## 4.5 The Factory Reception Stage

The next stage is the reception silo at the factory where the tankers deposit their loads. The model allows for multiple dairy processing sites. As with the tankers we will be treating these as part of the factory reception pool but only one silo worth of product can be passed on to the processing stage in any single time step. The possible events in the factory reception stage are collection, rejection, passing on for processing, or holding. Figure 4.4 is a flow diagram of the value entering and leaving the factory reception stage. This is essentially a close up view of the factory reception stage in figure 4.2 Because at this stage the milk is now the responsibility of the factory there is a disposal cost associated with the rejection of a delivery $D_F$.

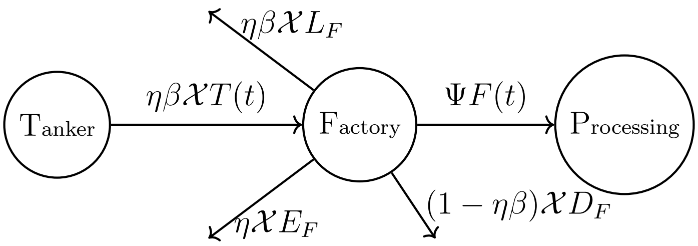

Figure 4.4: The rate of value flow into and out of the factory reception stage.

Including traceability means this stage has two possible rejection scenarios, which we will refer to as primary and secondary rejections. A primary rejection occurs when the milk is rejected due to the results of instantaneous testing conducted when the tanker arrives at the factory. The cost of the test $E_F$ is incurred along with the disposal cost $D_F$. If however, milk is rejected due to previous long term test results, then a secondary rejection occurs. There is no testing cost associated with a secondary rejection, as this cost has already been incurred at an earlier stage. Each milk tanker has both a trailer and a truck compartment. If these two compartments can be kept separate they can be accepted and rejected separately. We define $\lambda$ as the traceability factor, having a value of 1 or 0 depending on the whether we can distinguish between individual compartments on a tanker or not.

$$\lambda = \begin{cases} 1 \text{ With traceability} \\ 0 \text{ Without sufficient traceability} \end{cases}$$

Milk that is accepted into the factory reception stage, moves on to be processed as required. In this model milk leaves the Factory reception stage at a rate of $\Psi$ units per day.

Figure 4.5 shows a probability tree for how the probability of each acceptance and rejection combination can be calculated. The rejection of a tanker compartment is denoted by **r**, while **a** denotes its acceptance up to that point. We end up with three main possibilities; total acceptance, partial acceptance or total rejection. Introducing traceability gives us many more pathways and potential outcomes compared with those presented in Welsh et al. (2016)[4]. When there is insufficient traceability, $\lambda = 0$, this means we cannot between that distinguish between the tanker compartments until instantaneous tests are conduct upon arrival at the factory reception. In this case milk cannot be rejected due to previous long term tests and the model for the factory reception stage reverts to that presented in Welsh et al. (2016)[4].

The probability that one tanker compartment needs to be rejected is not totally independent of the other compartment's status. For example, if each tanker is collecting from three farms, the first farm's milk should fit comfortably in the first tanker compartment. Milk from the second farm visited will also be pumped into the first tank as well. It is likely however, that there is not enough room in the first tanker compartment, milk from the second farm will be then pumped into the second tank. Even if the milk from the second farm fits fully into the first tank, as most recent collection, any residue left in the pipe will likely be from farm 2 and have the potential to contaminate the next load of milk pumped through it into the second tank. In the case of an even number of farms $2n$, the farm potentially contaminating both tanks will be farm $n$ i.e. with four farms this will be farm 2. If, after milk collection is complete, we learn that one of the tanks on the tanker now contains contaminated milk, given that only two of the farms contributed milk to this tank there is a 1 in 2 chance that this was the middle farm in the collection run. There is the possibility that each tanker compartment is contaminated independently of the other by a different farm, with probability $\eta$. In figure 4.5, these conditional acceptance probabilities are represented by $\varsigma$ and $\varpi$ for primary and secondary rejections respectively. The example presented above would imply $\varpi = 0.5\eta$ and $\varsigma = 0.5\beta$.

### 4.5.1 Total acceptance

In this event both tanker compartments pass all tests (both instantaneous and long term) up to this point and are accepted. Therefore the probability of a milk delivery being accepted at the factory reception stage is simply the probability of a delivery attempt multiplied by the probability that the milk is not

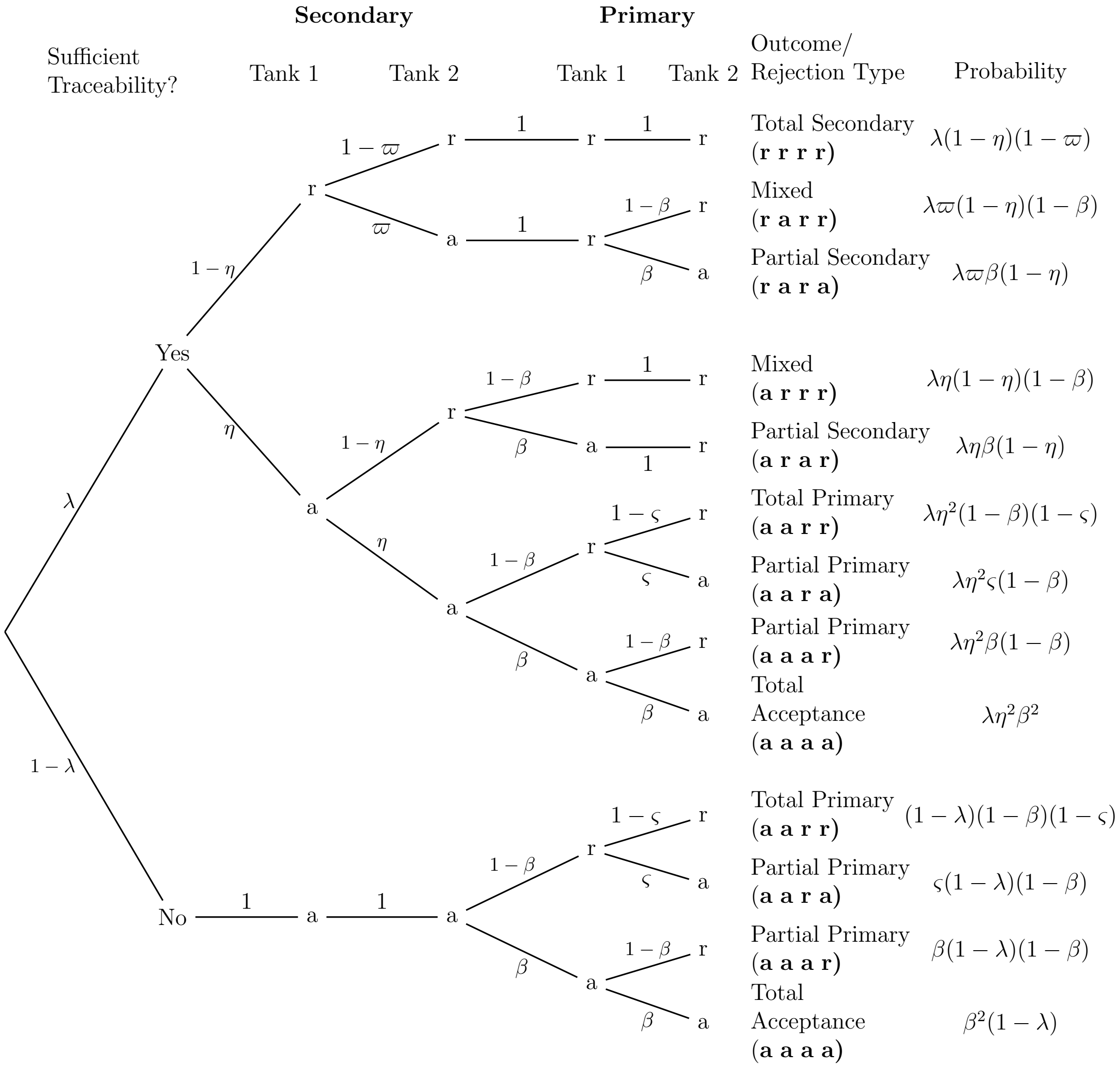

Figure 4.5: Probability tree showing the possible outcomes when milk is delivered to the factory.

rejected in either a primary or secondary rejection scenario. The probability of a delivery attempt is given in equation 4.4. The acceptance probabilities are given in figure 4.5 as $\lambda\eta^2\beta^2$ and $\beta^2(1-\lambda)$. Thus the probability of total acceptance, given that delivery is attempted, is given by

$$\begin{aligned}&\left(\lambda\eta^2\beta^2+\beta^2(1-\lambda)\right)\\ =&\beta^2\left(1+\lambda(\eta^2-1)\right)\end{aligned}\tag{4.11}$$

Where $\beta$ is the probability of a tanker compartment passing all instantaneous tests conducted at this stage (primary acceptance), and $\eta$ is the probability that no previous long term tests have detected any contamination (secondary acceptance). The resulting value change is

$$C_T - E_F - L_F$$

Where $C_T$ is the average value of milk in a tanker, $E_F$ is the cost of conducting tests at this stage and $L_F$ is the traceability cost incurred by accepted milk.

### 4.5.2 Partial acceptance

This occurs when just one tanker compartment is accepted by the factory. The other compartment could be rejected in either the primary or secondary rejection scenario. This will result in a slightly different value change, due to the testing costs only associated with a primary rejection.

A **Primary partial rejection** occurs with probability

$$\begin{aligned}&\left[\lambda\eta^2\varsigma(1-\beta) + \lambda\eta^2\beta(1-\beta) + \varsigma(1-\lambda)(1-\beta) + \beta(1-\lambda)(1-\beta)\right]\\ &= (1-\beta)(\varsigma+\beta)\left(\lambda(1+\eta^2-1)\right)\end{aligned} \tag{4.12}$$

This is the sum of the probabilities that the rejection occurs in either tank, as given in figure 4.5. The order of rejection has no effect on the outcome in this case. Because only one tanker compartment is rejected, half of the milk that was delivered is added to that currently contained in the factory reception stage. The full testing and tracing costs still apply however. The value change in the Factory reception stage in the case of a primary partial rejection is therefore

$$\frac{C_T - D_F}{2} - L_F - E_F$$

The probability of a **Secondary partial rejection** is given by

$$\begin{aligned}&[\lambda\varpi\beta(1-\eta) + \lambda\eta\beta(1-\eta)]\\ =&\lambda\beta(1-\eta)(\varpi+\eta)\end{aligned} \tag{4.13}$$

A secondary partial rejection may occur when the results of a previous long term test require one tanker compartment to be rejected. Because the milk has been rejected, no further testing is required for that tank. Testing is still required for the other tank however, this incurs a cost of $\frac{E_F}{2}$. The resulting total value change is

$$\frac{C_T - E_F - D_F}{2} - L_F$$

### 4.5.3 Total rejection

There are three ways a total rejection could occur. Both tanks could be rejected by previous long term tests in a total secondary rejection, both by instantaneous tests in a total primary rejection, or one of each resulting in a composite rejection.

A **Total primary rejection** requires both tanks to pass all earlier testing, then both be rejected due to the results of testing on arrival at the factory.

The probability of this occurring is given by

$$\begin{aligned}&\big[\lambda\eta^2(1-\beta)(1-\varsigma)+(1-\lambda)(1-\beta)(1-\varsigma)\big]\\&=(1-\beta)(1-\varsigma)\big(1+\lambda(\eta^2-1)\big)\end{aligned}\tag{4.14}$$

The value change in this case is simply the cost of testing and disposing of both tanker compartments

$$-E_F - D_F$$

In a **Total secondary rejection** both compartments are rejected due to earlier long term testing. The resulting value change consists solely of the disposal cost $-D_F$. The probability of this scenario occurring is given by

$$\lambda(1-\eta)(1-\varpi)\tag{4.15}$$

There is also the chance that one tanker compartment will be rejected based on earlier long term tests, while the other is rejected by instantaneous tests conducted upon arrival at the factory, resulting in a **composite rejection** scenario. The probability of this happening in a given time step is

$$\begin{aligned}&[\lambda\varpi(1-\eta)(1-\beta)+\lambda\eta(1-\eta)(1-\beta)]\\=&\lambda(1-\eta)(1-\beta)(\varpi+\eta)\end{aligned}\tag{4.16}$$

The change in the factory reception stage in this case is

$$\frac{-E_F}{2} - D_F$$

In this scenario there are no traceability costs incurred as no milk is accepted. Because of the within compartment mixing that will take place during tanker transport, the main contribution made by traceability at the factory reception stage is reduced testing costs. The traceability implemented up to and at this stage however will have an impact on the precision possible in later stages.

### 4.5.4 Passing on

Passing material on to the initial processing stage is the other possible event that can occur in a given time step. The probability that milk will leave the factory reception stage and move on for processing is dependent on the value of milk contained in the factory reception stage. The probability that milk will be passed on for processing is given by

$$\frac{F(t)}{N_F}\Psi\Delta t\tag{4.17}$$

There is now an upper limit on the movement each time step, based on the capacity of a reception silo. The change in value, in the factory reception stage, when milk is passed on for processing is $-C_F$, the average capacity of a reception silo.

### 4.5.5 Summary Equation

Equation 4.18 summarises the transition probabilities for the factory reception stage, where $u$ is the value of milk in the stage at time $t$ and $v$ is the amount at time $t + \Delta t$. The probability of accepting or rejecting a delivery is dependent on there being a delivery in the first place, which occurs with probability $\frac{T(t)}{N_T}\left(1 - \frac{F(t)}{N_F}\right)\mathcal{X}\Delta t$. This is reflected in the probabilities summarised in equation 4.18

$$p_{uv}(\Delta t) =$$

$$\begin{cases}
\beta^2(\lambda\eta + 1 - \lambda)\dfrac{T(t)}{N_T}\left(1 - \dfrac{F(t)}{N_F}\right)\mathcal{X}\Delta t & v = u + C_T - E_F - L_F \qquad \text{Acceptance} \\
(1-\beta)(\varsigma + \beta)(\lambda\eta + 1 - \lambda)\dfrac{T(t)}{N_T}\left(1 - \dfrac{F(t)}{N_F}\right)\mathcal{X}\Delta t & v = u + \dfrac{C_T}{2} - E_F - D_F - L_F \qquad \text{Partial Rejection I} \\
\lambda\beta(1-\eta)(\varpi + \eta)\dfrac{T(t)}{N_T}\left(1 - \dfrac{F(t)}{N_F}\right)\mathcal{X}\Delta t & v = u + \dfrac{C_T - E_F}{2} - D_F - L_F \qquad \text{Partial Rejection II} \\
(1-\beta)(1-\varsigma)(\lambda\eta + 1 - \lambda)\dfrac{T(t)}{N_T}\left(1 - \dfrac{F(t)}{N_F}\right)\mathcal{X}\Delta t & v = u - E_F - D_F \qquad \text{Rejection I} \\
\lambda(1-\eta)(1-\varpi)\dfrac{T(t)}{N_T}\left(1 - \dfrac{F(t)}{N_F}\right)\mathcal{X}\Delta t & v = u - D_F \qquad \text{Rejection II} \\
\lambda(1-\eta)(1-\beta)(\varpi + \eta)\dfrac{T(t)}{N_T}\left(1 - \dfrac{F(t)}{N_F}\right)\mathcal{X}\Delta t & v = u - \dfrac{E_F}{2} - D_F \qquad \text{Composite Rejection} \\
\dfrac{F(t)}{N_F}\Psi\Delta t & v = u - C_F \qquad \text{Passing on} \\
1 - \left[\dfrac{T(t)}{N_T}\left(1 - \dfrac{F(t)}{N_F}\right)\mathcal{X} + \dfrac{F(t)}{N_F}\Psi\right]\Delta t & v = u \qquad \text{Holding}
\end{cases} \tag{4.18}$$

### 4.5.6 Time Step Size

Similar to what was done in the tanker section 4.4.6, in keeping with what was discussed in section 4.2, the time step size must be small enough that the transition probabilities in equation 4.18 are each less than 1, and all sum to 1, allowing only one event to take place in each time step. This means we need to choose $\Delta t$ such that:

$$\left[\frac{T(t)}{N_T}\left(1 - \frac{F(t)}{N_F}\right)\mathcal{X} + \frac{F(t)}{N_F}\Psi\right]\Delta t \leq 1 \tag{4.19}$$

Again there is no global maximum for the left hand side of 4.19 but depending on whether $\Psi < \mathcal{X}$, $\Psi > \mathcal{X}$ or they are equal we can estimate local maximums and then solve for the maximum allowable value of $\Delta t$.

For $\Psi \geq \mathcal{X}$ the left hand side of equation 4.7 is maximised when $T(t) = 0$, and $\mathcal{X} = N_F$ giving

$$\begin{aligned} \left[0\left(1-\frac{N_F}{N_F}\right)\mathcal{X}+\frac{N_F}{N_F}\Psi\right]\Delta t &\leq 1 \\ \implies \Psi\Delta t &\leq 1 \\ \implies \Delta t &\leq \frac{1}{\Psi} \end{aligned} \tag{4.20}$$

If $\Psi < \mathcal{X}$ , the left hand side of equation 4.19 is maximised when $T(t) = N_T$ and $F(t) = 0$

$$\begin{aligned} \left[\frac{N_T}{N_T}(1-0)\,\mathcal{X}+0\Psi\right]\Delta t &\leq 1 \\ \implies 0+\mathcal{X}\Delta t &\leq 1 \\ \implies \Delta t &\leq \frac{1}{\mathcal{X}} \end{aligned} \tag{4.21}$$

Combining the two we can write

$$\Delta t \leq \frac{1}{\max\{\Psi, \mathcal{X}\}} \tag{4.22}$$

## 4.6 The Processing Stage

Here we look at how the value contained in the processing stage is changing over time. Processing of milk begins with separation, followed by standardisation. After these steps every dairy product undergoes pasteurisation[27,28]. The possible events include collection, rejection, passing on, and production. In this part of the model we also have both primary and secondary rejection and both result in a disposal cost. Only primary rejection incurs any testing cost, as secondary rejection is dependent on previous tests. After milk has undergone the initial processing stages of separation, standardisation and pasteurisation, it is passed on to different production processes depending on the intended end product. These further steps of processing are not included in our model.

At this stage of the production chain, the level of traceability can determine how much product is lost in the case of a contamination. With perfect traceability we will know which tanker, and which tank on this tanker the unsatisfactory material came from. We will also know what time it entered the reception silo and how much milk went in before and after it. A certain degree of mixing will occur, so we must allow for this, but it should be possible to reject only the unsatisfactory tank load along with a mixing allowance either side of this. Figure 4.6 shows a probability tree for the possible outcomes when milk enters this stage. Primary rejection, via “instantaneous” test results occurs with probability $\gamma$. The probability that a contamination is detected via a “long term ”test from a previous stage is given by $\theta$. $\mu$ takes the value 0 or 1 depending on how much of the silo may be salvageable, and

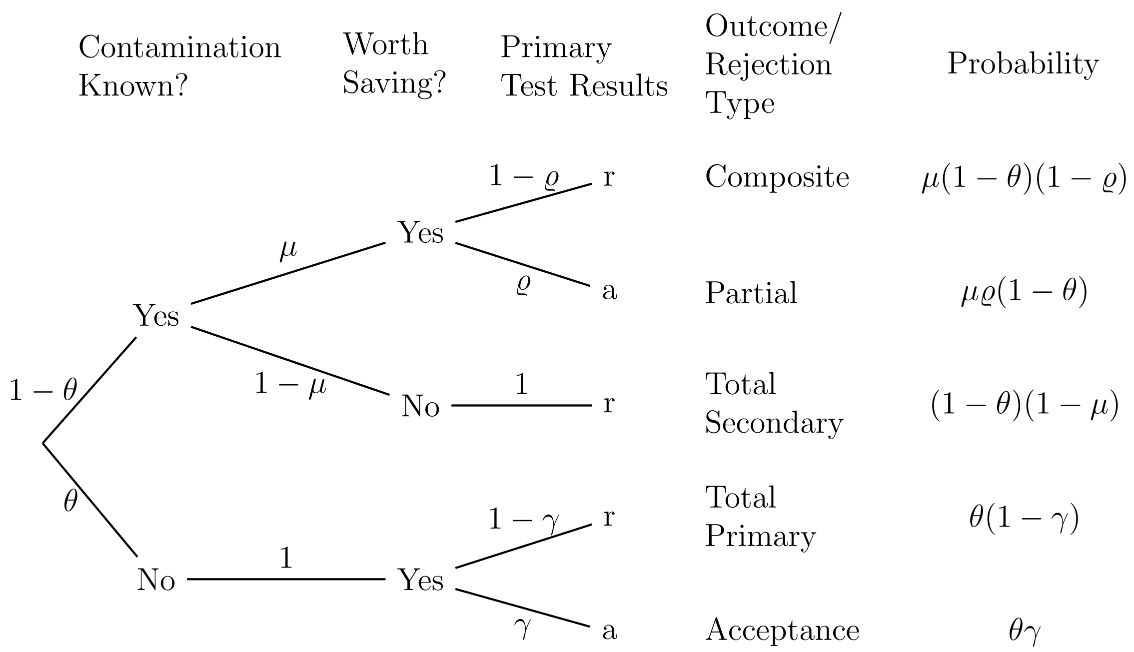

Figure 4.6: Probability tree for the outcomes when passing product from the factory reception stage to processing.

if this is worth more than the costs associated with accepting it. Retesting of the remaining product is necessary in this case to ensure any contamination has been successfully removed. The probability that a retest detects contamination is given by $\varsigma$.

### 4.6.1 Acceptance:

The probability that all of the milk in a reception silo is accepted for processing is

$$\frac{\theta\gamma F(t)}{N_F}\Psi\Delta t \tag{4.23}$$

The value change, now limited by the capacity of a reception silo, becomes

$$C_F - E_P - L_P \tag{4.24}$$

where $E_P$ is the cost associated with testing at this stage and $L_P$ is the traceability cost.

### 4.6.2 Partial Acceptance

This occurs when some milk is able to be salvaged following a secondary rejection. In the secondary rejection scenario, we are dealing with test results that have come from either the tanker collection or the factory reception stage. For a secondary rejection to be possible in the processing stage $\theta < 1$. The closer

$\theta$ is to 0, the more likely we are to reject product via secondary rejection. The value change in this scenario largely depends on the traceability in previous stages. A factory silo has a capacity of $C_F$. If a secondary rejection occurs it means some of the milk in this silo has come from a tanker identified as containing contaminated milk after it delivered its load. If we have sufficient traceability at the tanker level ($\lambda = 1$) we will be able to identify which compartment of the affected tanker contained the contaminated milk. Remember $\lambda = 0$ when traceability is insufficient to distinguish between individual tanks on a milk tanker. The level of traceability at the factory reception level, $\ell$, determines how precisely we can locate the contaminated tank load within the silo. $\ell$ must be between 0 and 1. If $\ell = 1$, we have perfect traceability and can identify the contaminated load exactly. Obviously, since milk is liquid some mixing will occur. We model this mixing error using the parameter $\varepsilon$. This can take any value from 0, implying no mixing error, to $C_F(1+\lambda) - C_T$, where one contaminated tank load contaminates the whole silo. Once the contaminated milk is removed from the silo, the remaining product left in the silo is given by

$$\ell\left(C_F - \frac{C_T + \varepsilon}{1+\lambda}\right) \tag{4.25}$$

The $1 + \lambda$ in the denominator becomes either 2 or 1 depending on whether we can just reject one tank, or must reject the whole tanker load. Equation 4.25 assumes that only one contaminated taker load may be identified in a reception silo. While it is theoretically possible that there may be more than one contaminated load in a single silo, for the sake of simplicity in our model we only allow for one to be the cause of a partial rejection. The actual incidence of contamination will generally be low enough that we can safely deal with any potential multiple contaminations through the composite rejection scenario described below in equation 4.31.

The value contained in the processing stage will increase by the amount of milk accepted and decrease by the costs of retesting, tracing and disposal. We also assume the test cost $E_P$, and the tracing cost $L_P$ are the same regardless of the volume being tested. The probability of a partial acceptance is given by

$$\frac{\mu\varrho(1-\theta)F(t)}{N_F}\Psi\Delta t \tag{4.26}$$

A disposal cost of $D_P$ is incurred for each unit of milk that must be discarded. The total value change to the processing stage, in the event of a partial secondary rejection is

$$\begin{aligned}&\ell\left(C_F - \frac{C_T + \varepsilon}{1+\lambda}\right) - \left(C_F - \ell\left(C_F - \frac{C_T + \varepsilon}{1+\lambda}\right)\right)D_P - E_P - L_P\\ =&\ell\left(C_F - \frac{C_T + \varepsilon}{1+\lambda}\right)(1 + D_P) - C_F D_P - E_P - L_P \end{aligned} \tag{4.27}$$

In order for the partial acceptance to be cost effective, equation 4.27 must be greater that 0. Therefore

$$\mu = \begin{cases} 0 & \text{for } \ell\left(C_F - \frac{C_T+\varepsilon}{1+\lambda}\right) \leq \frac{C_F D_P + E_P + L_P}{1+D_P} \\ 1 & \text{for } \ell\left(C_F - \frac{C_T+\varepsilon}{1+\lambda}\right) > \frac{C_F D_P + E_P + L_P}{1+D_P} \end{cases} \tag{4.28}$$

This value that is salvaged must still undergo instantaneous test for entry into the processing stage and may potentially be rejected, resulting in a composite rejection.

### 4.6.3 Total rejection

There are three different scenarios in this stage that could lead to total rejection; primary rejection, secondary rejection (where any salvageable milk is not worth the cost), or a composite rejection where some milk is salvaged following a secondary rejection but fails subsequent testing.

The probability of a **Primary** rejection is given by

$$\frac{\theta(1-\gamma)F(t)}{N_F}\Psi\Delta t \tag{4.29}$$

In the case of a primary rejection, it is not known when the unsatisfactory material entered the silo as no previous tests have picked it up. The whole reception silo is lost and the value change becomes $-E_P - D_P$.

In a **Secondary** rejection, traceability will determine what proportion of the reception silo must be disposed of. If the value of milk to be accepted following a partial rejection is less than the costs associated with accepting it, the whole silo will be rejected. The probability of this occurring is

$$\frac{(1-\theta)(1-\mu)F(t)}{N_F}\Psi\Delta t \tag{4.30}$$

The value change in this scenario is $-D_P$.

The probability of the retest detecting a problem is $(1-\varrho)$.
Thus a **Composite** rejection will occur with probability

$$\frac{\mu(1-\theta)(1-\varrho)F(t)}{N_F}\Psi\Delta t \tag{4.31}$$

In this case we lose both the costs of testing and disposal, $-E_P - D_P$. The disposal cost is the same in each of these situations as the whole tanker is rejected in each of them.

### 4.6.4 Passing on:

The probability that material is passed on from this processing stage is given by

$$\Omega\Delta t \tag{4.32}$$

There are no costs associated with passing material on to the next stage, but the value of material in the processing stage will decrease by $Q$, the value of milk that moves in each passing on event.

### 4.6.5 Producing

If no milk is coming into, or leaving the processing stage, we assume that all factories are busy producing processed milk ready to pass on to the next stage. The probability that all the factories are producing is the probability that none of them are doing anything else, that is

$$1 - \left[\frac{F(t)\Psi}{N_F} + \Omega\right]\Delta t \tag{4.33}$$

When all factories are producing only, there is no value change in the processing stage.

### 4.6.6 Summary of Transition Probabilities

In this stage of the supply chain, $g$ is the amount of product in this stage at time $t$ while $h$ is how much is contained at time $t + \Delta t$. Equation 4.34 shows the probability of each possible value of $h$ given the starting value $g$

$$p_{gh}(\Delta t) =
\begin{cases}
\dfrac{\theta\gamma F(t)}{N_F}\Psi\Delta t & h = g + C_F - E_P - L_P & \text{Acceptance} \\
\dfrac{\mu\varrho(1-\theta)F(t)\Psi}{N_F}\Delta t & h = g + \ell\left(C_F - \dfrac{C_T+\varepsilon}{1+\lambda}\right)(1 + E_P + D_P) & \\
 & \quad - C_F(E_P + D_P) - L_P & \text{Partial Acceptance} \\
\dfrac{\theta(1-\gamma)F(t)}{N_F}\Psi\Delta t & h = g - E_P - D_P & \text{Rejection I} \\
\dfrac{(1-\theta)(1-\mu)F(t)}{N_F}\Psi\Delta t & h = g - D_P & \text{Rejection II} \\
\dfrac{\mu(1-\theta)(1-\varrho)F(t)}{N_F}\Psi\Delta t & h = g - D_P - E_P & \text{Retest \& Reject} \\
\Omega\Delta t & h = g - Q & \text{Passing on} \\
1 - \left[\dfrac{F(t)\Psi}{N_F} + \Omega\right]\Delta t & h = g & \text{Producing}
\end{cases} \tag{4.34}$$

### 4.6.7 Time Step Size

As in both the tanker and factory reception stages, $\Delta t$ must be small enough that only one event can take place in the processing stage in any time step. The transition probabilities in equation 4.34 must all sum to 1, and individual have a value between 0 and 1. In the case of the processing stage this reduces to:

$$\left[\frac{F(t)\Psi}{N_F} + \Omega\right] \Delta t \leq 1 \tag{4.35}$$

The left hand side of equation 4.35 is maximised when $F(t) = N_F$, giving

$$\left(\frac{N_F\Psi}{N_F} + \Omega\right) \Delta t \leq 1 \tag{4.36}$$

$$\implies \Delta t \leq \frac{1}{\Psi + \Omega} \tag{4.37}$$

The final time step value chosen must fit the restrictions derived in all three stages, given in equations 4.10, 4.22 and 4.37. This means $\Delta t$ will be such that

$$\Delta t \leq \frac{1}{\max\{\Phi, \mathcal{X}, (\Psi + \Omega)\}} \tag{4.38}$$

## 4.7 Traceability and Interaction Between Stages

We have developed and described the model in three separate stages. While each stage has its own set of transition probabilities, the stages all interact with each other and affect how each other's transition probability values change from time step to time step. The state vector for time $t$ is $(T(t), F(t), P(t)) = (i, u, g)$, this becomes $(j, v, h)$ over the timestep $\Delta t$. When we consider the value output by the model, we will be looking at the value coming out the end of the supply chain which is the result of all three stages working together. In assessing the value of traceability we will be analysing the value it contributes across all three stages of the supply chain, regardless of what stage the traceability parameters and effects are directly influencing.

# 5 Model Simulation Results and Discussion

In this section we simulate the model developed in section 4 to investigate the effects of traceability. It is assumed that the dairy producer wants to minimise product loss due to contamination, and thus maximise overall product output. Welsh et al.(2016)[4] explores the value of milk flow from the farm to the factory. This paper extends this model by modifying it to include traceability parameters. Throughout this section we compare the results obtained with the additional feature of traceability, to those obtained in Welsh et al.'s (2016)[4] milk flow model.

In comparing the simulations we will obtain a value for certain levels of traceability, given that the desired affect is achieved. If traceability is perfect, then the location of any unit of product is always known perfectly at any point in time. This means in the event of product rejection only product likely to be contaminated is rejected. For our simulations in this section, if we invest in traceability it is perfect traceability.

If we are able to predict rejection in later stages based on tests done when the milk is collected from the farm, then we can shift the rejection on arrival at the factory from primary to secondary. We will estimate how much such a preemptive rejection is worth based on how much it reduces loss via primary rejections.

Product loss due to contamination could occur at any of the factory reception or processing stages, or both. The loss could also be due to contamination occurring at any stage prior to detection. If a contamination can be identified earlier in the supply chain, product loss can be reduced. If we increase surveillance and traceability, we will increase the rate of secondary rejection but in the process, the number of primary rejections will be reduced. In this section we explore the impact of using traceability to reduce loss at each of these stages.

## 5.1 Parameter Values for Dairy in New Zealand

In order to explore the impact of traceability, we use the model developed in section 4 to simulate a variety of different scenarios. To ensure the simulations reflect reality, we base the parameter values for the model on data from Fonterra, the largest dairy company in the New Zealand industry. Some parameter values, such as collection frequency, will remain the same throughout the simulations, however some will be varied in order to explore the impact of traceability in different scenarios. The parameters are all summarised in table 5.1.

### 5.1.1 Milk Tanker Parameters

The number of dairy herds in New Zealand has been steadily declining since 1980, but has recently begun to increase again slightly, beginning in the 2007/08 season. The number of herds increased by 43 in the 2014/15 season to 11970[29].

The capacity of an on-farm silo is based on each cow producing 25 litres of milk each day at the peak of the season. Fonterra currently requires their suppliers to have a minimum of 400 litres available at each collection[12]. We estimate an average collection amount per day, during the main season, based on herd size and cow output data. The details of this are given in the appendix.

Fonterra's tanker fleet operates 24 hours a day, with a 10-12 hr day shift involving 3-6 runs per tanker in Darfield, one of Fonterra's key factories. There is a 1-2 hr turnover before the night shift starts with a similar pattern to the day shift[30]. We assume that a similar structure applies to tanker operation throughout New Zealand. Each run involves delivering to the factory once

| **Parameter** | **Description** | **Initial values** |
|---|---|---|
| $V$ | Amount of milk collected from an on-farm vat | \$3050 |
| $\Phi$ | Frequency of collection a by tanker | 11970 |
| $\mathcal{X}$ | Frequency of delivery to factory | 3990 |
| $\Psi$ | Frequency with which milk enters processing | 343 |
| $\Omega$ | Frequency of production | 300 |
| $E_T$ | Costs of testing milk at collection site | \$1.90 |
| $L_T$ | Cost of tracing collected milk | 0 |
| $E_F$ | Cost of testing milk upon delivery | \$1.90 |
| $L_F$ | Cost of tracing accepted milk | 0 |
| $D_F$ | Cost of disposing of unwanted milk | 0 |
| $E_P$ | Cost of testing prior to processing | \$1.90 |
| $L_P$ | Cost of tracing milk accepted for processing | 0 |
| $D_P$ | Cost of disposing of unwanted milk at factory level | 0 |
| $\alpha$ | Probability of acceptance by tanker | 0.9999 |
| $\beta$ | Probability of passing tests upon arrival at factory | 0.99 |
| $\gamma$ | Probability of passing pre-processing tests | 0.99999 |
| $\eta$ | Probability there is no secondary rejection | 0.9999 |
| $\theta$ | Probability of no secondary rejection | 0.9899 |
| $\varsigma$ | Type 1 conditional 2nd tank acceptance probability | 0.495 |
| $\varpi$ | Secondary conditional 2nd tank acceptance probability | 0.49995 |
| $\varrho$ | Partial silo acceptance probability | 0.99999 |
| $\lambda$ | Factory Traceability Coefficient | 1 |
| $\varepsilon$ | Silo mixing error | \$42820.8 |
| $\ell$ | Processing traceability factor | 1 |
| $C_T$ | Capacity of one milk tanker | \$10705.20 |
| $N_T$ | Capacity of tanker stage | \$5620230 |
| $C_F$ | Capacity of one reception silo | \$89000 |
| $N_F$ | Capacity of factory reception stage | \$8811100 |
| $C_P$ | Capacity of one separator unit | \$561755 |
| $Q$ | Process exit amount | \$187,230 |
| $N_P$ | Capacity of Processing stage | \$18537830 |
| $\Delta t$ | Time step (days) | 0.00008102 |

Table 5.1: Parameter values in the perfect traceability scenario. All frequencies are the average number of occurrences per day.

every run, a tanker completes an average of 7.6 deliveries per day.

The price Fonterra pays farmers in \$ per kilogram of milk solids (kg MS), is calculated based on the Global Dairy Trade (GDT) prices for whole milk powder (WMP), skim milk powder (SMP), anhydrous milk fat (AMF), butter and buttermilk powder (BMP). Because these prices are in US dollars, the exchange rate must be taken into account before Fonterra subtracts the Lactose cost and the cash and capital cost[31]. The farm gate milk price for the 2014/2015 season was \$4.40[32]. This price takes into account fixed costs such as transport and manufacturing as well as allowing for appropriate returns on investment[33].

$V$ is the average amount of milk collected from a farm vat. Based on the information in Table A.1, during the main milking season each farm is producing an average of 693 kg MS per day, during the peak months of the year this jumps to 808 kg MS. Using a price of \$4.40 per kg MS we can estimate $V$, the average value of milk produced by and collected from each farm per day. $V = \$4.40 \times 693 = \$3049.20$.

$\Phi$ is the frequency of collection attempts. We can estimate this as the number of on farm vats that are collected from each day. There were 11970 herds supplying Fonterra in the 2014/2015 season[29]. If we assume all herds are being collected from everyday during the main production season $\Phi = 11970$ collections each day.

$E_T$ is the estimated cost of testing milk when it is collected by the tanker. LIC charges a rate of \$1.99 per animal, to conduct a suite of tests for milk quality[34]. Given that Fonterra conducts most of their testing themselves, we expect test not to cost the any more the the LIC price, we estimate $E_T = \$1.90$ NZD.

$L_T$ is the cost associated with traceability implemented at the tanker stage. While we have include this term in out model, in the following simulations we are trying to estimate the overall value of traceability, from which this would be a direct subtraction. It is simpler in this case to estimate the total value, to which the cost of the whole traceability system can be compared. Therefore we set $L_T = 0$.

$\alpha$ is the probability that the milk passes all testing and is accepted by the tanker. Information obtained though discussion with Fonterra staff suggests we set $\alpha = 0.9999$ (T. Kirk, oral communication, November 2015). This translates to an everyday rejection rate of 0.01%, or approximately 120 vats each day.

$C_T$ is defined as the average capacity of one milk tanker. Each milk tanker, truck and trailer unit can hold 28,800 L of milk[35], this is equivalent to 2433kg MS, therefore $C_T = \$10705.20$.

$N_T$ is the capacity of the entire fleet of tankers. Fonterra operates a fleet of 525 tankers[11] so $N_T = \$10705.20 \times 525 = \$5620230$.

$\mathcal{X}$ is the frequency with which milk tankers deliver milk to the factory. If each tanker collects from an avererage of 3 farms during each run, then we need $\frac{11970}{3} = 3990$ tanker runs every day. Given there is one delivery at the end of each run $\mathcal{X} = 3990$.

### 5.1.2 Factory Reception Parameters

Fonterra has the capacity to process about 70,000,000 litres of milk per day during the peak season[36]. Milk reception silos range in size from 225,000 to 500,000 litres.

$E_F$ is the cost of testing milk as it arrives at the factory. Similar to $E_T$ in section 5.1.1, we estimate $E_F = \$1.90$.

$L_F$ is the cost associated with traceability implemented at the factory reception stage. for the reasons outline in section 5.1.1 regarding $L_T$ we set $L_F = 0$.

$\beta$ is the probability that a tanker load is accepted by factory. This is the stage with the greatest rate of rejection. Based on conversations with Fonterra personnel, an average of 1% of milk is discarded upon arrival at the factory (T. Kirk, oral communication, November 2015). This gives us $\beta = 0.99$.

$\varsigma$ is the probability the second tank of a tanker will be accepted, given that the first tank was rejected. Because each tanker visits an average of 3 farms,there is about a 50% chance that the contaminated load spans both tanks, as explained in section 4.5. Taking this into account along with the possibility there is a second unrelated contamination we can estimate $\varsigma = 0.5\beta = 0.495$

$\lambda$ is the traceability coefficient. It represents whether we have sufficient traceability to distinguish between tanks on a tanker or not. In this scenario we can distinguish between tanks, therefore $\lambda = 1$.

$\eta$ is the probability that a tank load of milk is not rejected by a secondary rejection upon delivery to the factory. This is essentially a delayed rejection of farm vats and the rejection rate reflects that. We use $\eta = \alpha = 0.9999$

$\varpi$ represents the conditional probability that the second tank is accepted given the first tank of the tanker is rejected by a secondary rejection. Because we have perfect traceability, the second tank will only be rejected if milk from contaminated farm vat was loaded into both tanks. As outlined in section 4.5, with perfect traceability the second tank can be accepted in 50% of situations. So we let $\varpi = 0.5\eta = 0.49995$.

$D_F$ is the costs associated with disposing of rejected milk at the factory reception level. Most rejected milk can be used as calf feed or sprayed on crops as fertiliser. Fonterra does contract tankers from outside their own fleet to transport this rejected milk, but the associated costs can generally be recouped in the price paid for this rejected product. Because of this we set $D_F = 0$.

$C_F$ Each processing site has multiple reception silos, as mentioned above, a typical paediatric site has silos of 225,000 Litre capacity. This equates to 20228kg MS therefore, $C_F = \$89000$.

$N_F$ The typical paediatric processing site has three reception silos, with 33 processing sites around the country this gives a total capacity of $33 \times 3C_F = 2,002,572$ kg MS leading to $N_F = \$8,811,100$.

$\Psi$ is the rate at which milk moves into the processing stage from the reception silos. Fonterra operates 33 processing sites[37] and processes 70,000,000 litres of milk per day. $\frac{70000000}{33 \times 225000} \approx 9.43$ therefore, each site would need to process 9 or 10 silos of raw milk each day, a total of 311 silos each day. So $\Psi = 311$.

### 5.1.3 Processing Parameters

Aside from small quantities of on farm sales, the first steps in production required for all dairy products produced in New Zealand are separation, standardisation and pasteurisation[27,28]. In this model we will focus on the first of these stages, separation and standardisation. Typically a factory has a bank of several separators which feed into several silos for cream and skim milk. Each separator bowl has a volume of 50 litres and is capable of separating 33,000 litres every hour[38].

$E_P$ Assuming the range of tests conducted pre-processing is similar to those conducted before acceptance into the factory, we set $E_P = \$1.90$

$L_P$ is the cost associated with traceability implemented at the processing stage. Because any impact of traceability implemented at this stage would not be seen till later stages not currently modelled, and we are assessing the value contributed by traceability, we set $L_P = 0$.

$\gamma$ The rate of rejection before entry into the processing stage, was also discussed with staff at Fonterra. The processing stage has the lowest rejection rate of the three stages (T. Kirk, oral communication, November 2015). Once the milk is inside the factory the environment is much more controlled, the potential for contamination or spoilage is greatly reduced. We set the chance of rejection at 0.001%, implying $\gamma = 0.99999$.

$\theta$ is the probability that a factory reception silo does not have any of its contents rejected by a secondary rejection. $\theta$ reflects the value of $\beta$ and that of $\alpha$, the probability that material is not rejected instantly at the the tanker and factory reception stages. Therefore we have $\theta = 0.9999 \times 0.99 = 0.9899$.

$\varrho$ is the probability that product leftover following a partial rejection is accepted following retesting. A partial acceptance can only take place in a secondary rejection situation. Therefore, in a perfect traceability scenario $\varrho = \gamma = 0.99999$.

$\ell$ determines what portion of a contaminated silo can potentially be accepted based on the level of traceability employed in the factory reception stage. Because we have perfect traceability in this scenario $\ell = 1$.

$\varepsilon$ represents the mixing of milk in the reception silo, and the amount either side of a contamination that must be rejected along with the contaminated tank volume. Because milk is liquid this value is relatively large, we set this at $\varepsilon = 4C_T = \$42820.8$

$D_P$ As discussed in reference to $D_F$, disposal cost is negligible so we can set $D_P = 0$.

$C_P$ The typical paediatric processing site has a bank of 3 separators feeding into 2 cream silos, 3 skim milk silos and 2 excess silos. This gives a total capacity of $3 \times 50 + 2 \times 95,000 + 3 \times 350,000 + 2 \times 90,000 = 1,420,150$ Litres[39], equivalent to $127,670$ kg MS $C_P = \$561,755$.

$N_P$ is the total capacity of the processing stage. As there are 33 processing sites around the country $N_P = \$18,537,830$.

$Q$ is the value of milk leaving the processing stage each time step. because in reality there is continuos flow through this stage we have some freedom in the value we choose for Q. We really just require that Q is large enough to keep up with Fonterra's rate of production, but smaller than $C_P$. For our simulations we use the sum of the silo capacity in $C_P$ divided by 3 9the number of silos feeding in. This gives us $\frac{1,420,000}{3} = 473333$ kg MS, which is equivalent to $Q = \$187,230$.

$\Omega$ is the rate at which product leaves the initial processing stage and moves on to further processing. To allow for enough product flow each day we require that $\Omega \times Q > \$27,689,200$, the value of milk produced by Fonterra every day in peak season[36]. We use $\Omega = 300$.

### 5.1.4 Timestep

Using equation 4.38, we can calculate the maximum allowable value for the time step as

$$\begin{aligned} \Delta t &\leq \frac{1}{\max\{11970, 3990, (343 + 7100)\}} \\ \implies \Delta t &\leq \frac{1}{11970} = 0.00008354 \text{ days or } 7.218 \text{ seconds} \end{aligned}$$

This suggests we use a time step size of 7 seconds, giving $\Delta t = 0.00008102$. This will allow a tanker to be delivering or collecting milk almost every time step.

## 5.2 Reducing Factory Reception Rejection

The first point in the supply chain where the effect of traceability may be seen is the factory reception stage. Initially we will analyse the effect of traceability on total output when it effects this stage alone. Figure 5.1 shows the 24 hour production output value, in a scenario where $\beta = 0.75$, $\lambda = 0$ and $\eta = 1$. In this scenario 25% of product needs to be rejected upon arrival at the factory, due to some previously undetected contamination, or issue with the milk. The difference in production when we transfer this rejection from primary to secondary via traceability (where $\lambda = 1$, $\beta = 1$ and $\eta = 0.75$) is also shown. In this simulation we eliminate primary rejection upon arrival at the factory reception stage, by increasing the secondary rejection rate (due to tests conducted prior to collection by the tanker) to 25%. That is, through increased traceability we allow for more contaminated product to be detected before testing, resulting in a higher secondary rejection rate, but a lower primary rejection rate, meaning less product is discarded overall. This results in us retaining \$1.8 million of product per day that would otherwise be disposed of. In each case the acceptance rate of milk from farm vats is held constant at $\alpha = 0.9999$.

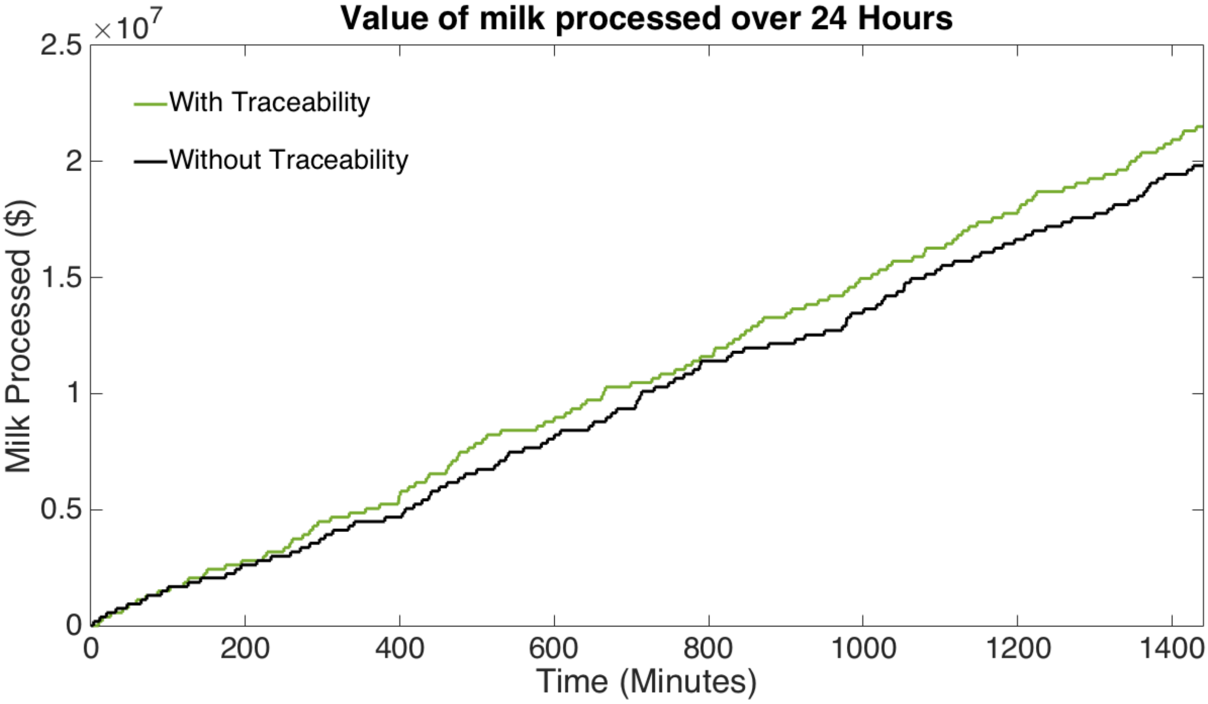

Figure 5.1: Simulations of milk produced over a 24 hour period, with a 25% rejection rate of product entering the Factory reception. One simulation of each scenario is shown.

Table 5.2 summarises the results of 500 simulations, transferring primary rejection to secondary, for a variety of $\beta$ values. The new minimum $\beta$ is the rate of primary acceptance required to ensure a positive traceability impact. For example we started with $\beta = 0.85$, when we introduce traceability to the effect that $\eta = 0.85$ instead, the result of this must be that the primary acceptance rate increases to at least $\beta = 0.96$ to achieve a grater output value than the scenario without traceability. The potential gain if we are able to eliminate primary rejection altogether, is given in the last two columns of table 5.2 as a total dollar value and a percentage increase from the no traceability scenario. Each value given is the average total 24 hour output over 500 simulation runs. For all of the simulations we hold the collection acceptance rate constant at $\alpha = 0.9999$.

We can see that as the initial primary acceptance rate increases, the potential for improvement is reduced. Even though the percentage of potential improvement decreases, overall the output value still increases. The improvement in primary acceptance rates is also reduced as the initial primary acceptance rate increases. The last row of table 5.2 shows the outcome if we are able to eliminate all primary rejection. The total output value becomes $\$26,181,496 + \$200,090 = \$26,381,586$. Given that, while paying for testing

| Factory Reception Implementation only | | | | |
|---|---|---|---|---|
| No traceability | | With Traceability | | |
| Initial $\beta$ | 24 Hour Output Value | Minimum New $\beta$ | Potential Gain | |
| | | | $ | % |
| 0.75 | $20,144,014 | 0.93 | $1,902,164 | 9.4% |
| 0.8 | $21,352,034 | 0.94 | $1,561,426 | 7.3% |
| 0.85 | $22,592,592 | 0.96 | $1,377,442 | 6.1% |
| 0.9 | $23,933,756 | 0.97 | $908,446 | 3.8% |
| 0.95 | $25,177,680 | 0.99 | $576,708 | 2.3% |
| 0.99 | $26,181,496 | 1 | $200,090 | 0.8% |

Table 5.2: Simulation outcomes for various rejection and traceability scenarios, where product is only rejected upon delivery to the factory reception stage.

and current traceability standards, Fonterra can produce over $26,000,000 of product per day[36], so this model produces the output we would expect.

Figure 5.2 summarises the results from the 500 simulation runs with box plots, for each primary rejection scenario, and for each scenario where all of the rejection is managed through traceability and secondary rejections. All of the plots are fairly symmetrical, they each have quite a large spread overall but the interquartile ranges are relatively small. In each case there is overlap between the value processed in the with and without traceability scenarios, but as the acceptance rate decreases, and the potential for improvement increases, the plots become more distinct. If we are able to mostly remove primary rejection, without increasing secondary rejection to the same rate, we can have scenarios with no overlap at all. For example in a scenario where we are rejecting 20% of product as it arrives at the factory via primary rejection, if we are able to eliminate this primary rejection by improving traceability and product identification such that 10% of product is rejected in secondary rejections, we will improve total production value everyday. Even if there is some overlap between scenarios, remembering that these are simulations of one day's production, the 95% confidence interval for the mean in each traceability vs no traceability comparison is distinct, meaning on average traceability is an improvement in each case. We only show box plots for scenarios up a 95% acceptance rate as, while the 95%confidence intervals are still distinct, the boxplots already show a significant amount of overlap which will only get worse as the potential for improvement decreases. The values for the 99% acceptance rate are still given in table 5.2 however.

## 5.3 Reducing Product loss at Processing Entry

When milk enters the processing stage it can be rejected via either primary or secondary rejection as it enters processing. The earlier in the supply chain a contamination is detected, the less product that is potentially contaminated

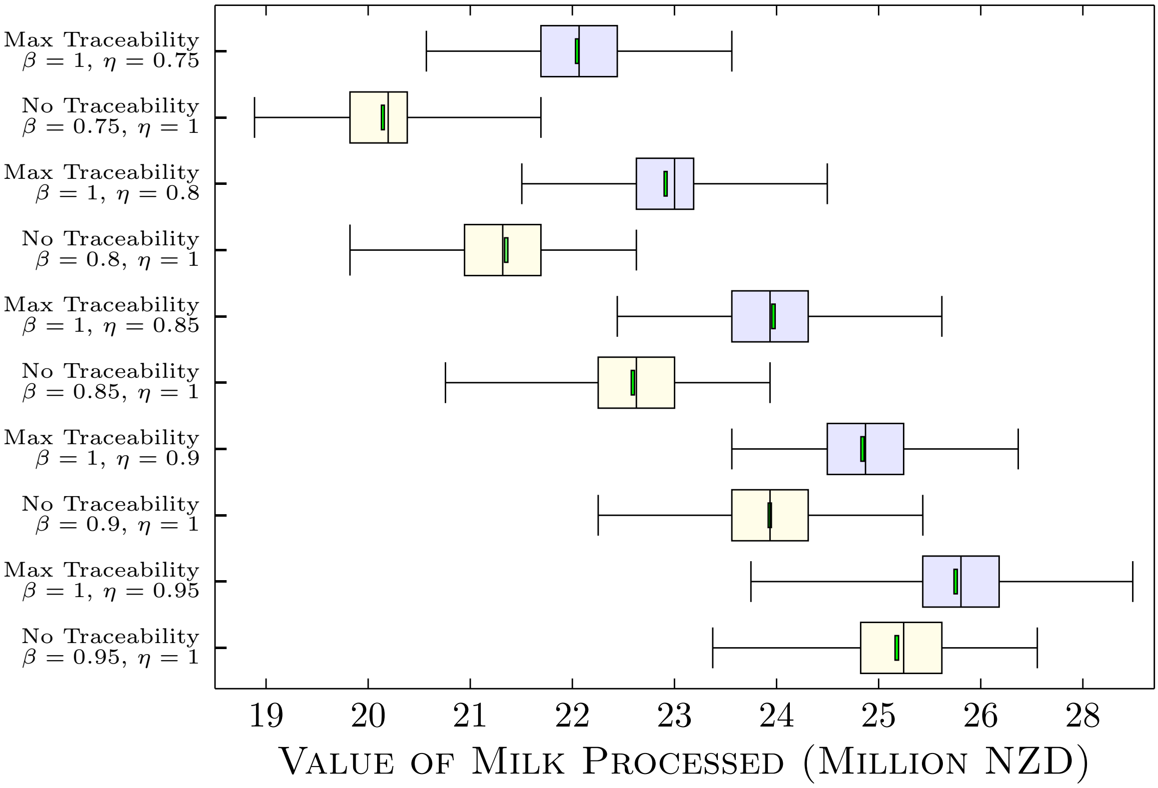

Figure 5.2: Box plots for the value of milk processed over a 24 hour period in various scenarios, with and without traceability over 500 simulations. The confidence interval of the mean is also shown as a small green box within each plot.

and necessarily disposed of. With a good traceability system, even if the contamination is not known until the product is ready to enter the processing stage, we can identify the original contaminated product, and any product that contamination may have spread too. This allows us to dispose of only product likely to be contaminated, thus reducing losses. For the scenarios simulated in this section, information relevant to traceability, is collected at the tanker and factory reception stages, but this information is only applied as material enters the processing stage. We run simulations for various values of $\gamma$, the rate of primary rejections at the point of processing entry. We compare these results to scenarios including traceability to investigate how much of an impact it will have on loss reduction and thus, overall production. Figure 5.3 shows one simulation each of before and after the implementation of traceability effects, using an acceptance rate of $\gamma = 0.75$ and $\eta = 1$. In this scenario, 25% of product entering the processing stage needs to be rejected, due to some previously undetected contamination. If we can identify more of this product for secondary rejection, as in the with traceability simulation where $\gamma = 1$ and $\eta = 0.75$, less will need to be rejected via primary rejection, meaning we reduce losses overall. The potential value of traceability in this scenario, as simulated in figure 5.3, is NZ \$3,600,000. Table 5.3 shows the simulation results for various rejection rates $\gamma$. Each value given is the average 24

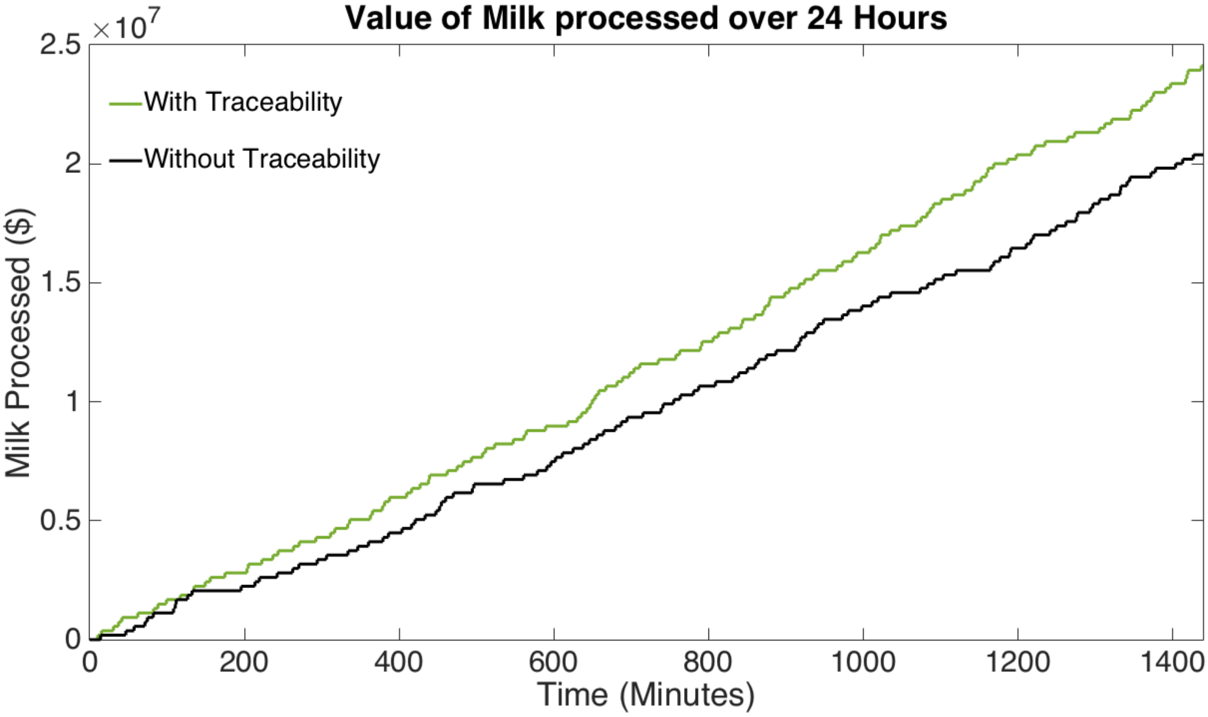

Figure 5.3: A single simulation of milk produced over a 24 hour period, with a 25% rejection rate of product entering the processing stage.

hour production value over 500 simulations. The outcome with no traceability, is given along with the potential gain if we are able to use traceability to eliminate primary rejections. The minimum primary acceptance rate needed to make an output improvement when we increase secondary rejection rates (the minimum new $\gamma$) is also given. Again the biggest potential gain is seen with the smallest acceptance rate, as this is logically where we will have the most room to improve.

| Processing Entry Implementation only | | | | |
|---|---|---|---|---|
| No traceability | | With Traceability | | |
| Initial $\gamma$ | 24 Hour Output Value | Minimum New $\gamma$ | Potential Gain 24 Hour Value | % Gain |
| 0.75 | $21,567,212 | 0.86 | $2,575,236 | 11.9% |
| 0.8 | $22,568,772 | 0.9 | $2,012,378 | 8.9% |
| 0.85 | $23,600,896 | 0.93 | $1,435,786 | 6.1% |
| 0.9 | $24,516,822 | 0.95 | $979,506 | 4% |
| 0.95 | $25,426,016 | 0.98 | $566,984 | 2.2% |
| 0.99 | $26,069,670 | 1 | $126,786 | 0.5% |

Table 5.3: Simulation results (average of 500 simulation runs) for scenarios where product is only potentially rejected as it enters the processing stage.

Comparing the implementation of traceability at the factory reception stage with the processing entry, over all we see more potential value retention. As the initial acceptance rate increases, the gap between these values closes. The new value for the primary acceptance rate after the implementation of traceability also does not need to be as high in the processing entry in order to see an improvement.

Figure 5.4 shows box plots summarising the results from 500 simulation runs; for each of the primary rejection scenarios given in table 5.3, along with plots for the scenarios where all primary rejection is eliminated via traceability and secondary rejection. Again most of the plots seem fairly symmetrical. The plots for scenarios with traceability seem to separate from the no traceability plots, more noticeably in this stage as rejection rates increase, when compared with the plots in figure 5.2. In fact the overall production value is consistently better in a scenario where we are rejecting 25% of product in secondary rejections ($\theta = 0.75$), with no primary rejection, than in the scenario where only 15% of product is rejected in primary rejections ($\gamma = 0.85$) with $\theta = 1$. The difference is still quite small with larger acceptance rates, however the 95% confidence interval for the mean is still distinct in each case.

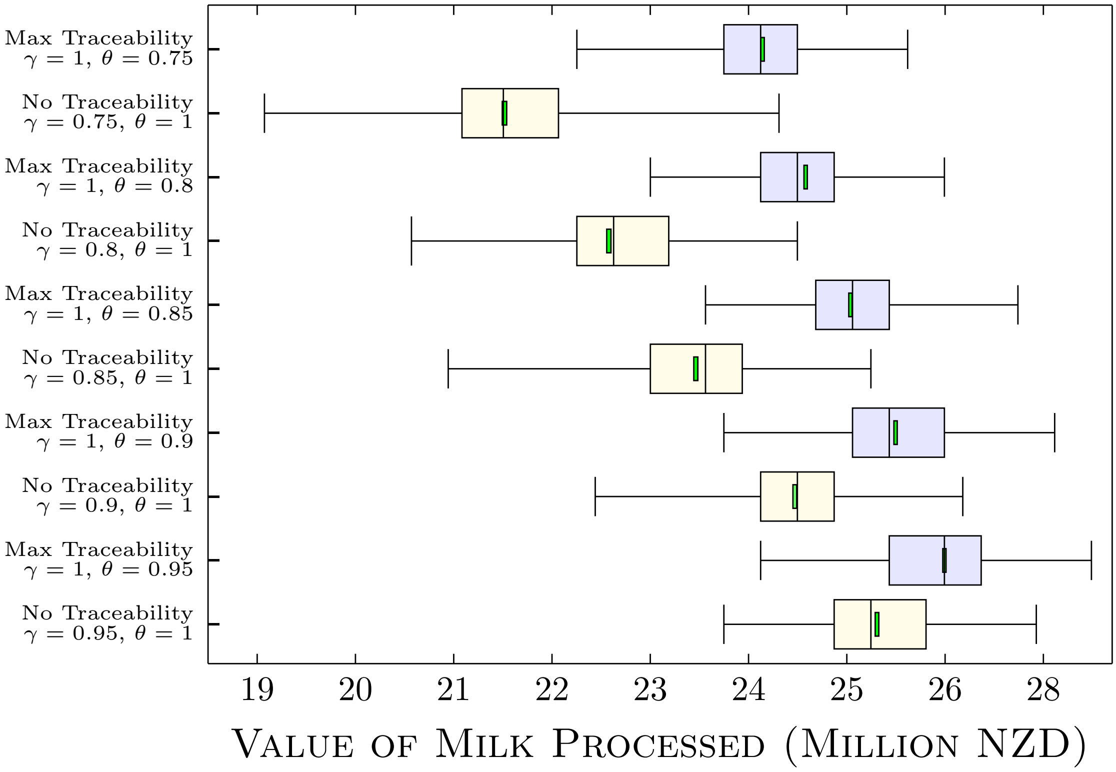

Figure 5.4: Box plots for 500 simulations of each scenario given in table 5.3. The 95% confidence interval for the mean is also shown as a green small box within each plot.

## 5.4 Reducing Product Loss Throughout the Supply Chain

As mention in the previous sections, the earlier a contamination is detected, the more it can be contained and losses limited. In this section, we allow that some contaminations may still take time to show up, and thus not be detected until the processing entry stage, but we also allow that other contamination will be detected earlier, and thus be able to be dealt with earlier. For simplicity we set the initial primary rejection rates $\beta$ and $\gamma$ equal to each other in the following simulations. We then investigate the impact that introducing traceability through secondary rejection can have, when applied at both the factory reception and processing stages.

Figure 5.5 shows a pair of simulations where the rejection rate of 25% is affecting both the factory reception and processing stages. One simulation without traceability and one simulation with traceability effects is shown, over a 24 hour production period. In the scenario without traceability $\beta = \gamma = 0.75$. In this particular set of simulations, including traceability increases total output by NZ \$4,960,000. Table 5.4 shows the potential value of traceability over the whole model for various, initial primary rejection rates $\beta$ and $\gamma$. In each case the value given is the average of 500 simulation results. The minimum primary acceptance rate required for traceability to have a positive effect (the minimum new $\beta = \gamma$) is given, along with the potential value of loss reduction if primary rejection is able to be eliminated altogether. Each

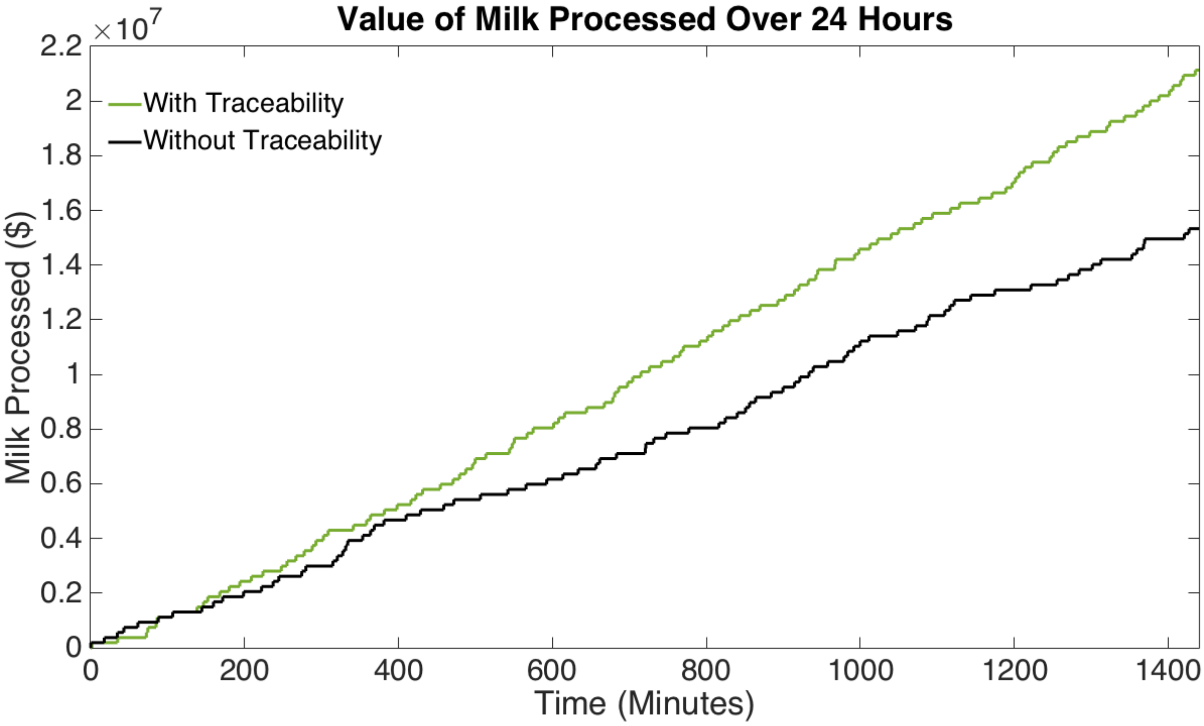

Figure 5.5: A simulation of milk produced over a 24 hour period, with a 25% rejection rate of product entering the factory reception and processing stages.

value shown in 5.4 is the total value of milk passing through all three stages and moving out to the next stage over a 24 hour period.

| Factory Reception and Processing Entry Implementation | | | | |
|---|---|---|---|---|
| No traceability | | With Traceability | | |
| | | | Potential Gain | |
| Initial $\beta$ and $\gamma$ | 24 Hour Output Value | Minimum New $\beta$ and $\gamma$ | 24 Hour Value | % Gain |
| 0.75 | \$15,969,426 | 0.88 | \$4,322,318 | 27.1 % |
| 0.8 | \$17,924,698 | 0.91 | \$3,603,116 | 20.1 % |
| 0.85 | \$19,980,202 | 0.94 | \$2,805,000 | 14% |
| 0.9 | \$22,052,910 | 0.96 | \$2,019,974 | 9.2% |
| 0.95 | \$24,299,154 | 0.98 | \$1,037,102 | 4.3% |
| 0.99 | \$26,091,175 | 1 | \$1,778,3705 | 0.7% |

Table 5.4: Average simulation results, over 500 runs, for rejection and traceability scenarios allowing rejection upon entry to both the factory reception stage and the processing stage.

While we see large improvements in production value with the introduction of traceability effects in cases where the rejection rate would have been high, such rejection rates are not typical of everyday dairy production. Milk tanker deliveries are generally accepted 99% of the time, while processing entry has a higher acceptance rate of 99.999%. The traceability system that is used needs to react to contamination scares and minimise their impact, while not influencing day to day production negatively. As seen in the simulation results above there is potential, for even day to day production to be improved through traceability.

We see also a larger improvement due to traceability when we apply it across multiple stages of the supply chain. As shown in table 5.4 the effect of traceability applied at both the factory reception and processing stages is greater than the sum of their effects individually. The potential for improvement still drops off quite steeply as the initial acceptance rate increases, though this is to be expected. Figure 5.6 shows box plots for each row in table 5.4. The larger potential improvement with traceability is noticeable even for the 95% acceptance rate scenarios.

# 6 Conclusion

We have extended the model developed in Welsh et al. (2016)[4] to include traceability effects. Using this model, we investigated the impact of traceability in several different scenarios. We have shown there is significant value to be gained when we allow increased secondary rejection via traceability, if this means we can reduce primary rejection rates. Separately, traceability has a larger impact when effects are implemented upon entry to the processing

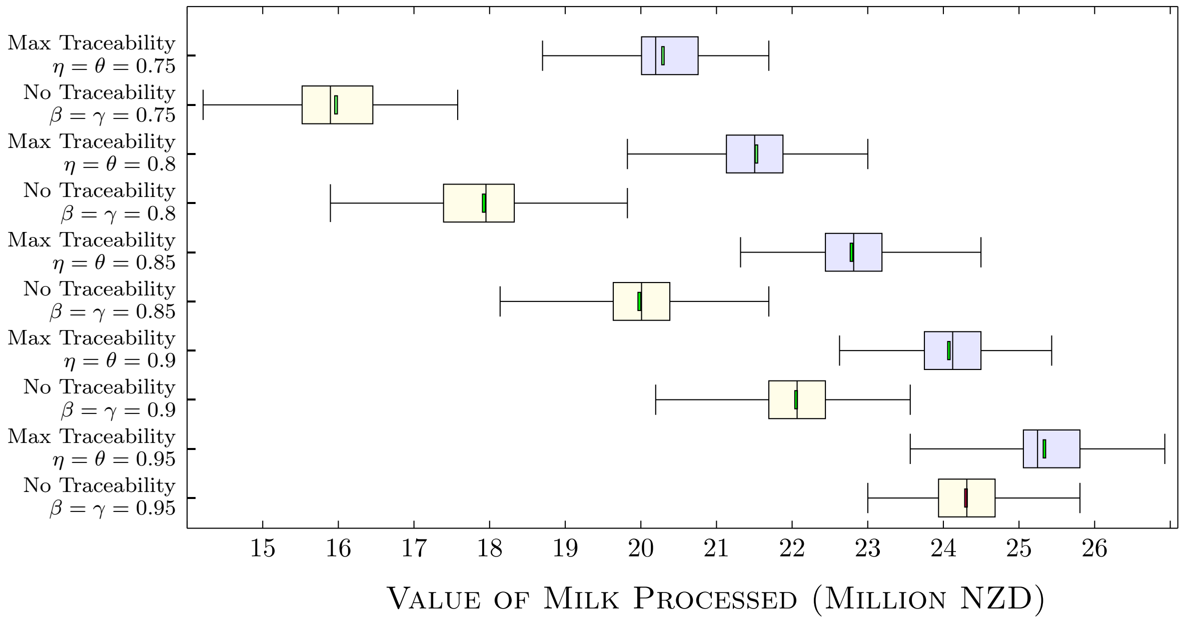

Figure 5.6: Box plots, produced after 500 simulations, for the rejection scenarios given in table 5.4. Both traceability and non-traceability scenarios are shown. The small green box within each plot shows the 95% confidence interval for the mean.

stage. The largest impact however is seen when traceability effects are applied across all the stages of the model.

The model we have developed is a useful tool for theoretically assessing the value of traceability in the early stages of the supply chain. It is still just a model however and limited by the information that was accessible during its development. We also model all of the tankers as one value pool, similarly the factory reception silos and processing sites, this may limit accuracy to some extent. This model also does not account for inter-site transfers that may happen between the factory reception and processing stages. We only model the first few stages in the dairy supply chain in this paper, the network of product flow becomes more complicated as we progress through the supply chain and more ingredients and products begin to interact. This model also does not include the costs of lost “goodwill” and reputation, focusing solely on the cost of product loss and reducing this. If we were to include reputation effects, the value of traceability may in fact be higher.

This paper fills a gap in traceability literature by using a stochastic model to investigate the value of traceability to a supply chain. There is still plenty of scope for future research into the effects of traceability throughout the supply chain, possibly extending the model to follow products to completions, retail and potentially all the way to the customer. Increasing the resolution of the model, to follow individual product locations more closely, i.e. modelling each tanker individually, is another direction future research could take. This model serves as a good starting point to further investigate traceability in dairy, extending this research in either direction.

# A Appendix

Here we provide some extra background information and data regarding milk production. Particularly seasonality in production and how we estimate the value we give to each litre of milk.

The average herd size has tripled over the last 30 seasons and is still increasing. For the 2014/2015 season, the average herd size was 419[29].

The average output, per herd in the 2014/2015 season was 1,775,501 litres, containing 157,885kg MS (Kilogram milk solids)[29]. Because milk price in New Zealand is measured in \$/kg MS, it is handy to have a conversion estimate. The amount of milk solids per litre of milk varies throughout the year. If we choose to cover only the main season, from August to April, when most farms are regularly producing milk, we should use the estimate that best applies that time period for conversion. The relevant values are given in Table A.1. This table also gives us values for the daily production per farm or per cow for each month of the year in the 2014/2015 season.

A total of 1,614,000,000 kg MS was collected by Fonterra in the 2014/2015 season ending in May 2015[40]. This equates to an average daily production 4,421,918kg MS per day. Though this is skewed by the fact that very little production is taking place for three months of the year. Using the information in table A.1 we can estimate daily production values for the main season when most farms are producing, as well as just the peak season. Given there were 11970 herds supplying Fonterra in the 2014/2015 season[29], the average daily production from August to April can be estimated as $11970 \times 639.49 = 8,301,075$kg MS, equivalent to 92,336,763 litres. The average daily production in just the peak season from August to October was $11970 \times 807.59 = 9,666,859$kg MS, the equivalent of 114,808,222 Litres of milk.

$$1\text{kg MS} \approx \frac{1775501}{157885} = 11.25\ l \tag{A.1}$$

$$1L \approx \frac{157885}{1775501} = 0.089 \text{ kg MS} \tag{A.2}$$

Table A.1 shows the average milk production per cow per day, and per herd per day, by month from June 2014 to May 2015. The average kg MS is also given for each month. Average production values are calculated for peak production season between August and October and the whole of the main production season from August to April. May to July is the off season, generally only farms with special winter milk contracts are producing during these months, which may make the data during this period less reliable[29].

|  | Production per day |  |  |  |  |  | kg MS |
|---|---|---|---|---|---|---|---|
|  | per cow |  |  |  | per herd |  | per litre |
| **Month** | **Litres** | **Milkfat kg** | **Protein kg** | **kg MS** | **Litres** | **kg MS** |  |
| **June** | 17.26 | 0.83 | 0.66 | 1.49 | 7231.94 | 624.31 | 0.0863 |
| **July** | 18.34 | 0.87 | 0.72 | 1.59 | 7684.46 | 666.21 | 0.0866 |
| **Aug** | 22.01 | 1.04 | 0.84 | 1.88 | 9222.19 | 787.72 | 0.0854 |
| **Sep** | 23.50 | 1.08 | 0.89 | 1.97 | 9846.50 | 825.43 | 0.0838 |
| **Oct** | 23.66 | 1.08 | 0.90 | 1.98 | 9913.54 | 829.62 | 0.0836 |
| **Nov** | 21.33 | 1.01 | 0.82 | 1.83 | 8937.27 | 766.77 | 0.0857 |
| **Dec** | 20.12 | 0.96 | 0.78 | 1.74 | 8430.28 | 729.06 | 0.0864 |
| **Jan** | 17.41 | 0.86 | 0.67 | 1.53 | 7294.79 | 641.07 | 0.0878 |
| **Feb** | 15.30 | 0.80 | 0.61 | 1.41 | 6410.70 | 590.79 | 0.0921 |
| **Mar** | 13.19 | 0.74 | 0.56 | 1.30 | 5526.61 | 554.70 | 0.1003 |
| **Apr** | 12.24 | 0.72 | 0.56 | 1.28 | 5128.56 | 536.32 | 0.1045 |
| **May** | 13.05 | 0.74 | 0.59 | 1.33 | 5467.95 | 557.27 | 0.1019 |
| **Peak (Aug - Oct)** |  |  |  |  |  |  |  |
| **Ave** | 23.05 | 1.06 | 0.87 | 1.94 | 9660.74 | 807.59 | 0.0842 |
| **Main Season (Aug - Apr)** |  |  |  |  |  |  |  |
| **Ave** | 18.75 | 0.92 | 0.73 | 1.65 | 7856.71 | 693.49 | 0.0899 |
| **Full Season (Aug - July)** |  |  |  |  |  |  |  |
| **Ave** | 18.11 | 0.89 | 0.71 | 1.61 | 7591.23 | 674.10 | 0.0903 |

Table A.1: Average daily milk production summary of the 2014/2015 season, based on data obtained from DairyNZ and LIC[29].

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