Reserve Bank of New Zealand Analytical Notes

Short-term risk premiums and policy rate expectations in the United States

AN2016/07

*Leo Krippner and Michael Callaghan*

September 2016

NON-TECHNICAL SUMMARY

In recent years, Federal Funds Futures rates in the United States have been persistently lower than the Federal Reserve's projections and analysts' surveyed expectations of the Federal Funds rate. Figure 1 shows a recent example.

We present a case for the difference based on risk premiums, the compensation that holders of securities demand for bearing risk, or return they are prepared to forego to avoid risk. In particular, it may be that market participants are at present willing to pay an insurance value to own high-quality interest rate securities (i.e. accept a negative risk premium) because such securities would outperform in the event of an unexpected economic downturn. Financial market factors, such as the expanded Federal Reserve balance sheet from quantitative easing, may also be contributing to negative risk premiums.

We estimate risk premiums from a term structure model and find they are of a sign and magnitude that would readily account for the differences mentioned in the first paragraph, and are plausible economically. Besides providing a rational reconciliation for the differences, a further implication from our negative risk premium estimates is that the expected path of the Federal Funds rate in the United States may currently be materially higher than might be inferred directly from the prevailing rates on the current series of Federal Funds Futures contracts.

*Figure 1: July 2016 FFF rates and analysts' expectations*
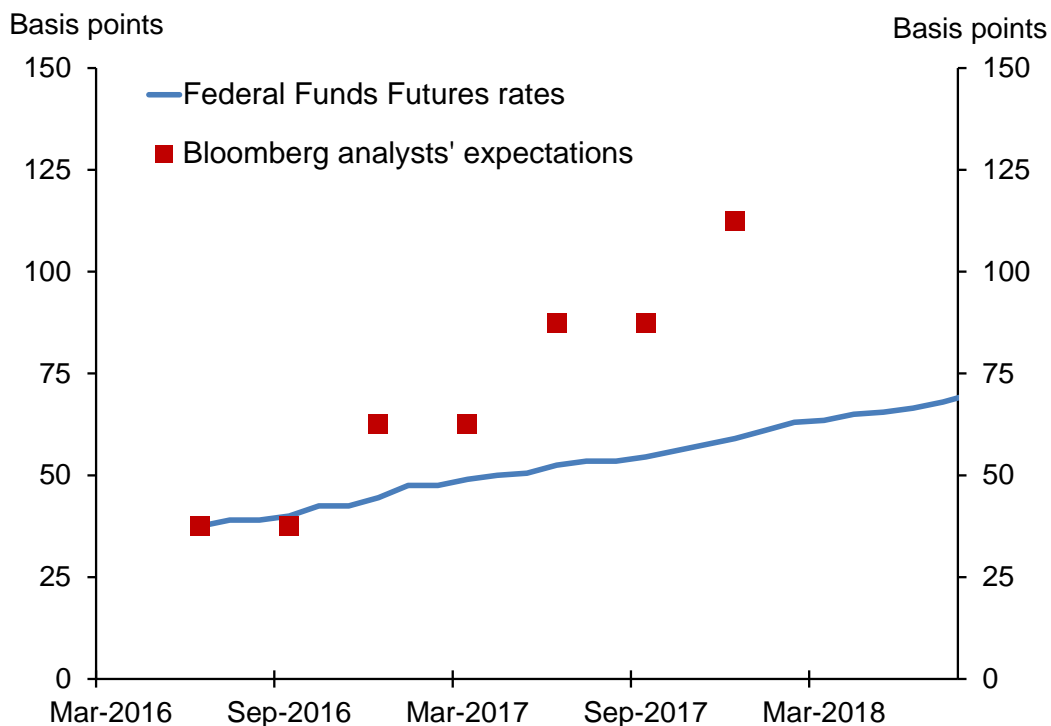
Source: Bloomberg.

1. INTRODUCTION

Federal Funds Futures (FFF) rates have been substantially lower than the Federal Reserve's dot plot projections for Federal Funds (FF) rates in recent years, and also lower than analysts' surveyed expectations. For example, in July 2016 the average expectation of analysts was for two 25bp interest rate increases by mid-2017. Taken verbatim as expectations of FF rates, FFF rates did not imply a 25bp rate increase until 2018.

One potential explanation is that the expectation of the marginal FFF investor might genuinely be lower than the average of survey respondents. For example, prevailing FFF rates may be allowing for low probability/high impact downside events, which the analyst group might be underweighting. In general, traders have a direct and powerful financial incentive to reflect any nuances that might be relevant for FF outcomes, while the incentives are not necessarily as strong for survey respondents.

Figure 2: Expected year-end 2017 Federal Funds Rate, July 2016 Survey
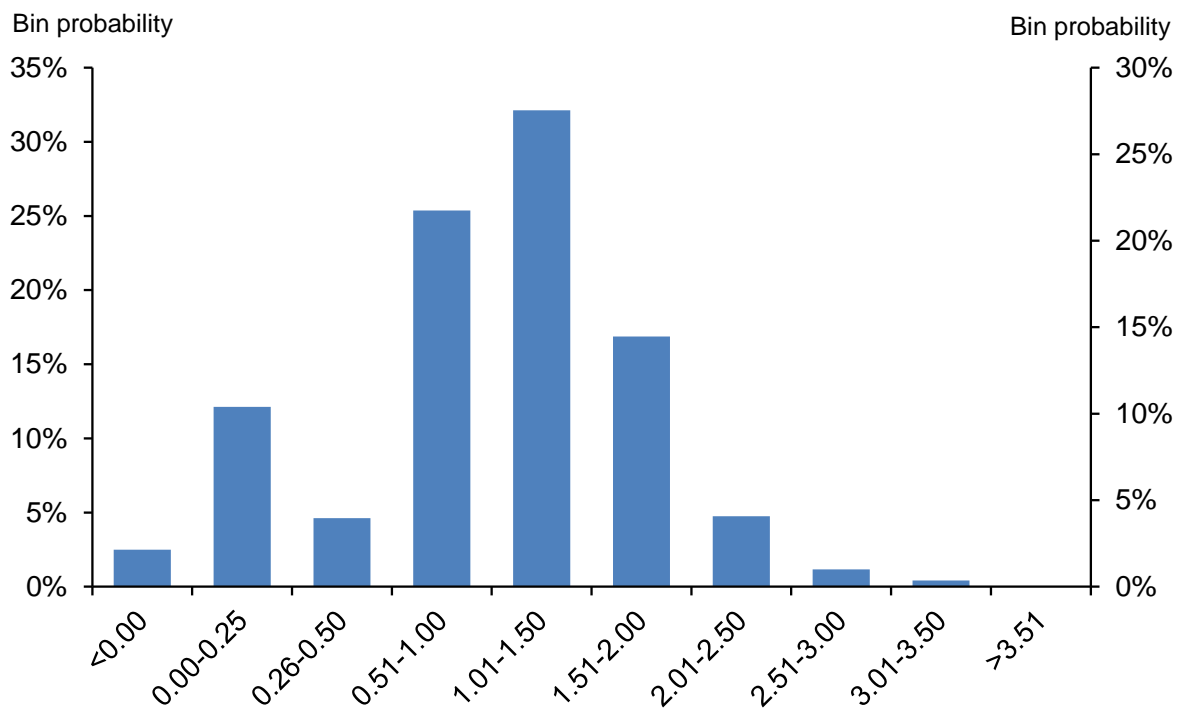
Sources: New York Fed Survey of Primary Dealers, New York Fed Survey of Market Participants, RBNZ staff calculations.

However, the same information set is always available to both groups and both can infer information from each other. Hence, one would not expect differences to persist over long periods of time. Furthermore, the distribution of the most recent analyst survey of expectations shown in figure 2 contains a distinct negative skew, indicating that analysts are also aware of downside tail risks, presumably including those that the market has factored in. Indeed, the surveyed expectations in figure 2 are from US primary dealers and market participants whose

institutions trade and invest in interest rate securities. So the question remains: why is the average surveyed expectation for the FF rate materially higher than the FFF rate trading in the market?

An alternative explanation is to recall that prices and interest rates in financial markets are not determined solely by expectations of future outcomes. Market prices and interest rates also need to compensate investors for the risk they are taking on with any investment. That compensation, known as the risk premium, can shift market prices materially away from those that would otherwise prevail if expectations of future outcomes were the only aspect considered by investors.

From that perspective, the expectations of investors and analysts for the future path of FF rates may actually be very similar, and the differences between FFF rates and average analyst expectations may reflect material and persistent negative risk premiums. In other words, FFF rates may be lower in the current environment because the marginal FFF investor with a long position is prepared to forgo some yield (pay an insurance value) in return for holding safer securities. We discuss this principle in section 3.[1] The marginal investor FFF with a short position takes on a higher risk and receives the insurance premium as compensation.

A negative risk premium in FFF rates would also be consistent with the same phenomenon already estimated for US government bond yields across the yield curve for recent years,[2] and for the same reasons. That is, for owning a safer asset, bond investors are prepared to accept a lower expected return than from a rolling investment in the expected policy rate or closely related short-maturity interest rate securities.

The models used to estimate bond risk premiums provide a means of checking if the negative risk premium explanation for recent FFF rates is plausible. Hence, in section 2, we use such a model and find that risk premium estimates are negative for the shorter horizons relevant to FFF rates, and that they are about the appropriate magnitude.

In section 3, we outline the economic intuition for our risk premium estimates, both recently and over history. Recent negative risk premiums are consistent with economic uncertainty and quantitative easing following the global financial crisis in 2008/09, while positive risk premiums before then are consistent with perceived risks around inflation.

We discuss the implications of negative risk premiums in section 4, making two main points. First, the material and changing risk premiums obtained over our sample suggests that routinely adjusting FFF rates for risk premiums would obtain a better estimate of FF rate expectations. Second, the expected path of the FF rate may currently be materially higher than that implied by simply using the prevailing rates on the current series of FFF rates.

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[1] Brodsky, Del Negro, Fiorica, LeSueur, Morse, and Rodrigues (2016a,b) also discuss this phenomenom with respect to FFF rates, and also use a term structure model to gauge the size of the risk premiums.

[2] Examples for the United States are the up-to-date estimates from the Adrian, Crump, and Moench (2013) and Kim and Wright (2005) models. The model we outline in section 2 also produces negative risk premium estimates for the United States, and for other major economies that we have investigated.

2. SHORT-HORIZON RISK PREMIUM ESTIMATES

To check the plausibility of a negative risk premium for FFF rates, we first require a quantitative gauge of risk premiums for short horizons. In section 2.1, we outline the model we use to obtain those estimates. That section can be skipped by readers who are more interested in the results, which we present and discuss in section 2.2.

2.1 Term structure model

The yield curve model on which our risk premium estimates are based is currently internal to the Reserve Bank. Full details will be released once the documentation is complete. However, as the following overview explains, the model and its estimation are based on standard approaches in the term structure literature for obtaining estimates of the expected policy and risk premium components in observed interest rate data.

The model specification for the nominal yield curve is the shadow/lower-bound term structure model specified in Krippner (2015). It uses three factors (Level, Slope, and Bow [curvature]) to describe the shadow term structure at any point in time, and the framework from Krippner (2011, 2015) to impose the lower bound. Christensen and Rudebusch (2015, 2016) have also applied the same model specification, and it accords closely with other shadow/lower-bound term structure models in the literature.

The state variables and parameters for the model are estimated using the iterated extended Kalman filter, as detailed in Krippner (2015), applied to a time series of yield curve data and survey data for expected interest rates of different maturities for a wide range of future horizons.[3] As discussed in Kim and Orphanides (2012), including survey data is important when trying to obtain precise and robust estimates of the expected policy and risk premium components of interest rates, and this is common practice in the literature; e.g. see Priebsch (2013). Long-horizon survey data on nominal output growth are also included to help inform expected policy rates for long horizons. This approach is used in term structure modelling at the European Central Bank.

The advantage of using a model is that it represents all of the yield curve data and survey data consistently over time and by time to maturity. Hence, any idiosyncratic elements in the individual elements of yield curve data and survey data will be appropriately downweighted by the model. It is worthwhile noting two points in that respect. First, the model naturally accords much more weight to the yield curve data than the survey data. This result suggests that market data have less idiosyncratic influences than survey data, which is as one might anticipate given the comment in section 1 about the incentives of traders versus analysts. Nevertheless, the non-zero weight accorded to the survey data shows that it does provide some useful systematic information. The second point, related to the first, is that our risk premium estimates at any point time for any time to maturity are based on the estimated systematic component of the survey data, not the survey data itself.

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[3] The yield curve data contains maturities from 3 months out to 30 years. The data are zero-coupon government bond rates from Bloomberg up to January 2006, and then zero-coupon overnight indexed swap rates thereafter (when the full range of data from 3 months to 30 years first became available on Bloomberg).

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2.2 Summary of risk premium results

Our model, like its counterparts in the associated literature, is designed primarily for explaining movements in longer-maturity interest rates with respect to expected policy and risk premium decompositions. Nevertheless, we can mechanically obtain the decomposition estimates for shorter-maturity yield curve data from the model, which are relevant for our discussion of FFF rates.

Figure 3 shows the risk premium estimates from our model for the 3-month rate, the 3-month rate three months forward, and the 3-month rate twelve months forward. The forward risk premium estimates are respectively calculated from the risk premiums for the 3-month and 6-month rates and the 12-month and 15-month rates.

The first point to note on our risk premium estimates is that, over recent years, they are of the correct sign and order of magnitude to account for FFF rates being lower than the Federal Reserve's projections and analysts' surveyed expectations of the Federal Funds rate. We will discuss the implications of this in section 4.2.

Figure 3: Risk premium estimates
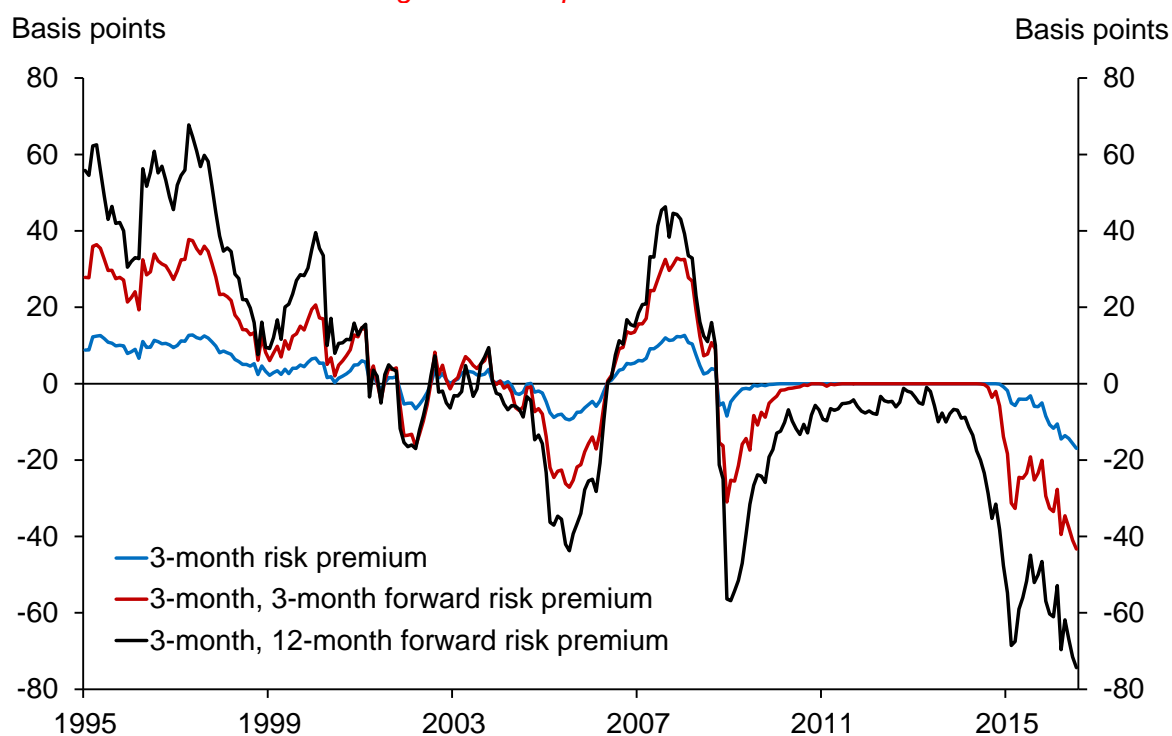
The second notable feature of our risk premium estimates is that they show marked variation over time. That is, the risk premiums are materially positive in the early part of the sample, they cycle around zero in the middle of the sample, and they become materially negative in the latter part of the sample. We discuss the risk premium variations in the following section.

3. ECONOMIC DISCUSSION OF RISK PREMIUMS

To improve our confidence about the risk premium estimates reported in the previous section, in this section we discuss their plausibility from an economic perspective. In section 3.1, we discuss the principles underlying risk premiums in economics and finance, and in section 3.2 we relate those principles to bond investments within the environments that prevailed over the evolution of our risk premium estimates. Section 3.3 discusses how bond risk premiums relate to risk premiums within FFF rates.

3.1 Risk premiums in principle

Financial market prices always include a risk premium component in addition to the expected return component. As a general principle, this is because investors are, on average, risk-averse; for a given expected return, they prefer less rather than more variability in the range of potential returns.

More precisely, according to economic and finance theory (e.g. see Cochrane 2001), the risk of an investment is related to how the unanticipated variability in its returns co-vary with the utility that economic agents get from additional consumption in different economic states. In particular, assets that perform well in an economic downturn will be more desirable as part of a diversified portfolio, because they would increase in value at just the time when additional wealth is useful to smooth the consumption of economic agents. Such assets will perform poorly in an economic upturn, but economic agents will already have higher income at those times. Those hedging properties would lead the marginal investor to pay a higher price (accept a lower yield) for such assets compared to the price if only expected returns were taken into consideration.

Conversely, assets that perform poorly in an economic downturn will be less desirable as part of a diversified portfolio, because they would decrease in value and therefore work against economic agents trying to smooth their consumption. Such assets will perform well in an economic upturn, but economic agents do not require the additional wealth at those times. The marginal investor will therefore pay a lower price (demand a higher yield) for such assets compared to the price based on expected returns alone.

The adjustment in the market price of a security due to risk considerations is a risk premium, which can be positive or negative. Hence, even with a given expectation from a known range of possible outcomes, risk aversion and the associated risk premium means that probabilities inferred directly from the market price will differ from the actual probabilities. The appendix offers a simple numerical example to illustrate this idea.

3.2 Discussion of our risk premium estimates

Our model obtains positive risk premium estimates up to around the year 2000, which is consistent with a heightened focus on inflation up to that time. That is, markets could reasonably anticipate that upside surprises to inflation would prompt a sharper response from the Federal Reserve, in the form of higher policy interest rates, than downside surprises.

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Hence, economic activity would slow at the same time that bonds provided unanticipated negative returns (i.e. prices would decline as bond yields rose in response to higher policy rates). That positive covariance is an "anti-hedge" for consumption smoothing, so investors would demand compensation in the form of higher yields, i.e. a positive risk premium.

A similar covariance in the context of positive risk premiums obtained over a longer historical period is discussed in Chen, Engstrom, and Grishchenko (2016). That is, 'supply-side' shocks (e.g. oil price spikes) were prevalent in the 1970s and early 1980s, and these simultaneously lowered real economic activity, raised inflation, and lowered bond prices.

Our model obtains negative risk premium estimates during and following the Global Financial Crisis (GFC). This is consistent with unanticipated returns on bonds now having a positive covariance with economic downturns. That is, markets can reasonably anticipate in the prevailing economic environment that downside surprises to economic activity would prompt a sharper response from the Federal Reserve than upside surprises. That could take the form of lower policy interest rates or unconventional easing actions, both of which would produce lower bond yields and hence unanticipated positive returns on bond prices. The negative covariance with economic activity is consistent with a negative risk premium.

Chen, Engstrom, and Grishchenko (2016) discusses the same covariance in the context of negative risk premiums that they obtain over recent history. That is, compared to earlier periods, 'demand-side' shocks have been more prevalent before and after the GFC, and negative shocks lower real economic activity, inflation, and increase bond prices.[4]

3.3 Risk premiums in FFF rates

The discussion in section 3.2 is, for clarity, in the context of bond prices and yields. FFF contracts are actually derivatives written on the FF rate, but they can essentially be viewed as the sale of a bond by the "short" counterparty and a purchase of that bond by the "long" counterparty. More precisely, FFF contracts are equivalent to a notional sale of a short-maturity bond to settle at a future date, like the forward horizons for 3-month tenures that we used in section 2.2 (although FFF contracts actually have monthly tenures). The forward rates are themselves calculated from bond rates, so the discussion of bond risk premiums from an economic perspective in section 3.2 also applies to a long position in FFF contracts.

With a negative risk premium, the long counterparty pays an insurance premium by accepting a lower yield, and the short counterparty receives that premium. In the earlier period, a short FFF position hedged against inflation risk, and so those with short positions paid an insurance premium to those with long positions.

It is also worth noting that the risk premium estimates from our model are highly applicable to FFF rates since before the GFC. That is because, as mentioned in section 2.1, we use

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[4] From that perspective, it is interesting to note that our first instance of negative risk premiums is around the deflation scare in the early to mid-2000s, and the risk premium turned positive once that scare passed and the Federal Reserve started raising the FF rate from the then historic low of 1 percent. However, during that period, risk premiums are thought to have been influenced by a range of factors (e.g. global savings, contributing to the so-called "bond conundrum" of bond yields not rising as the policy rate increased). Hence, we believe it's best to focus on the broad evolution from positive to negative risk premiums over the sample, which is all we require for our discussion, rather than specific events.

overnight indexed swap (OIS) data in our term structure model from January 2006. OIS contracts are settled on FF rates just like the FFF contracts. Prior to January 2006, as a proxy for OIS data, we have used US government bond data. These are essentially default-free and are therefore, like FFF rates and OIS rates, largely determined by policy rate expectations and risk premiums.

Indeed, FFF rates, OIS rates, and US government bond rates are typically closely connected. Hence, the Federal Reserve's purchases and ongoing holdings of US government bonds since the GFC is another potential contribution to the negative risk premiums we have estimated, even though we have not used government bond data in our sample during that time.

4. RECENT NEGATIVE RISK PREMIUM AND IMPLICATIONS

In this section, we discuss the implications of risk premiums in FFF rates. In section 4.1 we discuss the routine monitoring of FF rate expectations using FFF rates, and in section 4.2 we return to discussing the current situation from the perspective of our negative risk premium explanation.

4.1 Implications for monitoring FF rate expectations

If only expected returns were important to investors, FFF rates would be an unbiased predictor of the future path of the FF rate. However, the presence of material and time-varying risk premiums in FFF rates means that FFF rates should not necessarily be taken verbatim as the pure expectation of the future FF rate. Specifically, a negative risk premium would mean that the FFF rate would understate the FF rate actually expected by the market, and a positive risk premium would mean that FFF rates would overstate actual expectations.

Adjusting the FFF rates for risk premiums at each point in time should provide improved ex-ante estimates of the likely path of the FF rate at each point in time. We can readily undertake a preliminary test of this proposition by comparing the ex-post performance of unadjusted and adjusted FFF rates against realised FF rates over our sample. Our analysis shows that FFF rates adjusted for risk premiums have, on average, been better predictors of the FF rate than the FFF rates over the whole sample period. These results are consistent with those of Piazzesi and Swanson (2008), who show that over the period 1988 to 2006 (where our risk premium is positive on average), FFF rates have on average overestimated future realised FF rates.

However, we stress the 'on average' and 'ex-ante' caveats in the previous paragraph. The ex-post performance of FFF rates adjusted for risk premiums at any point in time depends not only on the accuracy of the ex-ante prediction, but also any new information realised subsequent to that prediction. In particular, over the post-GFC period so far, we found that unadjusted FFF rates have been more accurate predictors of the realised FF rate than their risk-adjusted counterparts. This is not surprising, because unanticipated negative shocks have resulted in the FF rate staying at zero for much longer than the Federal Reserve and markets initially expected.

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Nevertheless, in general, our results suggest that it is best to adjust FFF rates with estimated risk premiums. Given that risk premium estimates include the systematic information from analyst surveys, as discussed in section 2.1, this suggests that it is useful to use both FFF rates and analyst surveys when assessing FF rate expectations. FFF rates have the advantage of being timely, but they are obtained from financially-traded securities and so we need to consider the role risk premiums may play before using them to infer 'pure' expectations. Analysts' expectations, reported by Bloomberg, Thomson Reuters or Consensus Forecasts, have the advantage of not including a risk premium, but they are imperfect samples of the 'true' expectation for the path of interest rates that the market holds, and they are only available periodically.

One approach to combining the information would be to use the analyst survey data when they become available along with the FFF rates at the time to infer the risk premium component. The risk premium component could then be used with the subsequent FFF rates to provide a timely update with less bias than if the FFF rates were used alone. The risk premium estimate could then be updated when the analyst survey data next become available. This approach would involve some modelling and/or assumptions on how best to use the data and we are investigating the feasibility of such a project.

Finally, the risk premium itself provides useful information. That is, the sign indicates the direction of risks that the market is most concerned about (i.e. most willing to insure against), and the magnitude of risk premium indicates the depth of that concern.

4.2 FF rate expectations at present

Our estimates of the risk premiums for FFF rates are currently negative. Hence, actual expectations of the future FF rate path are likely to be higher than implied by FFF rates.

As an order of magnitude, we use the point estimates of our risk premium from the end of our sample, i.e. mid-July 2016, contained in figure 3. Our risk premium for the 3-month rate suggests that the average FF rate expected over the next three months (i.e. mid-July to mid-October) is around 15 basis points higher than the average of the FFF rates over that period. For the following three months (mid-October to mid-January 2017), actual expectations are around 45 basis points higher than the average of the FFF rates over that period (as suggested by our risk premium for the 3-month rate three months forward).

In the three months from a year ahead (i.e. from mid-July to mid-October 2017), our risk premium for the 3-month rate twelve months forward indicates an average FF rate expectation 75 basis points higher than implied by FFF rates over the same period.

Figure 3 shows that our monthly point estimates of the risk premiums are subject to some noise, and we have not shown the estimation uncertainty around the estimates. Hence, the numbers we have mentioned above should not necessarily be taken at face value. Nevertheless, the numbers clearly illustrate that the risk premium can readily account for a path of FF expectations that is materially higher than implied directly from FFF rates.

So, what are the implications if actual expectations of future FF rates are above FFF rates, or

equivalently, if future FF rates are likely to end up higher on average than currently implied by FFF rates? In principle, the implications should not necessarily be large, but it depends on how well the risk premium explanation applies and how aware markets are of its effect on FFF rates. Those with long positions in FFF contracts would lose money on higher realizations of the FF rate, but that represents the cost of insuring against downside risks in the economy and not having such events realized. An analogy, for example, is that a homeowner with fire insurance is not at all upset when no payout occurs in the event of no house fire.

In practice, some of the long positions in FFF contracts are certainly due to speculation on the FFF rate remaining low, and many market participants use FFF rates as a proxy for actual expectations. For these reasons, there is likely to be some volatility if/when FF rates change or are expected to change in upcoming Federal Reserve meetings.

While it is not the focus of this note, it is also worthwhile mentioning the implications of an unwind of negative risk premiums in FFF rates and fixed interest markets in general, which could result from several catalysts; e.g. an improving economic outlook, increasing inflation expectations, any normalization of the Federal Reserve's expanded balance sheet, or fiscal stimulus funded by government bond issuance. Once again, the resulting adjustment in fixed interest markets and other assets could be benign in principle, but the practical effect would probably be periods of heightened volatility in financial markets.

5. CONCLUSION

US Federal Funds Futures rates have been persistently lower than the Federal Reserve's projections and analysts' expectations of the Federal Funds rate in recent years. In this paper, we have presented evidence suggesting that the difference could reflect negative risk premiums. Negative risk premiums would be consistent with market participants being more worried about downside risks to inflation and the economy, and being willing to pay an insurance value (give up yield) to hold interest rate securities that would perform well in an economic downturn. Financial market factors such as quantitative easing could also be contributing to negative risk premiums.

One implication of our negative risk premium explanation is that the expected path of the Federal Funds rate may be materially higher than that implied from the prevailing rates on the current series of Federal Funds Futures contracts.

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APPENDIX

This numerical example illustrates how the market price of a security can underpredict the true probability of an event occurring, if it provides hedging/insurance properties for investors' portfolios.

Assume that we have a ten-sided dice that has nine sides of a 'good' state occurring, and one side of a 'bad' state. We also have a security that respective pays $10 or $100 depending on the outcome. The expected value of the security is therefore calculated as in the following table.

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Security	'Good' state	'Bad' state
Probability of occurrence	0.9	0.1
Security payoff	$10	$100
Expected value of security	0.9 x 10 + 0.1 x 100 = $19	

Now assume that investors have a portfolio that returns $100 in a 'good' state and $-100 in a 'bad' state. Investors can choose to leave their portfolio unhedged, or to hedge their portfolio by purchasing the security at a price of $19. This represents the world where investors are risk neutral (i.e. indifferent to the volatility of returns) because the following tables show that the unhedged and hedged portfolios have the same expected return of $80.

Unhedged portfolio	'Good' state	'Bad' state
Portfolio payoff	$100	$-100
Expected return	0.9 x 100 + 0.1 x -100 = $80	

Hedged portfolio	'Good' state	'Bad' state
Portfolio payoff	100	-100
Security payoff	10	100
Cost of security	-19	-19
Total payoff	$91	$-19
Expected return	0.9 x 91 + 0.1 x -19 = $80	

However, on average, investors are risk averse. Therefore most investors would prefer the hedged portfolio. Specifically, its returns are more desirable because the downside risk is greatly reduced, at a net cost of $9 = $-19 + $10 in the 'good state'. As a result, there would be a greater demand for the security, due to its hedging/insurance properties within the portfolio. This demand means that the price of the security would be bid up in the market.

For the purposes of our example, let's assume that the marginal investor is willing to pay a market price at $30.[5] The following table summarizes the new hedged portfolio.

Hedged portfolio	'Good' state	'Bad' state
Portfolio payoff	100	-100
Security payoff	10	100
Cost of security	-30	-30
Total payoff	$80	$-30
Hedged expected return	0.9 x 80 + 0.1 x -30 = $69	

The assumed price of $30 represents the risk-adjusted world where the marginal investor, and hence the market, is indifferent between the hedged and the unhedged portfolio. The expected return on the hedged portfolio is lower by $11, but the marginal investor is willing to sacrifice

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[5] Any number greater than $19 would serve for the illustration. The value would depend on the degree of risk-aversion for economic agents, but it is assured to be greater than $19 in any case because the marginal investor is risk-averse.

that expected return for the associated lower volatility in returns.

Based on the security's market price of $30 and its known payoffs in the 'good' and 'bad' states, the following table summarizes the estimation of the probability **p** of the 'good' state occurring.

	'Good' state	'Bad' state
Market-implied probability	**p**	**1-p**
Security payoff	$10	$100
Value of security	**p** x 10 + **(1-p)** x 100 = $30	

Calculating out the last line gives an implied market-pricing probability of **p = 0.78**. However, we specified at the start that the actual probability of the 'good' state occurring is 0.9, and all investors are aware and accept the probability on the dice. That is, we would expect the 'good' state to occur nine times out of ten, but the probability implied from the market price of the security underestimates that with a value of around eight times.

The reason that the market-implied probability of a 'good' state understates the true probability is that, in our specific example, market participants are willing to pay a premium for the security for its insurance properties. In other words, for the benefit of lower volatility in returns, investors sacrifice expected returns. They therefore accept a lower yield, which is a negative risk premium.

The general message from our example is that, because all securities in markets trade at risk-adjusted prices/yields, the probabilities inferred directly from them will provide a biased estimate of actual probabilities. An adjustment for the risk premium embedded in the prices/yields of market securities is required to provide an unbiased estimate.